Math 408 • Section 3963xD • Mathematical Statistics
Quiz # 1 Solution, January 22, 2013 Teaching Assistant: Greg Sokolov

This was a 20 minute quiz worth 10 points.

A student comes to a multiple-choice test. The test contains 10 questions, each with 4 options and only one correct answer. The student answers each question at random, independently of each other.

1.) (5 points) Are the events

$$A = \{\text{the student got full score}\} \quad \text{and} \quad B = \{\text{the student made at most one mistake}\}$$

independent?

$$A = \{\text{the student made no mistakes}\},$$
$$B = A \cup \{\text{the student made exactly one mistake}\}.$$

Note that $B \cap A = A$, and $0 < P(A) < 1,\ 0 < P(B) < 1$. That means

$$P(B \cap A) = P(A) \neq P(B) \cdot P(A),$$

so A and B are not independent.

2.) (5 points) What is the conditional probability that the student has got the first problem right if he has made exactly one mistake?

Neat solution.

$$P\{\text{first right}|\text{one mistake}\} = 1 - P\{\text{first wrong}|\text{one mistake}\} = 1 - \frac{1}{10} = \frac{9}{10}.$$

Long solution.

$$\begin{aligned} P\{\text{one mistake}\} &= P\{\text{mistake in } 1^{\text{st}}\} + \cdots + P\{\text{mistake in } 10^{\text{th}}\} \\ &= \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^9 + \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^8 + \cdots + \left(\frac{1}{4}\right)^9\left(\frac{3}{4}\right) = 10 \cdot \frac{3}{4^{10}}, \\ P(\{\text{one mistake}\} \cap \{\text{first right}\}) &= P\{\text{one mistake } \textit{and} \text{ first right}\} \\ &= P\{\text{mistake in } 2^{\text{nd}}\} + P\{\text{mistake in } 3^{\text{rd}}\} + \cdots + P\{\text{mistake in } 10^{\text{th}}\} \\ &= \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^8 + \cdots + \left(\frac{1}{4}\right)^9\left(\frac{3}{4}\right) = 9 \cdot \frac{3}{4^{10}}. \end{aligned}$$

Finally,

$$P\{\text{first right}|\text{one mistake}\} = \frac{P(\{\text{one mistake}\} \cap \{\text{first right}\})}{P\{\text{one mistake}\}} = \frac{9}{10}.$$