Math 408 - Mathematical Statistics

Lecture 27+. The Bootstrap Method: Simulation Results

April 8, 2013

Example: Gaussian Model

Suppose that:

- $X_1, \ldots, X_n \sim \mathcal{N}(\mu, \sigma^2)$, true values: $\mu = 1$ and $\sigma = 2$
- Exact Confidence Intervals:

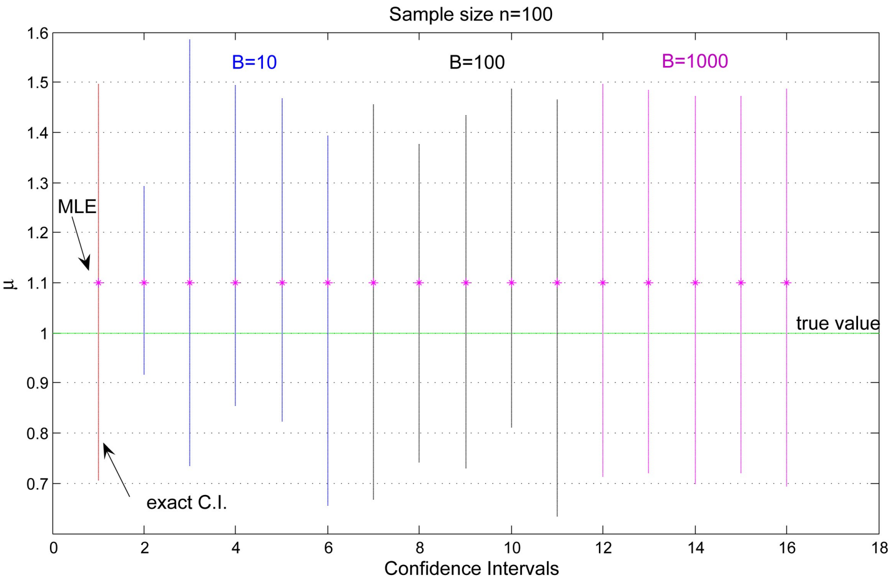
$$\mu : \quad \hat{\mu}_{\mathrm{MLE}} \pm \frac{1}{\sqrt{n-1}} \hat{\sigma}_{\mathrm{MLE}} t_{n-1}(\alpha/2) \qquad \sigma^2 : \quad \left(\frac{n\hat{\sigma}^2_{\mathrm{MLE}}}{\chi^2_{n-1}(\frac{\alpha}{2})}, \frac{n\hat{\sigma}^2_{\mathrm{MLE}}}{\chi^2_{n-1}(1-\frac{\alpha}{2})} \right)$$

```
%---- Data:
mu0=1;                      % true mean
sigma0=2;                   % true sigma
n=100;                      % sample size;
X=mu0+sigma0*randn(1,n); % data
%---- MLEs:
mu_mle=mean(X);
sigma_mle=std(X,1);
%---- Level of Confidence:
alpha=0.05;                 % 100(1-alpha) CI
%---- Exact Confidence Intervals:
CImu_exact=[mu_mle-sigma_mle*tinv(1-alpha/2,n-1)/sqrt(n-1),
mu_mle+sigma_mle*tinv(1-alpha/2,n-1)/sqrt(n-1)];
CIsigma_exact=[sqrt(n*sigma_mle^2/chi2inv(1-alpha/2,n-1)),
sqrt(n*sigma_mle^2/chi2inv(alpha/2,n-1))];
%[phat,pci] = mle(X);
```

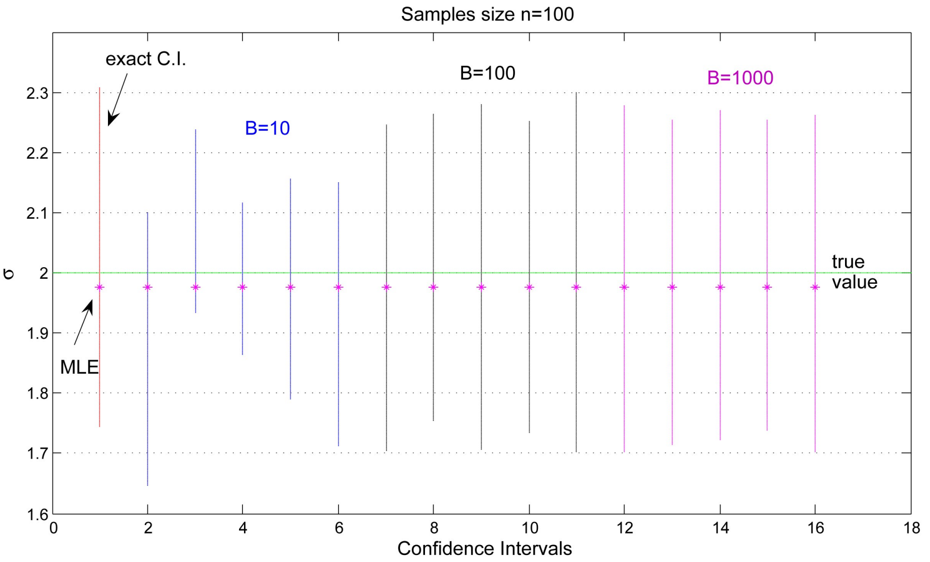
Bootstrap

```
%---- Bootstrap Confidence Intervals:
B=10;     % number of the bootstrap samples
for i=1:B
    Z(i,:)=mu_mle+sigma_mle*randn(1,n); % "bootstrap data"
    mu_b(i)=mean(Z(i,:));                % MLE from b-data
    sigma_b(i)=std(Z(i,:),1);            % MLE from b-data
    Delta_mu(i)=mu_b(i)-mu_mle;
    Delta_sigma(i)=sigma_b(i)-sigma_mle;
end
CImu_bootstrap=[mu_mle-quantile(Delta_mu,1-alpha/2),
    mu_mle-quantile(Delta_mu,alpha/2)];
CIsigma_bootstrap=[sigma_mle-quantile(Delta_sigma,1-alpha/2),
    sigma_mle-quantile(Delta_sigma,alpha/2)];
```

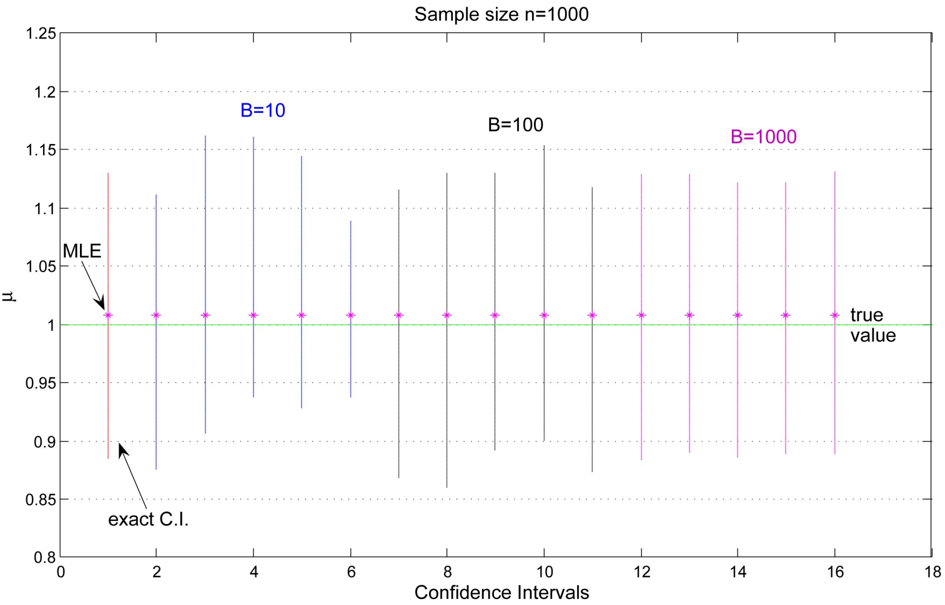
Confidence Intervals for μ, $n = 100$



Confidence Intervals for σ, $n = 100$



Confidence Intervals for μ, $n = 1000$



Confidence Intervals for σ, $n = 1000$