MA 157C: TOPICS IN GEOMETRY AND TOPOLOGY

SPRING 2025

Instructor: Seung-Yeon Ryoo (she/her, sryoo@caltech.edu)
Lectures: Monday, Wednesday, and Friday, 1:00 PM – 1:55 PM, Linde 387
Office hours: Thursday 3:00 PM – 5:00 PM, Linde 258

Course Description: An introduction to Lie groups and their representations.

Prerequisites: Ma 151 or equivalent, or instructor's permission.

Main textbook: *Lie groups, Lie algebras, and representations* by Brian C. Hall, Springer, 2003.

Supplementary textbook: *Representation theory: a first course* by William Fulton and Joe Harris, Springer, 2004.

Grading scheme: Problem sets (100%).

Tentative Course Schedule:

- Week 1 (3/31 - 4/4): Introductions and motivation. The classical groups, matrix Lie groups, Lie groups, Lie subgroups (Hall Chapters 1 and 2)
- Week 2 (4/7 - 4/11): Lie algebras, vector fields, exponential map, Baker–Campbell–Hausdorff formula, the Lie theorems (Hall Chapters 3 and 5)
  Problem Set 1 due on 4/11.
- Week 3 (4/14 - 4/18): Representations of Lie Groups and Lie algebras. The Schur lemma and Haar measure. Representations of $\mathfrak{sl}(2,\mathbb{C})$ and the spherical Laplace operator (Hall Chapter 4)
- Week 4 (4/21 - 4/25): Structure theory of Lie algebras: ideals of Lie algebras, solvable/nilpotent Lie algebras, Lie and Engel theorems, Bilinear and Killing forms, semisimple Lie algebras.
  Problem Set 2 due on 4/25.
- Week 5 (4/28 - 5/2): Complex Semisimple Lie algebras. Root systems, positive and negative roots. Dynkin diagrams. (Hall Chapters 6 and 7)
- Week 6 (5/5 - 5/9): Representations of semisimple Lie algebras. Verma modules, universal enveloping theorem and the PBW theorem. (Hall Chapters 8 and 9)
  Problem Set 3 due on 5/9.
- Week 7 (5/12 - 5/16): Further topics on representations. Weyl formulas, Kostant multiplicity formula. (Hall Chapter 10)
  5/12: Last day for seniors to remove conditions and incompletes
- Week 8 (5/19 - 5/23): Compact Lie groups and maximal tori (Hall Chapter 11)
  Problem Set 4 due on 5/23.
  5/21: Last day for dropping courses, exercising pass/fail option, and changing sections
- Week 9 (5/26 - 5/30): The compact group approach to representation theory, fundamental groups of compact Lie groups (Hall Chapter 12, 13)
  5/26: Memorial day (institute holiday)
  5/30: Last day of classes for seniors and graduate students

(Last updated March 30, 2025)