MA 157B: RIEMANNIAN GEOMETRY

SPRING 2024

Instructor: Seung-Yeon Ryoo (she/her, sryoo@caltech.edu)
Lectures: Monday, Wednesday, and Friday, 1:00 PM – 1:55 PM, Linde 387
Office hours: Monday 2:00 PM – 4:00 PM, Linde 258

Prerequisites: Ma 151 or equivalent, or instructor's permission.

Supplementary textbooks:

- *Sobolev Spaces on Metric Measure Spaces* by Juha Heinonen, Pekka Koskela, Nageswari Shanmugalingam, and Jeremy Tyson, Cambridge University Press, 2015.
- *A course in metric geometry* by Dmitri Burago, Yuri Burago, and Sergei Ivanov, American Mathematical Society, 2001.
- *Lectures on Nonsmooth Differential Geometry* by Nicola Gigli and Enrico Pasqualetto, Springer Nature, 2020.
- *Control of nonholonomic systems: from sub-Riemannian geometry to motion planning* by Frédéric Jean, Springer, 2014.
- *Gradient flows in metric spaces and in the space of probability measures* by Luigi Ambrosio, Nicola Gigli, and Giuseppe Savaré, Springer Science and Business Media, 2005.
- *Degeneration of Riemannian metrics under Ricci curvature bounds* by Jeff Cheeger, Edizioni della Normale Pisa, 2001.
- *Lectures on Differential Geometry* by Richard Schoen and Shing-Tung Yau, International Press, 2010.

References:

- Colding, T. and Minicozzi, W. Harmonic functions on manifolds. *Ann. of Math.*, 146(3):725-747, 1997.
- Kleiner, B. A new proof of Gromov's theorem on groups of polynomial growth. *J. Amer. Math. Soc.*, 23(3):815-829, 2010.
- Cheeger, J. Differentiability of Lipschitz functions on metric measure spaces. *Geom. Funct. Anal.* 9:428-517, 1999.
- Klainerman, S. and Rodnianski, I. A geometric approach to the Littlewood–Paley theory. *Geom. Funct. Anal.* 16:126-163, 2006.
- Pansu, P. Métriques de Carnot–Carathéodory et quasiisométries des espaces symétriques de rang un. *Ann. of Math.*, 1-60, 1989.
- Semmes, S. Bilipschitz mappings and strong A_∞ weights. *Ann. Fenn. Math.*, 18(2):203-210, 1993.

Grading scheme: Presentations (100%). I will not keep track of class attendance.

Tentative Course Schedule:

- Week 1 (4/1 - 4/5): Gromov–Hausdorff distance and convergence. Asymptotic cones. Nilpotent groups and Carnot groups.
- Week 2 (4/8 - 4/12): Subriemannian geometries. Pansu–Semmes nonembeddability theory. Basic extension and embedding theorems.
- Week 3 (4/15 - 4/19): Rectifiable curves in metric spaces. Modulus of a family of curves. Upper gradients.
- Week 4 (4/22 - 4/26): Sobolev spaces and Poincaré inequalities.

- Week 5 (4/29 - 5/3): Cheeger's differentiation theory and application to embedding theory. Finite dimensionality of harmonic functions and Gromov's theorem on groups of polynomial growth.
- Week 6 (5/6 - 5/10): Optimal transport on metric spaces and RCD spaces.
- Week 7 (5/13 - 5/17): Heat kernel estimates and geometric Littlewood–Paley theory.
- Week 8 (5/20 - 5/24): Student presentations.
- Week 9 (5/27 - 5/31): Student presentations.
  Memorial day 5/27.

(Last updated April 17, 2024)