SYLLABUS

MA 109C: Introduction to Geometry and Topology

Spring 2026

Instructor: Seung-Yeon Ryoo (she/her, sryoo@caltech.edu)
Lectures: Monday, Wednesday, and Friday, 10:00 AM - 10:55 AM, Linde 387
Office hours: Thursday 3:00 PM - 4:00 PM, Linde 258

Teaching Assistant: Shixiong Xu (sxu@caltech.edu)
Office hours: TBA, Location TBA

Official Course Description: An introduction to differentiable manifolds. Transversality, differential forms, and further related topics.

Prerequisites: Ma 2 or equivalent. Ma 5a and Ma 108a recommended.

Main textbook: "Differential Topology" by Victor Guillemin and Alan Pollack, American Mathematical Society, 2010.

Supplementary textbooks: The following will be referred to throughout the course.

- "Topology from the Differentiable Viewpoint" by John Milnor, Princeton University Press, 1997.
    - A 'crash course' on differential topology. It is a rather thin book, so I recommend you have a quick look through it before or at the start of the semester. I might talk about Chapter 7 (Framed cobordism and the Pontryagin construction) at the end of the course if time permits.
- "A Comprehensive Introduction to Differential Geometry, Volume 1" by Michael Spivak, Publish or Perish, 1999.
    - This book gives a nice introduction to smooth manifold theory. This volume, together with the other four volumes, indeed gives a comprehensive introduction to differential geometry (as advertised in its title), so I would suggest this collection to aspiring geometers.
- "Differential Topology" by Morris Hirsch, Springer, 1976.
    - A graduate-level textbook on differential topology. While you read Guillemin-Pollack, if you feel you can generalize some result or think some more abstract theory is hiding behind the curtains, have a look at Hirsch.
- "Lectures on the h-Cobordism Theorem" by John Milnor, Princeton University Press, 1965.

    - If time permits, I will start to cover this book. This is research-level topology, where we can find nice applications of the ideas covered in our course. As it is at the advanced graduate level, I do not expect you to read this from cover to cover during this quarter.

Further supplementary textbooks: The following can help with learning, but are completely optional. A student from a previous year said they found John Lee's book useful for self-study.

- "Calculus on Manifolds" by Michael Spivak, CRC Press, 1971.
    - A systematic treatment of differentiation and integration, culminating in Stokes' theorem.
- "Differential Forms and Applications" by Manfredo do Carmo, Springer, 1994.
    - A quick and compact reference for differential forms, with the Gauss-Bonnet theorem at the end.
- "Introduction to Smooth Manifolds" by John Lee, Springer, 2000.
    - Another comprehensive introduction to smooth manifold theory. Check out Lee's other collection, "Introduction to (Topological/Riemannian/Complex) Manifolds," if you want to learn manifold theory thoroughly.

Problem sets:

- Problem sets will be assigned weekly and posted on Canvas by each Friday. The solutions should be submitted to Gradescope, and the deadline is the Friday of the following week.
- Each student will have an allowance of seven late days, which can be used without prior permission from the instructor. E.g., if you have an emergency right before the deadline, you can submit late on Gradescope without notifying the teaching assistant or me, as long as you submit within seven days minus your remaining allowance. Gradescope will automatically keep track of your cumulative lateness, which will be measured in seconds. Once you exceed your lateness allowance, any late submission thereafter will receive no credit. Late submissions may take longer to be graded than regular submissions.
- The lowest problem set grade will be dropped. If you choose not to submit a problem set, that will count as your dropped problem set grade.
- You should write your solutions in your own words, while justifying all claims and stating the previous results that you are using. The arguments you use should be from the content learned in this course and the prerequisite courses. Tools beyond the scope of this course, e.g., algebraic topology arguments not covered in 109a or 109b, are not allowed.
    - Collaboration with other students on the problem sets is allowed, though thinking about the problem sets on your own is the best way to understand the material. If you do discuss the problem set with others, please indicate clearly the names of the people with whom you collaborated. The habit of giving credit where it is due will grow into a habit of acknowledging or even collaborating on research papers.

- Generative AI has reached a level where it might solve most of the problems I will throw at you. I discourage you from using generative AI to solve the problem sets, as I consider it an Honor Code violation.

  In general, regarding our attitude towards generative AI, I sometimes use it when learning new material and find it useful for discovering new references; however, it sometimes makes mistakes, and I have often seen it hallucinate facts and produce false arguments. I find it imperative to track down and peruse the original sources it gives, and to reproduce the arguments by my own. I would like to share an announcement that my friend, Will Kwon, made in his course regarding the use of generative AI, where he says my points better than I wrote here.

Midterm and Final exams: The midterm and final will be oral exams. The exact logistics will be announced later in the quarter.

Grading scheme: Weekly problem sets (30%), midterm exam (35%) and final exam (35%). The lowest problem set grade will be dropped.

Tentative Course Schedule

- Week 1 (3/30-4/3): Introductions and motivation, review of basic manifold theory.

  Reading material: Guillemin-Pollack Chapter 1.1 - 1.4.

- Week 2 (4/6-4/10): Transversality, Sard's theorem, Whitney embedding theorem.

  Reading material: Guillemin-Pollack Chapter 1.5 - 1.8.

  Problem Set 1 due on 4/10.

- Week 3 (4/13-4/17): Intersection theory mod 2, Jordan-Brouwer separation theorem, Borsuk-Ulam theorem.

  Reading material: Guillemin-Pollack Chapter 2.1 - 2.6.

  Problem Set 2 due on 4/17.

  4/17: Last day for adding courses and removing conditions and incompletes

- Week 4 (4/20-4/24): Differential equations and vector fields on manifolds, tangent bundle, orientations.

  Reading material: Guillemin-Pollack Chapter 3.1 - 3.2, Spivak (CIDG) Chapters 3 and 5.

  Problem Set 3 due on 4/24.

- Week 5 (4/27-5/1): Lefschetz theory, Poincaré-Hopf theorem.

  Reading material: Guillemin-Pollack Chapter 3.3 - 3.5.

  4/29-5/5: Midterm examination period

- Week 6 (5/4-5/8): Hopf degree theorem, Euler characteristic.

  Reading material: Guillemin-Pollack Chapter 3.6 - 3.7.

  Problem Set 4 due on 5/8.

- Week 7 (5/11-5/15): Tensors, differential forms, integration of differential forms.

  Guillemin-Pollack Chapter 4.1 - 4.5.

  Problem Set 5 due on 5/15.

  5/11: Last day for seniors to remove conditions and incompletes

- Week 8 (5/18-5/22): Stokes' theorem, Gauss-Bonnet theorem.

  Reading material: Guillemin-Pollack Chapter 4.6 - 4.9.

  Problem Set 6 due on 5/22.

  5/20: Last day for dropping courses, exercising pass/fail option, and changing sections

- Week 9 (5/25-5/29):[1] Framed cobordism, Pontryagin construction, Hopf theorem.

  Reading material: Milnor (TftDV) Chapter 7.

  Problem Set 7 due on 5/29.

[1] Some day in the neighborhood of this week might be Ditch Day (or perhaps not, I have no clue), which I will honor if it falls on a Monday/Wednesday/Friday.

5/25: Memorial Day (institute holiday)

5/29: Last day of classes for seniors and graduate students[2]

5/30-6/2: Study period for seniors and graduate students

- Week 10 (6/1-6/5): The h-cobordism theorem and an Overview of its proof.

Reading material: Milnor (LothCT)

Problem Set 8 due on 6/5.[3]

6/3-6/5: Final examinations for seniors and graduate students

6/5: Last day of classes for undergraduates

6/6-6/9: Study period for undergraduates

- Week 11 (6/9-6/13): No classes.

6/10-6/12: Final examinations for undergraduates

(Last updated March 10, 2026)

[2]In the final examination, seniors and graduate students will be tested on the material up to Week 9. Of course, they are welcome to join Week 10's lectures.

[3]Seniors and graduate students are exempt from Problem Set 8.