# MA 109C: INTRODUCTION TO GEOMETRY AND TOPOLOGY

SPRING 2025

**Instructor:** Seung-Yeon Ryoo (she/her, `sryoo@caltech.edu`)
Lectures: Monday, Wednesday, and Friday, 10:00 AM – 10:55 AM, Linde 387
Office hours: Thursday 3:00 PM – 5:00 PM, Linde 258 (additional hours available upon request)

**Teaching Assistant:** Sameer Kumar (he/him, `skumar5@caltech.edu`)
Office hours: Wednesday 1:00 PM – 2 PM, Location TBA

**Course Description:** An introduction to differentiable manifolds. Transversality, differential forms, and further related topics.

**Prerequisites:** Ma 2 or equivalent, and Ma 108 must be taken previously or concurrently.

**Main textbook:** *Differential Topology* by Victor Guillemin and Alan Pollack, American Mathematical Society, 2010.

When I cover material not in the main textbook, I will provide lecture notes on Canvas. The following supplementary textbooks can help with studying but are completely optional. A student from the previous year said that they found John Lee and Morris Hirsch's book useful.

Supplementary textbooks (in order of relevance):

- *Topology from the Differentiable Viewpoint* by John Milnor, Princeton University Press, 1997.
  - A 'crash course' on differential topology.
- *A Comprehensive Introduction to Differential Geometry, Volume 1* by Michael Spivak, Publish or Perish, 1999.
  - A rather "comprehensive" introduction to smooth manifold theory.
- *Introduction to Smooth Manifolds* by John Lee, Springer, 2000.
  - Another comprehensive introduction to smooth manifold theory.
- *Differential Topology* by Morris Hirsch, Springer, 1976.
  - A graduate level textbook on differential topology, which you can read for further studies after finishing this course. I do not expect you to read this book this quarter.
- *Calculus on Manifolds* by Michael Spivak, CRC Press, 1971.
  - A systematic treatment on differentiation and integration, culminating in Stokes' theorem.
- *Differential Forms and Applications* by Manfredo do Carmo, Springer, 1994.
  - A compact reference for differential forms.

**Problem sets:** Problem sets will be assigned weekly, and posted on Canvas by Monday. Solutions must be submitted to Gradescope by the Friday of the following week. Each student will have an allowance of five late days, which can be used without prior permission from the instructor. Collaboration with other students on the problem sets is allowed, though thinking about the problem sets on your own is the best way to understand the material. If you do discuss the problem set with others, please indicate clearly the names of the people with whom you collaborated, and write your solutions separately in your words while justifying all the claims and stating clearly all the previous results that you are using.

**Midterm and Final exams:** The midterm and final exams will be posted on Canvas, and the exact logistics will be announced later in the quarter. Collaboration with other students on the midterm and final exams is not allowed.

**Grading scheme:** Weekly problem sets (60%), midterm exam (20%) and final exam (20%). The lowest problem set grade will be dropped.

**Tentative Course Schedule:**

- Week 1 (3/31 - 4/4): Introductions and motivation, review of basic manifold theory.
  Relevant reading material: Guillemin–Pollack Chapter 1.1 - 1.4.
- Week 2 (4/7 - 4/11): Transversality, Sard's theorem, Whitney embedding theorem.
  Relevant reading material: Guillemin–Pollack Chapter 1.5 - 1.8.
  Problem Set 1 due on 4/11.
- Week 3 (4/14 - 4/18): Intersection theory mod 2, Jordan–Brouwer separation theorem, Borsuk–Ulam theorem.
  Relevant reading material: Guillemin–Pollack Chapter 2.1 - 2.6.
  Problem Set 2 due on 4/18.
- Week 4 (4/21 - 4/25): Differential equations and vector fields on manifolds, tangent bundle, orientations.
  Relevant reading materia: Guillemin–Pollack Chapter 3.1 - 3.2. Optional to read Spivak (CIDG) Chapters 3 and 5.
  Problem Set 3 due on 4/25.
- Week 5 (4/28 - 5/2): Lefschetz theory, Poincaré–Hopf theorem.
  Relevant reading material: Guillemin–Pollack Chapter 3.3 - 3.5.
  4/30 - 5/6: Midterm examination period
- Week 6 (5/5 - 5/9): Hopf degree theorem, Euler characteristic.
  Relevant reading material: Guillemin–Pollack Chapter 3.6 - 3.7.
  Problem Set 4 due on 5/9.
- Week 7 (5/12 - 5/16): Tensors, differential forms, integration of differential forms.
  Guillemin–Pollack Chapter 4.1 - 4.5.
  Problem Set 5 due on 5/16.
  5/12: Last day for seniors to remove conditions and incompletes
- Week 8 (5/19 - 5/23): Stokes theorem, Gauss–Bonnet theorem.
  Relevant reading material: Guillemin–Pollack Chapter 4.6 - 4.9.
  Problem Set 6 due on 5/23.
  5/21: Last day for dropping courses, exercising pass/fail option, and changing sections
- Week 9 (5/26 - 5/30): Frobenius integrability theorem, introduction to Lie groups.
  Optional reading material: Spivak (CIDG) Chapters 6, 7.5, 10.
  Problem Set 7 due on 5/30.
  5/26: Memorial day (institute holiday)
  5/30: Last day of classes for seniors and graduate students
  5/31 - 6/3: Study period for seniors and graduate students
- Week 10 (6/2 - 6/6): Framed cobordism, Pontryagin construction, Hopf theorem.
  Optional reading material: Milnor Chapter 7.
  Problem Set 8 due on 6/6.
  6/4 - 6/6: Final examinations for seniors and graduate students
  6/6: Last day of classes for undergraduates
  6/7-6/10: Study period for undergraduates
- Week 11 (6/10 - 6/14) No classes.
  6/11-6/13: Final examinations for undergraduates

(Last updated March 31, 2025)