Banjo Rim Sound

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Four different wood rims are equipped with the same kind of head, tension ring, hooks, nuts, and shoes. Recordings of taps on the rims and heads yield information on their stiffness and impact on head vibration. A single neck with its own strings, bridge, tailpiece, and co-rod is attached to one rim after the other. The rims produce different pluck-sound rise times and sustain, in accord with the physics of weight, stiffness, and coupled oscillators. Timbre variations are complicated. The combined head-rim systems show slight variations in head resonant frequencies but clear variations in the amplitudes of head motions in response to excitation by the bridge at different frequencies. The details presumably depend on the proximity in frequency of head and rim modes that can couple strongly because of their matching geometries. Thus, the rims give each banjo a different sound. Sound samples of taps, plucks, and played tunes are included. Some may conclude that the differences are not that different from what can be achieved with variations in playing technique and set-up.

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I. BACKGROUND

Players have opinions about the pro's and con's of different rims. But the differences are subtle and are not totally distinct from what can be achieved by various adjustments or swappable parts or playing technique. Furthermore, the player is in a privileged position relative to any other listener. The sound there is most dramatic and is compared immediately to an intention or expectation. In judging attack or rise time, only the player knows when the string was plucked. Underscoring the subtlety from a physics perspective, zeroth order modeling with an assumption of an ideal, perfectly rigid rim does a fair job of accounting for banjo sound, including adjustments and parts choices that are available post-manufacture.[1]

This study addresses rim sound by comparing four instruments that differ only by the species and construction of their wooden rims. A single neck, co-rod, set of strings, bridge, and tailpiece are mounted sequentially on the rims. The complete pots are assembled with the same kind of hardware (head, tension ring, hooks, nuts, and shoes) with current stock from Deering Banjos.

II. INTRODUCTION

A technique described in section §VII produces plucks that are highly reproducible in location and force. This is the sound of a single such pluck on the open 1st string on each of the assembled banjos:

https://www.its.caltech.edu/∼politzer/rims/1st-14th-singles-4sec.mp3

They sound different. (For the hearing challenged, turn up the volume and/or try an upgrade from laptop speakers.)

The differences are greatest in the earlier part of the sounds. So, the following plucks are trimmed to the first 0.25 seconds. (That is sedate relative to most banjo playing.) And, to drive the point home, you will hear a series of ten successive plucks on each banjo:

https://www.its.caltech.edu/∼politzer/rims/four-rims-1st-10-TRIMMED.mp3

Just as neck flexing effects the sound by its direct impact on string vibration[2], rim flexing effects the sound by its direct impact on head vibration. The goal of what follows is to characterize the sound differences with sufficient precision that some connection can be made to the physical properties of the rims.

The first three rims are about 9/16″ thick. They are similar but sufficiently different to sound somewhat different. The fourth rim is about 5/16″ thick and, therefore, much lighter and much more flexible. Interestingly, those differences are sufficiently large to put its physics description into a different realm. It works as a banjo, but it works differently.

A. jump to the conclusion?

The pluck sounds linked above aren't music. And the next two sets of sounds presented below help characterize the details of the rims, but they are even further from playing the banjo. If you simply want to hear some music played on the four rims, jump to section §VIII for some brief samples. However, some of the differences can be hard to discern and might not seem distinguishable from slight variation in playing technique or recording set-up.

B. outline

In section §IV, the sound of taps on the rims themselves allows a quantitative estimate of their relative stiffness.

In section §V, mounting the heads and tapping in a reproducible fashion reveals the small but discernible differences in head vibration produced by the different rims.

Banjos are assembled for section §VI. An impulse hammer gives reproducible taps on the bridge with strings both damped and open. The recordings simply sound like taps, albeit slightly different from each other. The damped string sounds are a reflection of the bridge admittance, a function that encodes the bridge motion that would be produced by the various frequency components of a properly plucked string. The open string sounds give a picture of the inharmonic partials that accompany every string pluck.[3]

The 1st string pluck sounds linked above are examined in detail in section §VII. Rise time and sustain are gross features rather than being frequency specific. The differences between the first three thick rims behave in these respects in ways that follow directly from the weights and stiffnesses. However, understanding the sound of the thin rim requires reference to its frequency spectrum, which is substantially lower than the corresponding resonances of the other rims because of its relative flexibility.

The spectrograms of the 1st string pluck sounds reveal many slight differences in frequency component intensity and time dependence. There are no simple patterns. Rather, they must be the consequence of the interplay of the head and rim resonances. Brief qualitative explanations are presented.

Played music samples are presented in section §VIII. There is also a set of alternate takes, presented without the rim identity as a test of whether one can actually identify the rims by their sound.

III. THE RIMS

In the figures and sound samples, the fours rims are always presented in the same order. It's actually by age, oldest first. The first is from a year 2000 Deering Goodtime and has 11 plies of maple. So, it is designated "11 ply." The second is from an example of the upgraded Goodtime, i.e., 3-ply maple — and labeled "3 ply." The third is from the limited edition 2023 "Cherry" Goodtime. It has three plies: cherry - maple - cherry — and labeled simply "cherry." These rims are all of the same dimensions. In particular, they are $\sim \frac{9}{16}''$ thick. The fourth is fashioned from a 12-ply cherry drum shell (from https://nordicshells.com/) that is $\sim$ 8mm ($\sim \frac{5}{16}''$) thick — and labeled "8mm." TABLE I lists the labels and descriptions for convenient reference.

label	description – see p. 1 photo
11 ply	laminated maple
3 ply	bent maple
cherry	3-ply bent cherry-maple-cherry
8mm	12-ply laminated cherry

TABLE I: identifying rim labels

IV. BANJO RIM TAPS

The rims are suspended from one of their hanger bolt holes and tapped at 45° from the suspension point. A directional mic is positioned close to the outer surface and at 45° on the other side of the suspension point. These locations emphasize exciting and recording the lowest mode. (See Appendix B regarding cylindrical shell normal modes.) For these recordings, the same current stock shoes (the things that hold the tension hooks & nuts) are attached to each rim. The 16 shoes and their screws together weigh 5 oz.

This is the actual sound, and FIG.s 1 & 2 are waveforms, spectrograms, and spectra of those sounds:

https://www.its.caltech.edu/∼politzer/rims/rim-taps-one-each.mp3

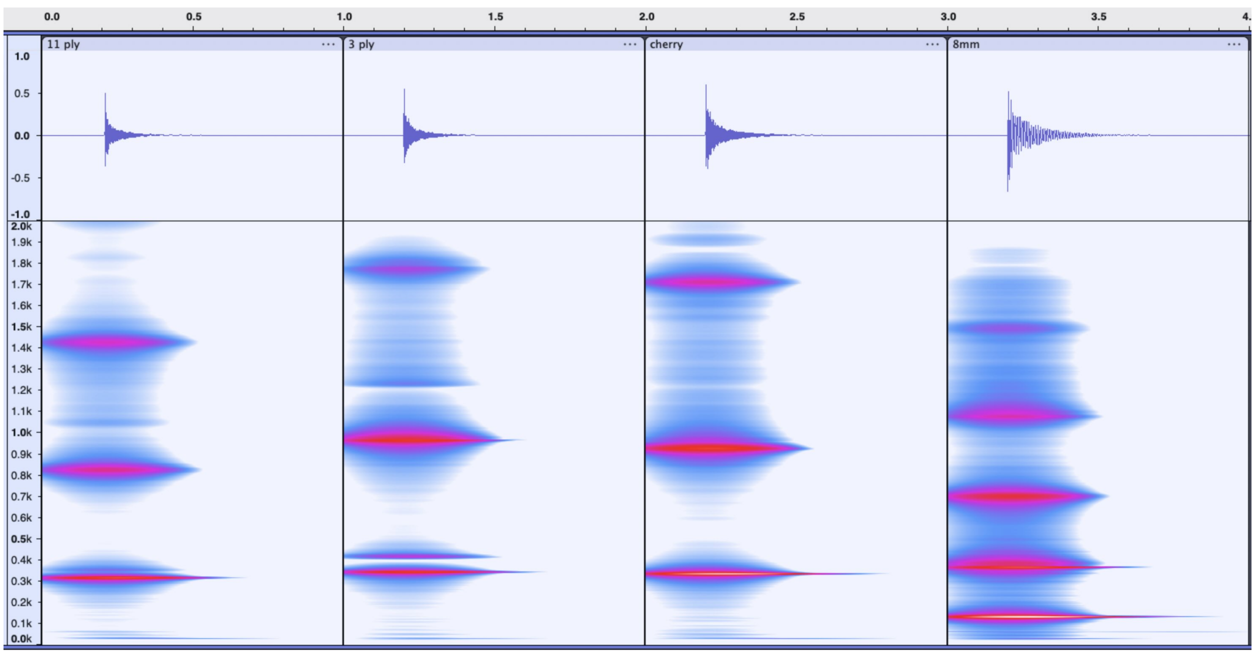

FIG. 1: taps on rims

A. lessons from a bit of physics

The actual sounds of the rims themselves have no direct relation to the sounds produced by the assembled banjos, where the rims are constrained to move with the tension ring and edge of the head. However, a bit of physics brings some order to the analysis of these rim sounds and produces some useful results. In particular, we can deduce the rims' relative stiffnesses.

First, recall the formula for the frequency f of the ideal mass on a spring:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

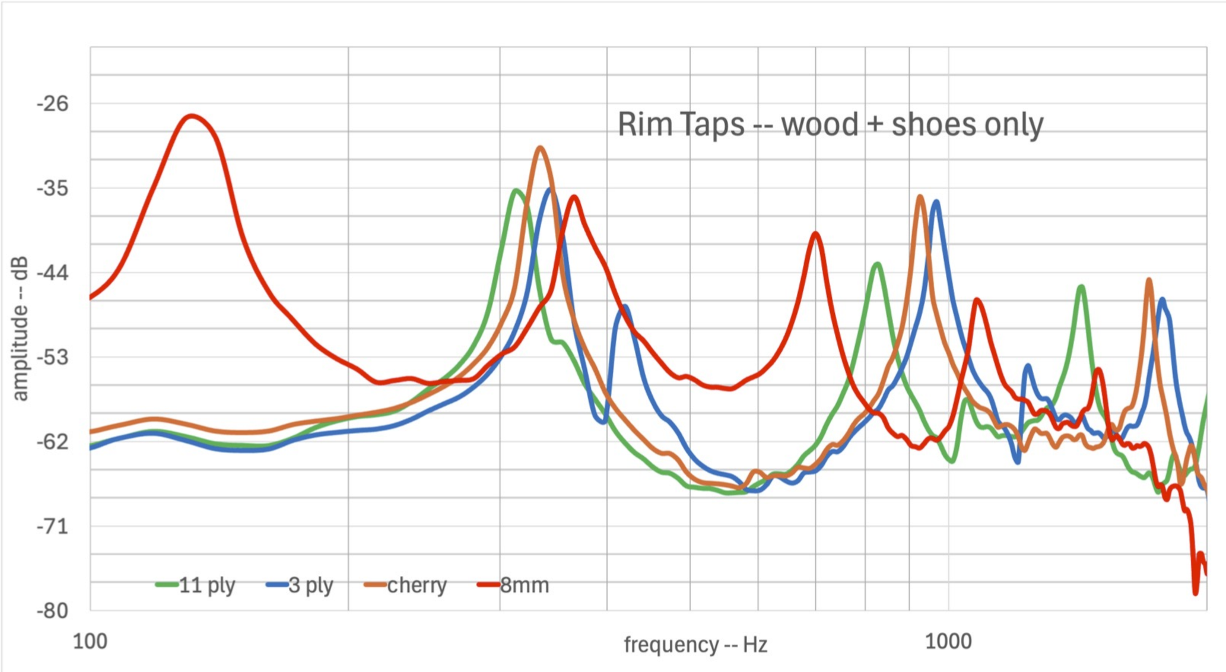

FIG. 2: rim tap spectra

which applies to any oscillation at small enough amplitude.

The resonant frequencies of a particular rim are given by a series of dimensionless (pure) numbers times a single frequency, which is obviously proportional to the square root of some measure of its stiffness divided by its total mass.

The four rims' observed fundamental frequencies and measured weights are displayed in TABLE II. The third column is the stiffness deduced from the frequencies and weights relative to the results for 3-ply maple. As long as the fundamental modes' motions are perpendicular to the thickness, this is legitimate because all four rims' dimensions are the same in directions perpendicular to the thickness. The rank order of stiffnesses are what many people might have expected.

Furthermore, the dimensionless numbers that give the series of resonant frequencies for a particular rim should be (approximately) the same for all rims that are sufficiently similar in their geometry and differ most in their weight and stiffness. For thin, squat cylinders, this will work best for the first few resonances, i.e., before the rank order of different types of motions gets scrambled by the differences in physical parameters. (See APPENDIX B.)

FIG.s 1 & 2 bear some resemblance to this expectation. In FIG. 1, the linear frequency axis of the spectrograms shows similar ratios of the spacing of first to second and second to third resonances. In Fig. 2 with a logarithmic frequency scale, this shows up as roughly equal spacing of peak 1 to 2 and 2 to 3.

rim	fundamental frequency – Hz	rim & shoe weight – oz.	relative stiffness
11 ply	316	23	0.83
3 ply	354	22	1
cherry	335	22	0.90
8mm	133	16	0.10

TABLE II: wood rims & shoes

The second relevant bit of physics is that, for a thin, uniform material, the stiffness to bending perpendicular to the thin direction is proportional to the cube of its thickness:

$$\text{stiffness} \propto \text{thickness}^3$$

Wood references give standard approximate values for stiffness. In particular, the ratio of Young's modulus for cherry to maple is 0.83 (accidentally approximately equal but totally unrelated to the relative stiffness of the 11-ply rim, as given in TABLE II). This allows a reasonable comparison of the two laminated rims, i.e. 8mm cherry to 11-ply maple. The stiffness ratio of the 8mm to 11 ply deduced from weight and fundamental frequency as in TABLE II is 0.12. This is to be compared to the ratio of Young's moduli times the cube of the ratio of thicknesses: $0.83 \times (5/9)^3 = 0.14$. *(In the realm of banjo physics, I say that's a triumph. Were those laminated with different glue? Is that relevant? If nothing else, the closeness of the Young's modulus-based estimate to the measured/deduced value demonstrates that the discussion above is not obviously wrong.*

Also, the photo on page 1 suggests that the 11 ply and 8mm rims are "plywood," i.e., with grain going in two different directions. That makes them more stable but inherently less stiff than were all layers with grain going around. Even though the bending moduli are different in different directions, the two laminated rims have roughly the same mix of the two directions. So, the ratio of rim stiffness may well be about right using just the along-the-grain value.)

B. shoes vs. no shoes

In retrospect, recording rim taps of the wood alone, i.e., without the shoes, would have been clearer in its interpretation — even if a bit further from the action of the assembled instruments.

The scaling of the resonant frequencies of each rim in proportion to, say, each lowest frequency, is only true if the densities of all rims are distributed in the same proportions. Inclusion of the 5 oz. of shoes & screws violates this. (That is over 30% of the weight listed for the 8mm rim and $\sim$ 23% for the others.) For these rims' profiles, the lowest mode presumably has four straight node lines spaced 90° apart around the circle and no circular node line going all the way around. (See Appendix B.) The next mode higher in frequency is likely one with six equally spaced straight lines. The next one might be the one with eight. Where the modes with circle node lines enter the series is difficult to say without a detailed numerical calculation. Their frequencies depend on how the mass is distributed in the vertical direction, i.e., perpendicular to the plane of the head. Because the same type of shoes were used on all rims, the *relative* mass distributions were different in the vertical direction.

In any case, deducing relative stiffness using the lowest mode frequency works just as well with shoes as it would without as long as the lowest mode for each rim has the same spatial shape, i.e., node line pattern suggested above. If so, the inertia of that mode is simply proportional to the total wood and shoe weight for each particular rim.

V. BANJO POT DRUM TAPS

Each rim was supplied with head, tension ring, hooks, and nuts from current Deering factory stock and tightened to 88 on a DrumDial. (The tensions were left to stabilize and readjusted over a period of a week.)

The next set of sounds are well-controlled taps on the head. Without neck, bridge, & strings, these are drums. The taps were performed by dropping a pencil from a position and height using a rigid frame pictured in FIG. 3 as a guide. I analyzed a set of ten clean ones for each rim. The differences from tap to tap were much smaller than the differences between rims. One consistency check was to repeat a set of taps on the first rim (the 11 ply) after doing all the others – to confirm that it actually was just like the first round and obviously different from the others.

FIG. 3: pencil/pot tap rig and mic

The following is the sound of one representative tap for each sequence of ten. Note that the 11 ply is repeated, the second version having been recorded after the other three to demonstrate the consistency of the tap mechanism. However, it is played here immediately after the first one — to demonstrate that they sound the same. So, there are five head taps: 11 ply, 11ply, 3 ply, cherry, and 8mm.

https://www.its.caltech.edu/∼politzer/rims/drum-taps-one-each.mp3

Some of the complexity and subtlety of rim physics is already apparent in these sounds. Rayleigh, who was seriously interested in the physics of drums, made the distinction of those with *definite* pitch to those of *indefinite* pitch. “Definite” required that the low lying resonant frequencies be integer multiples or nearly so. The discerned pitch frequency is that common factor. Examples include the kettledrum, which Rayleigh studied and the Indian tabla, whose heads sport a patch of flexible iron-weighted clay that C.V. Raman showed produced the magic ratios. More generally, a series of strong frequency components whose spacings are integer multiples of a single frequency can be heard as that spacing frequency even if they are all offset from that one as a base.[4]

It is more common for drums to have frequencies in rough agreement with the Bessel function zeros analysis of the "ideal" drumhead. Most human brains do not recognize that series as perfectly well-ordered. It is certainly not "musical."

Nevertheless, human brains struggle mightily to discern a particular pitch wherever possible — even when technically "indefinite." The wood rim taps in the previous sound sample likely seem to have pitches. Likewise, the indefinite pitch drums in the immediately preceding sound sample also likely have associated pitches. However, people do not all agree on which is higher and which is lower![5] For example, does the lowest pitch of each set belong to the same rim? And which in the sequence is, indeed, the lowest? People answer with confidence, but their answers might be different.

FIG. 4 shows the waveforms and spectrograms of those chosen representative pot drum taps. And FIG. 5 shows computed spectra for each rim, averaged over the set of 10 each.

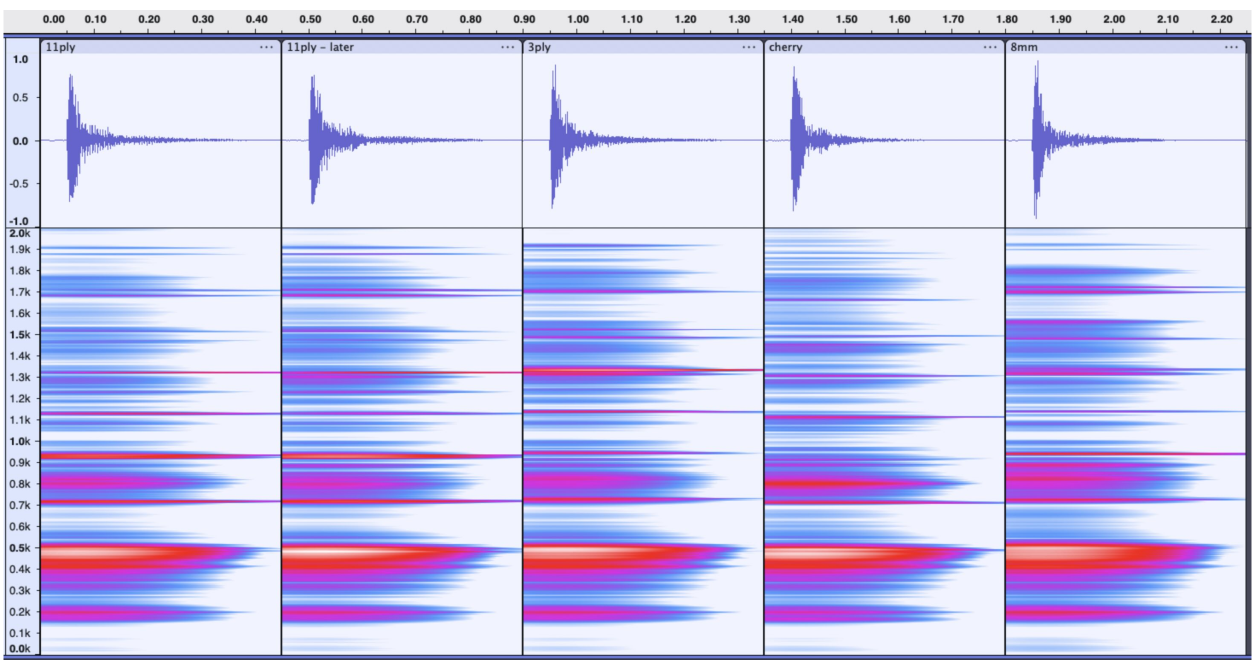

FIG. 4: spectrograms of the representative tap for each rim, with 11 ply repeated

So, the sounds, spectrograms, and spectra of the four pots are definitely different.

There is some qualitative physics in FIG.s 4 & 5 that will contribute to understanding some of what happens with the fully assembled banjos.

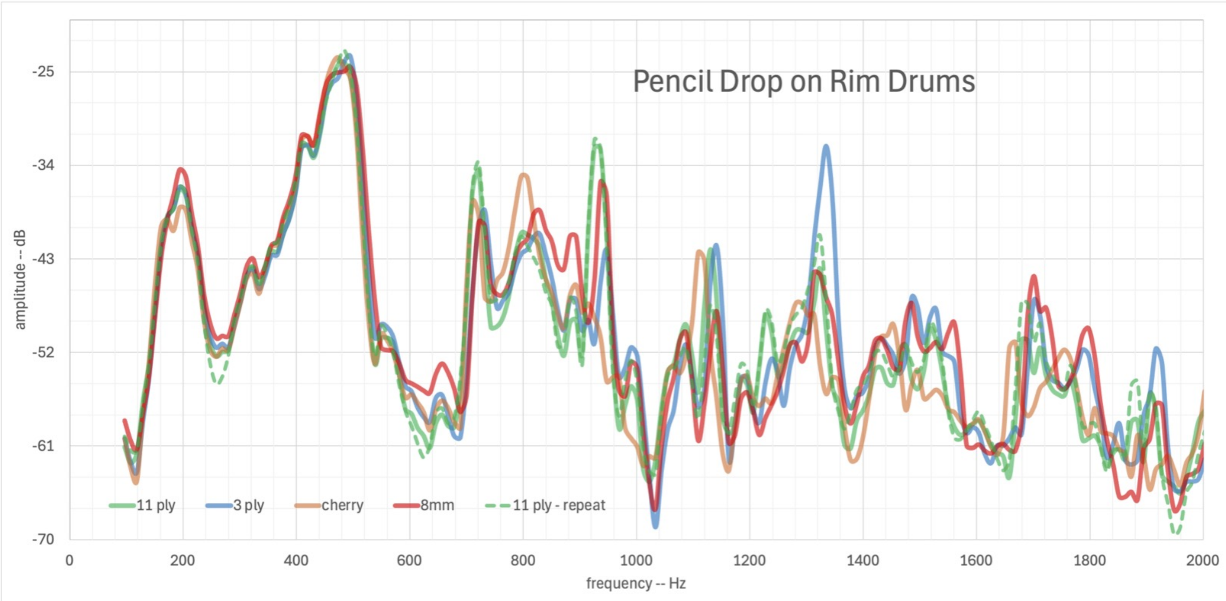

FIG. 5: pot drum tap spectra – average of 10 for each pot

The brain has the knack of listening to and analyzing many different aspects of heard sound before deciding and reporting "what it sounds like." No one spectrogram can capture that. Rather, the brain considers a great many analyses and criteria simultaneously. It also compares what it is hearing to what it heard in the past on all time scales, i.e., from immediately before to long ago.

FIG. 6 displays spectrograms of the same representative plucks as used to produce FIG. 4. However, the spectrogram analysis parameters are quite different. The frequency range goes to 6000 Hz, and the frequency resolution is much lower, which allows a much higher time resolution. For all string instruments, on average, the spectrum decreases substantially with increasing frequency. Audacity offers a feature that allows viewing high frequency ranges along with low. In particular, their "High Boost" amplifies the frequency analyzed signal by some number of decibels per decade of frequency. That is used in FIG. 6 to get meaningful comparisons up to 6 kHz of differences that certainly are apparent with normal, good hearing.

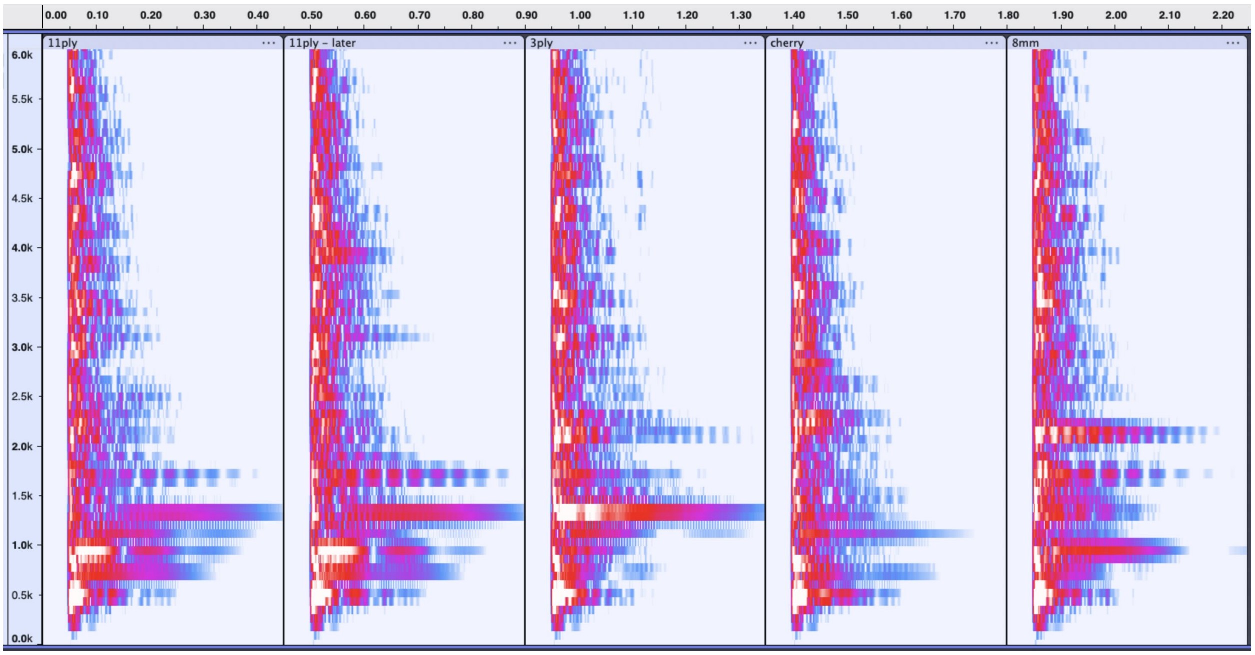

FIG. 6: spectrograms of the representative tap for each rim, with 11 ply repeated

A. coupling of the rim to the head

We can think of any head motion as a superposition of its normal modes, each with its own frequency and spatial pattern along the top of the rim. Considered individually, these each tug on the rim with their own frequency and pattern. Similarly, rim motion can be decomposed into normal modes. And we can imagine the total interaction as a pairwise sum. (This is an application of the approximate linearity of the physics for small amplitudes.)

It is natural to think of the head as a driver of the rim. The effect of a particular head mode on a particular rim mode depends on four things. 1) There's the tension of the head and the amplitude of the particular head mode. 2) The rim response is inversely proportional to a measure of its mode inertia. Those are equal to the total rim mass times a series of dimensionless numbers that depend on the mode number. As discussed in section §III (and Appendix B), that series depends on the spatial distribution of the mass. The four rims are sufficiently similar that these numbers are expected to be about equal for the lowest few modes. 3) There is an effective coupling which depends on the degree of spatial matching. This is greatest when the head and rim modes have the same number of nodes around the edge of the head and the top of the rim. There can be nearly complete cancellation if they do not. The situation is simple if both systems have exact circular symmetry. On real banjos, both are more complex. And 4) there is the resonant response as suggested by the curves in FIG.s 7 & 8. A given rim mode responds most to head modes of nearby frequencies. The effect of higher head frequencies goes to zero, while all lower head frequencies contribute to the rim motion.

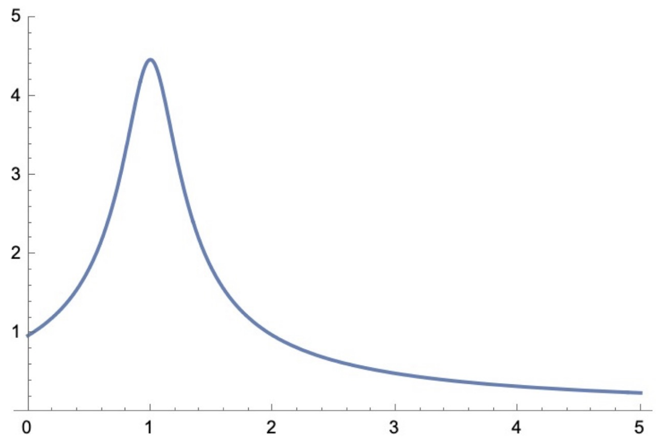

FIG. 7: generic shape of resonant amplitude response; note the behavior away from the peak. The horizontal axis is the driving frequency, and the peak is at the oscillator's natural frequency.

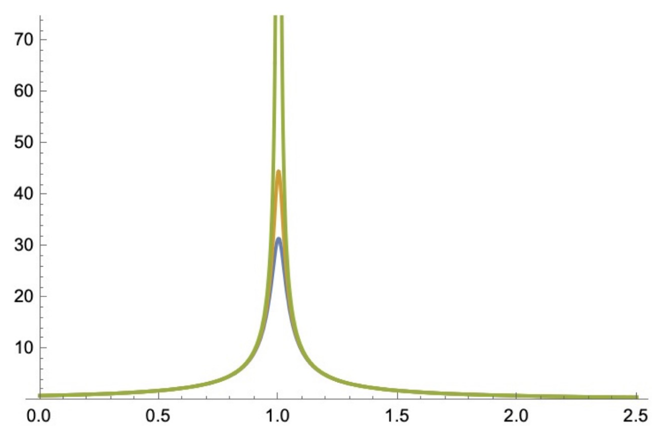

FIG. 8: amplitude for sinusoidal forcing of a harmonic oscillator, with and without damping, as a function of forcing frequency

FIG. 8 is a reminder of the effect of damping in the driven rim mode. The green curve is for zero damping and goes to infinity on resonance. The blue curve has twice the damping as the yellow one. When it is said that damping increases the width in frequency of the resonance, that is with respect to FWHM – full width at half maximum – as a measure of width. It is nevertheless true that damping reduces the maximum response amplitude but has little to no effect away from the resonant frequency.

Of course, the rim pushes back on the head, establishing joint modes of the coupled system. The sound of tapping on the head or plucking the strings is produced overwhelmingly by the head because the head is a much more efficient radiator of sound than the rim or strings. (Hunting down errant string buzz is an amusing example. Wherever its origin, the sound of the buzz comes off the head.)

In FIG.s 4 & 5, we see peaks that are generally similar from rim to rim. The strongest ones are presumably descendants of modes of an ideal head with a perfectly rigid rim. There are differences in frequency and intensity due to the way in which the complete set of rim modes alter the particular head mode due to their coupling. For the comparisons in the figures to be relevant in terms of intensities and not just frequencies, it is important that the excitation was in the same place and with the same impulse in each case. (That was the purpose of the pencil-drop guide structure pictured in FIG. 3.)

One interesting mode is the lowest one, i.e., just below 300 Hz. The lowest pure-head mode is simply up and down simultaneously over the whole head, with no node lines. This mode is expected not to couple to the rim in any appreciable way. Any rim mode for which the whole top edge goes in simultaneously and then goes out is what Rayleigh termed "extensional." The whole rim has to stretch and then compress. "Inextensional" modes involve bending. Their restoring force and, hence, frequency, is generally substantially lower. This, of course, depends on the actual dimension ratios. (See Appendix B.) In FIG.s. 4 & 5 the lowest frequencies of the four rims are essentially equal, reflecting the further fact that they were the same type and size of head, tensioned to the same DrumDial reading. FIG. 5 reveals that they are not quite the same intensity, which reflects the fact that the same total impulse gets divided among the modes as they see fit, and the higher rim modes do, indeed, couple noticeably to the head.

VI. BRIDGE TAPS

The next round involves fully assembled banjos. As stressed before, the same one neck (Goodtime-CNCed white oak), tension ring, bridge, string set, and tailpiece were used on all four rims. Still not normally playing, the protocol was to tap on the bridge, first with all strings damped and then with all strings open. The rig shown in FIG. 9 produced nearly identical taps — in the same place and with the same impulse.

Below are sounds, waveforms, and spectrograms of one representative tap for each rim.

FIG. 9: impulse hammer for bridge taps

https://www.its.caltech.edu/∼politzer/rims/hammer-strings-damped-one-each.mp3

https://www.its.caltech.edu/∼politzer/rims/hammer-strings-open-one-each.mp3

The sounds are different for each rim. Even the string component of the taps with open strings are impacted by the differing rim interactions with the head. While the dominant physics is the same for all rims, as in the rim drum discussion, head modes get combined with rim modes in ways that depend both on the details of the frequency spectra and on the spatial geometry of the modes. The result is variation from rim to rim that cannot be described as a single, simple trend.

Adding the strings has added further physics. To be sure, the static down-pressure of the bridge on the head has altered what might be thought of as the pure head modes. A big change due to the strings is that they significantly damp the head vibration, i.e., take away energy, compared to the "drum taps" without strings – whether the strings themselves are damped or not.

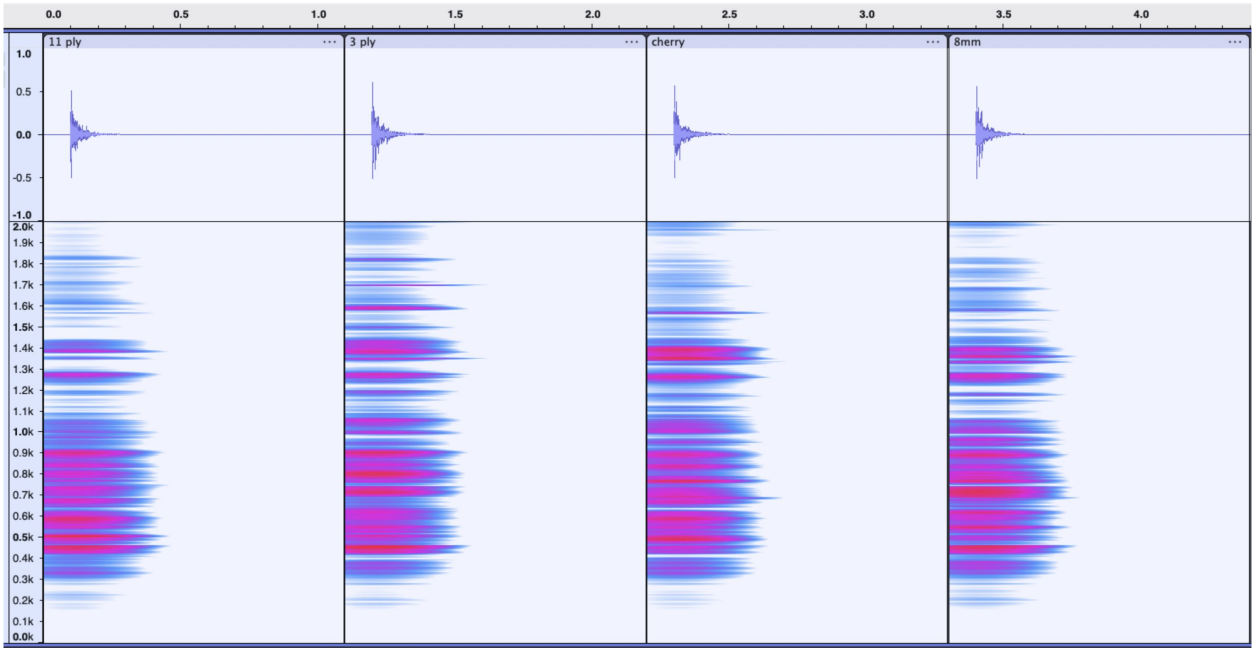

FIG. 10: bridge taps with strings damped

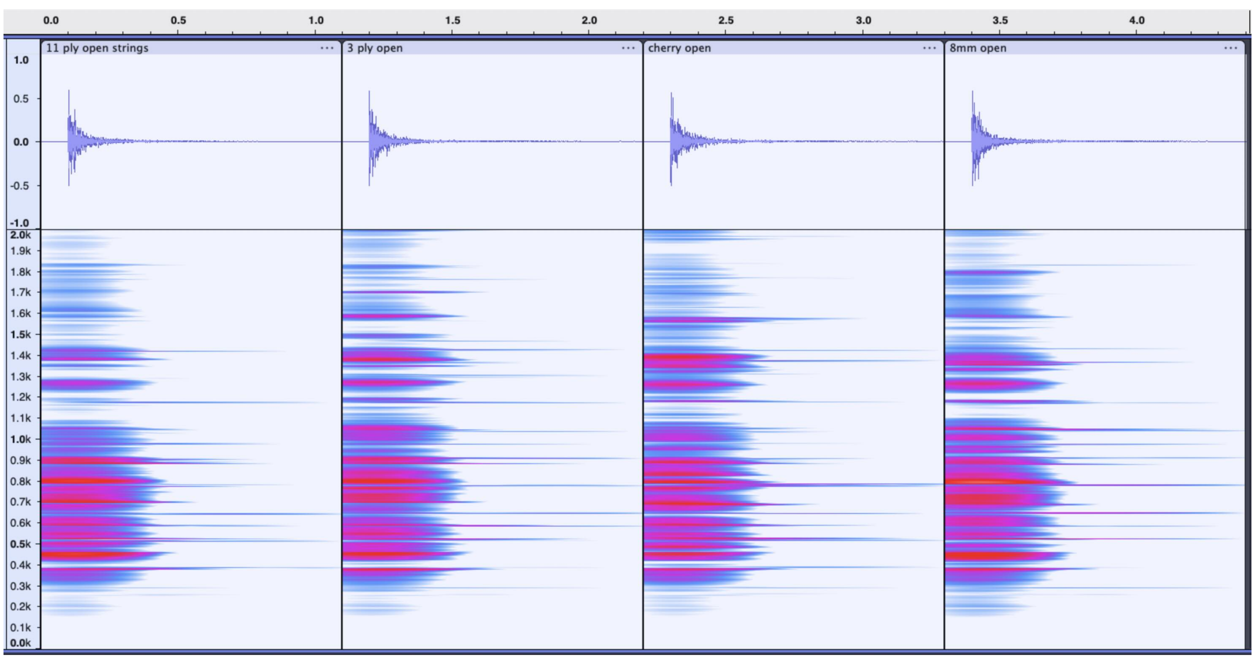

FIG. 11: bridge taps with all strings open

VII. 1st STRING PLUCKS

Not yet music... but at least the sound of plucked strings:

Highly reproducible plucks are performed on the 1st string at the 14th fret for each rim with all strings open. The technique is a wire break. #42 AGW magnet wire (0.0024″ diameter) is looped once around the string and pulled very gently until it breaks. Hence, the location, direction, and release tension are very close each time. So, the differences between successive plucks are tiny compared to the differences from rim to rim.

The following is a series of ten plucks for each rim (in the same order: 11 ply, 3 ply, cherry, and 8mm). The sounds die off after about 2.5 seconds. However, I found my impressions to be dominated by the long-time behavior, for which the rims were quite similar. When banjos are played, the notes almost always follow hard and fast. Four per second is not fast playing by any means. To better reflect that situation, I cut the sound samples to their first 0.25 sec to produce the second linked sound file on p. 2 (and repeated here for convenience). Hearing ten in a row might sound annoying, but it helps focus on what's relevant in played music. (For laptop speakers and/or impaired hearing, just turn up the volume. Good speakers help. The differences are obvious.)

https://www.its.caltech.edu/∼politzer/rims/four-rims-1st-10-TRIMMED.mp3

A. attack & sustain

Sometimes people's discussion of rims focuses on responsiveness and sustain. One possible interpretation of "responsiveness" is the rise time of the attack. And, in practice, "sustain" to a banjo player would not be how long a note hangs in the air, but how much it dies off between notes. Given the complex structure of any of the waveforms, there's really no unambiguous mathematical definition for these musically important concepts. The best I thought to do was to look at wave forms and estimate how long it took for any single pluck's sound to reach its maximum and how much it decayed in a typical time between notes. Even the concept of "maximum" is ambiguous when one looks closely at the waveforms at the highest possible time resolution.

Audacity's default sample rate is 441000 Hz. It is possible to view the recorded data in terms of individual samples, but a better idea of the sound comes from viewing at some much lower time resolution. Audacity has its own algorithms for mapping the data onto a graph with the available pixel size, and you can choose the time scale used for viewing.

To the ear, plucked note sounds often do not rise and then fall steadily. The net result of all this is that identifying a maximum amplitude is not unambiguous.

Nevertheless, I looked and gave it a try. TABLE III has the averages for the ten plucks for each rim of the time to perceived maximum, as viewed on the middling resolution waveforms shown in the upper part of FIG. 12. (The lower portion shows the first half of one of those waveforms at the lowest resolution that just barely shows the up and down variation that is unambiguous in the stream of time samples; in some sense, more data seems to get further away from what most brains convert to conscious impression.) Also tabulated is the percentage drop in amplitude from its value at maximum to its value at the end of the 0.25 seconds. Noting the time from the maximum to the end of the interval, I compute the rate of amplitude drop over that interval as a percentage per 0.10 seconds.

Admittedly, the numbers in TABLE III are rather subjective. On another day, I might have chosen differently. However, the associated physics stories likely have some merit.

rim	rise time – sec	amplitude % drop from max to 0.25 sec	% per 0.10 sec drop
11 ply	0.059	72 in 0.19 sec	38
3 ply	0.013	36 in 0.24 sec	15
cherry	0.076	45 in 0.17 sec	27
8mm	0.034	48 in 0.22 sec	22

TABLE III: attack & sustain – averages for 10 plucks each

B. physics perspectives

1. *rise time*

With the release of a pluck, the string starts applying an up-and-down force on the bridge, which, in turn, applies some version of that force to the head. And it's the head motion that moves enough air to produce sound. The amplitude rises quickly, but a lot is happening during the period from the start to the peak amplitude response. A disturbance in the head propagates outward from the feet of the bridge at the wave speed of the head. That disturbance encounters the rim and reflects back. As time goes on, more waves are created at the bridge, and there are more reflections. Moving on the same surface, these waves superpose. Constructive interference occurs to the extent that reflected waves are in

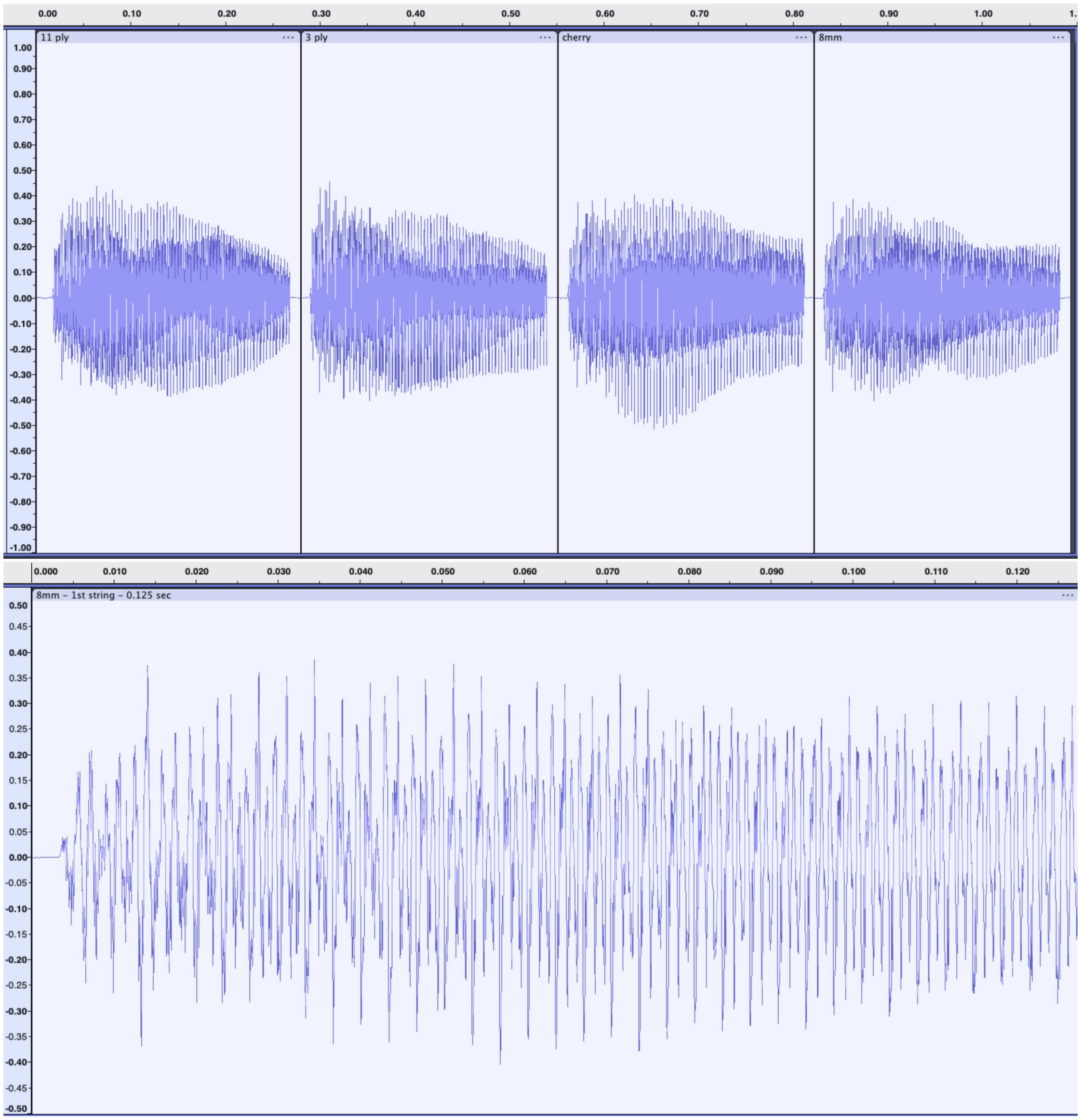

FIG. 12: wave forms of the first 0.25 sec of representative plucks AND the first 0.125 sec of the same pluck of the 8mm rim

phase with the waves generated by bridge motion. These total amplitudes contribute to a total that clearly has components that increase for early time.

That encounter and reflection of head waves at the rim is where rim variations play a role in the sound. If the rim doesn't move at all, the reflected amplitude of a given wave equals the incident amplitude. It is said that the reflection coefficient is 100%. Any rim motion will lower the reflection coefficient. And a lower reflection coefficient implies that the total amplitude will take longer to build up to its maximum value. As mentioned in section §IVA, rim motion in response to a given force is inversely proportional to its inertia. (Each rim mode has an effective inertial, but all are proportional to the total mass, and the proportionality constants will be the same for all rims with the same spatial mass distribution.) In conclusion, all other things being equal, a heavier rim produces a faster rise time.

Similarly, a stiffer rim will give a larger reflection coefficient because the rim will move less in response to a given impulse. This is born out by the first three rims (11 ply, 3 ply, and cherry), which have nearly the same weights but different stiffnesses (listed in TABLE II).

Something else must be going on with the much thinner 8mm rim, which is the lightest and most flexible of the four. The key is likely the much lower rim frequencies (as seen in FIG.s 1 & 2 and TABLE II). FIG. 7 is a reminder of the frequency dependence of the response of an oscillator to a driving force. In particular, driving frequencies far above the resonant frequency have a vanishing effect, while all driving frequencies far below resonance have a common, non-zero effect, as emphasized by plotting the standard resonance curve on a logarithmic scale of frequency, as in FIG. 13. Hence, head motion will be ineffective at moving the rim for head frequency components that are significantly higher than the relevant rim motion.

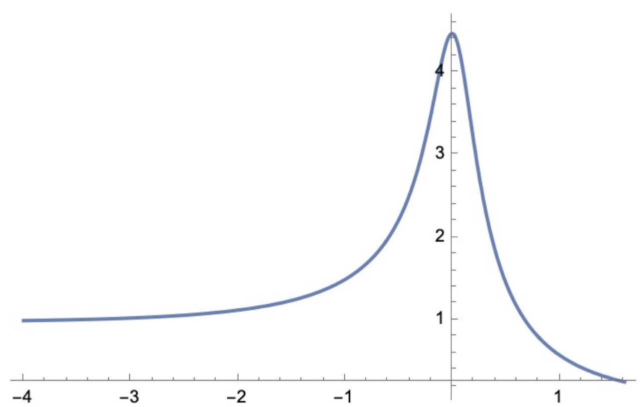

FIG. 13: the *exact same* function as in FIG. 7, plotted against the logarithm of the driving frequency, for which octaves are equally spaced

“Relevant” is also a crucial consideration here. It refers to the degree of spatial matching of the head and rim modes around the edge of the head and the top of the rim. The lowest rim modes presumably have four nodes roughly equally spaced around the circumference. These are best spatial matches to the third head mode, which also has four node lines ending at the edge. From FIG. 11, one might imagine that those head modes are around 500 Hz for all of the rims. The 9/16″ thick rims have their lowest modes between 300 and 400 Hz. However, the 8mm rim lowest mode is around 130 Hz. This makes the 8mm rim relatively unresponsive to head vibration.

Certainly, there is a series of higher and higher frequency rim modes. However, the first several, like the lowest one, do not match the higher frequency head mode geometries — in contrast to the thicker rims. Eventually, the head modes become significantly more dense as a function of frequency. So, inevitably there is some matching, but that may lie above the region of significant amplitudes and energies. A situation of this sort begs for a careful numerical analysis.

2. sustain

The sustain of a plucked string is the time appreciable vibration remains on the string. In ref. [1], the banjo's dominant loss of energy was shown to be sound radiation from the head. (In contrast, wood-topped instruments lose most of the input energy to heat in the wood.) Rims will differ in how much energy gets dissipated in the wood, with more dissipation resulting in less sustain. And the biggest difference results from how much they move once the repeated reflections of head waves establish a quasi-stationary amplitude. In rims with many laminations, the glue may well impact both stiffness and internal dissipation. Some such effect seems necessary accommodate the ordering of the numbers presented in TABLEs II & III.

C. a caution

Before looking closely at spectrograms of carefully controlled single plucks for each rim, it might be worth reflecting on the distance between perception of pitch and timbre and the results of mathematical analysis of digitized recordings. Perception evolved to help us navigate the everyday world successfully. And we learn to correlate our perceptions with those of other folks so that we can get on with our lives.

But it's clear that there are significant differences in people's hearing acuity as a function of frequency – even among "normal" hearers. For pitch, we make reference to an identified sinusoidal signal of definite frequency. But it is possible to hear a definite pitch when there is no sinusoidal component with the corresponding frequency. As noted earlier, in banjo head tap tuning, the frequency components of the sound are not even multiples of the perceived pitch.[4] As noted in section §V[5], there are complex sounds with definite pitches where people disagree which is the higher note. And in the domain of timbre, there is neither any agreed upon or standard terminology nor is it likely that people all perceive sounds in the same way.

Spectrograms display some aspect of the amplitudes of a sound's various frequency components as a function of time. Physics has a chance of connecting with those features. But I don't think we understand much of what different people perceive. This is particularly vexing if one wants to understand subtle differences.

D. pluck spectrograms

Fig.s 14 & 15 are spectrograms of the plucks on the open 1$^{\mathrm{st}}$ string, 14$^{\mathrm{th}}$ fret featured in the sound files on p. 1 & 17. The equally spaced, strong features are harmonics of the D(294). Note that the lowest ones (at 294 Hz) are very similar and are devoid of much extra stuff. This is a consequence of what is shown in FIG. 10. On their own, the heads with strings damped do not particularly want to vibrate at that frequency. They will, only if the vibrating string and bridge insist.

Since the heads and their tensions are virtually identical, their vibration characteristics differ because of the coupling to the top of the rims. And it is the coupled rim-head systems that respond differently to the forcing from the strings. (The pluck also initiates vibration in the other open strings, which produce weaker partials, some of which are distinctly inharmonic.[6]) The higher harmonics excite rim-head modes to the extent that they are nearby in frequency and willing to move.

This is what makes the rims sound different.

Repeat: This is what makes the rims sound different. And it is complicated.

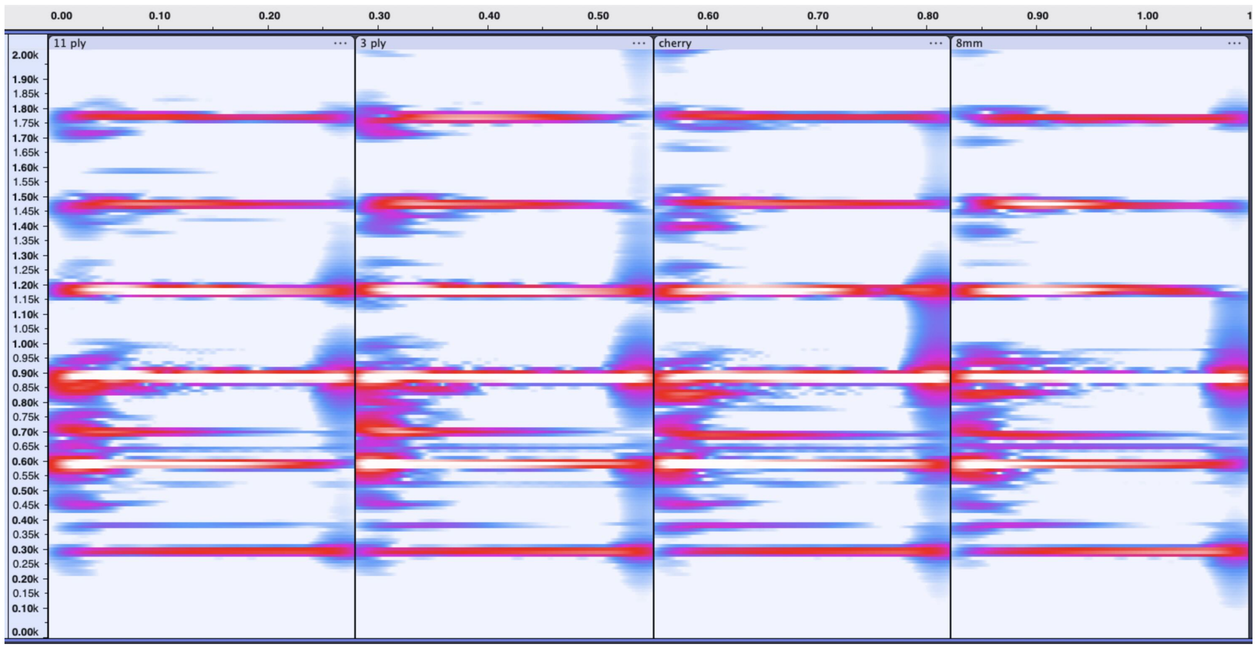

FIG. 14: a representative pluck of one of the ten for each rim in the sound file on p. 17 and in FIG. 15.

VIII. PLAYED TUNES

Recordings are given below of a tune plucked fingerstyle with each rim and another tune frailed. The playing technique and recording set-up were as identical as I could manage. However, they were separated in time to allow moving the neck and doing the bridge taps and single string plucks.

An apology: Listening turned up loud and using my hearing aids for this write-up, I was a bit appalled by more fret noise than I would have liked. I normally play without the hearing aids. So, the noise originally went unnoticed.

"Better" is clearly a matter of taste. Overwhelmingly, listeners respond to musicianship rather than subtleties of timbre. But many players imagine that hardware is the secret sauce that will lift their playing closer to perfection. Those goals are most often emulation of admired players and neglect the extent to which technique and musicianship were really the basis of the admiration.

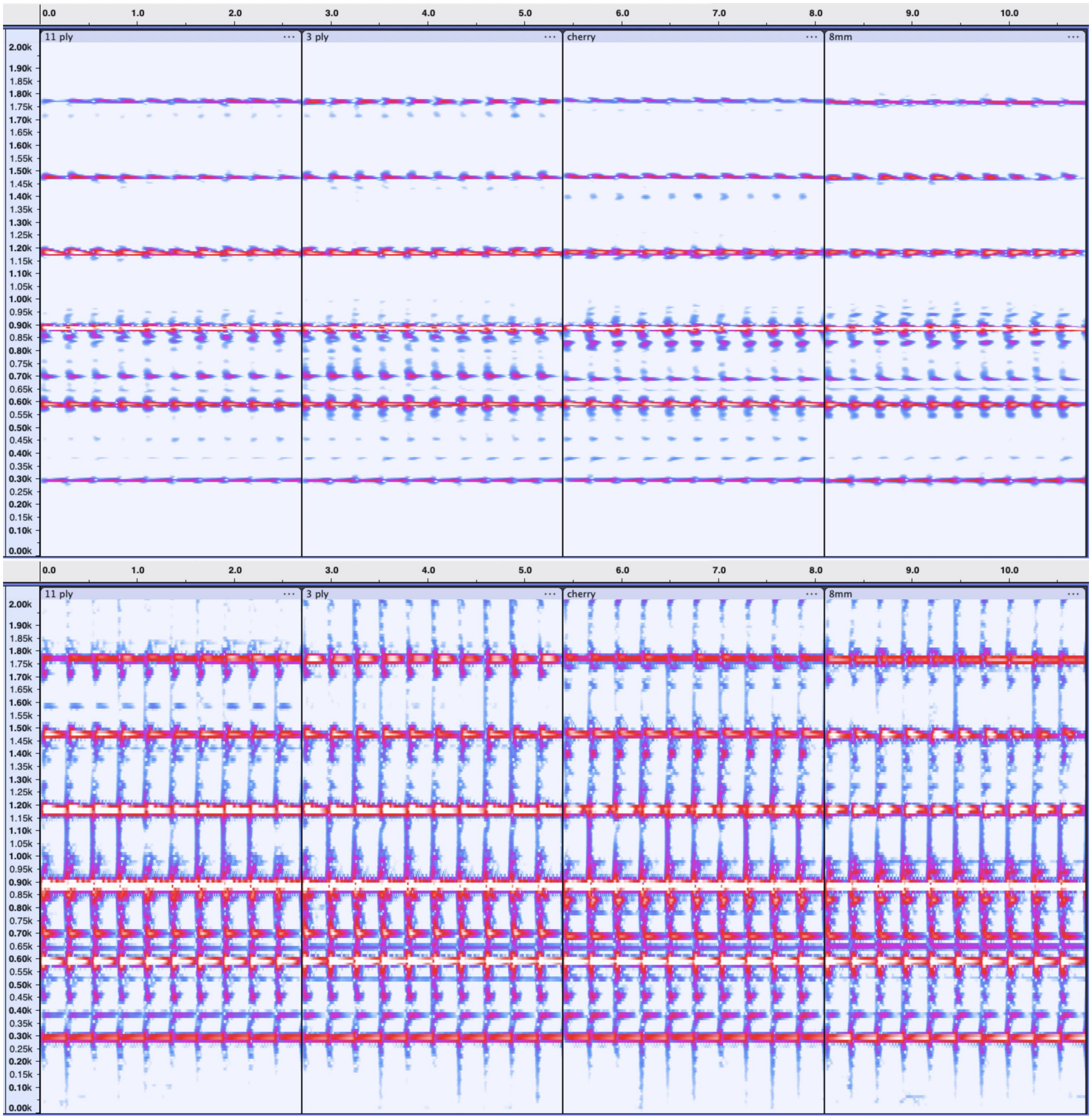

FIG. 15: two sets of spectrograms for the first 0.25 seconds of 10 plucks on the 1st string for each rim, displaying the same data but with a different map of amplitude onto color, thus highlighting different ranges of the computed spectrogram.

A. plucked fingerstyle

https://www.its.caltech.edu/∼politzer/rims/tunes/11ply-charlie-II.mp3

https://www.its.caltech.edu/∼politzer/rims/tunes/3ply-charlie-II.mp3 https://www.its.caltech.edu/~politzer/rims/tunes/cherry-charlie-IV.mp3

https://www.its.caltech.edu/~politzer/rims/tunes/8mm-charlie-II.mp3

B. frailed

https://www.its.caltech.edu/~politzer/rims/tunes/11ply-susana-IV.mp3

https://www.its.caltech.edu/~politzer/rims/tunes/3ply-susana-II.mp3

https://www.its.caltech.edu/~politzer/rims/tunes/cherry-susana-IV.mp3

https://www.its.caltech.edu/~politzer/rims/tunes/8mm-susana-V.mp3

C. ID challenge

These are alternate takes of the same selections without the rim labels — to see to what extent they can be identified by their sound:

1. plucked fingerstyle

https://www.its.caltech.edu/~politzer/rims/ID-challenge/charlie-I.mp3

https://www.its.caltech.edu/~politzer/rims/ID-challenge/charlie-II.mp3

https://www.its.caltech.edu/~politzer/rims/ID-challenge/charlie-III.mp3

https://www.its.caltech.edu/~politzer/rims/ID-challenge/charlie-IV.mp3

2. frailed

https://www.its.caltech.edu/~politzer/rims/ID-challenge/susana-I.mp3

https://www.its.caltech.edu/~politzer/rims/ID-challenge/susana-II.mp3

https://www.its.caltech.edu/~politzer/rims/ID-challenge/susana-III.mp3

https://www.its.caltech.edu/~politzer/rims/ID-challenge/susana-IV.mp3

The answer key is in Appendix A.

IX. CONCLUSION

Taps on the rims and assembled pots highlight the differences that impact how the banjos will sound. Carefully controlled single plucks of the assembled banjos demonstrate those differences. Some connections to the physics underlying the differences are simple while others are complicated. I can attest to significant differences for the player. Identifying differences by the listener is up to you.

X. HISTORICAL CODA

These four rims were sufficiently different in construction that the consequent differences in sound were discernible. However, the three $9/16''$ thick ones were *very* similar, and the thin one performed similarly. This is no accident. Musical instruments undergo something akin to Darwinian evolution over the years. Slight variations are widely adopted if deemed to improve the sound. The instruments don't generally tend to perfection because the context changes. For animals, the environment changes. For instruments, taste in music changes. Radically different rims would have shown greater variation but would have been of less interest to players generally.

Appendix A: answer key

The answer key is here: [7]

Appendix B: cylindrical shell normal modes

In vibration and material failure studies, "shell" is used to refer to something that is thin in the dimension of its motion and substantially extended in the perpendicular directions. A flat shell is a plate. And an effort is made to approximate the full 3-dimensional physics to approximate equations for transverse waves on a 2-dimensional surface. So, the thin dimension is treated very differently. The different 2-D wave equations are various compromises between accuracy and utility. The approximation for banjo heads is the simplest and most effective. The tension is so high and the wave amplitude so small that stiffness and other material properties can be safely ignored. The soundboard of wood-topped instruments requires serious work, both in finding a reasonable approximation and in actually using the resulting equations to analyze the motion.

Cylindrical shells are of interest here. Actually, there are many real-world situations where the vibrations, structural integrity, and failures of cylindrical shells are of practical importance. The existence of a rich literature in applied math and engineering for the past century attest to the fact that there is nothing simple about the problem.

For small amplitude vibrations of a thin shell, linearized theory determines normal modes, i.e., motions of a single, definite frequency throughout. These can be distinguished by their node lines. If the cylinder is the obvious straight, circular type, a given mode can have set of node lines that include some combination of circles going around and an even number of straight lines perpendicular to the circles that are equally spaced around the circle.

Rayleigh distinguished extensional and inextensional (aka flexural) cylinder motions. The issue is whether the motion involves bending relative to equilibrium or requires stretching along the whole length. This is unambiguous for vibrations of a straight board but somewhat ambiguous for a cylinder. However, if the cylinder is thin enough, flexing should be easier than stretching (even though flexing obviously requires stretching and compressing).

For a thin, squat cylinder, it seems reasonable to imagine that the lowest modes are flexural and do not have any circular node lines.

The lowest purely flexural mode will have four node lines. If suspended as described in section §IV, gravity breaks the rotational symmetry and determines the node locations. In particular, the suspension point lies on one of the four node lines. And 45° to either side are locations of maxima of motion. Hence, those are good places to tap and listen for the lowest mode.

Images of visualizations of actual drums might help the skeptical.[8]

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[1] J. Woodhouse, D. Politzer, H. Mansour: a technical exposition is available as open-access as *Acoustics of the Banjo: measurements and sound synthesis & theoretical and numerical modeling*, Acta Acustica, **5**, 15 and 16 (2021) https://doi.org/10.1051/aacus/2021009 and https://doi.org/10.1051/aacus/2021008); *Pickers' Guide to* Acoustics of the Banjo, HDP: 21 – 01, http://www.its.caltech.edu/∼politzer – APRIL 2021 is an informal account of some of the

salient results.

[2] *Four Banjo Necks and One Pot*,
https://www.its.caltech.edu/∼politzer/neck-wood/neck-wood.pdf

[3] *Inharmonic Partials and Banjo Ring*,
https://www.its.caltech.edu/∼politzer/banjo-ring/inharmonic-partials-banjo-ring.pdf

[4] A common method of tap-tuning the tension of a banjo head relies on such an identification of spectrum with pitch. See the Pickers' Guide Addendum, p. 17, cited as ref. [1].

[5] Psychoacoustics expert Diana Deutsch pioneered conclusive demonstrations of this phenomenon, most decisively and intriguingly in what she calls the "Tritone Paradox," described in *Musical Illusions and Phantom Words* and elsewhere.

An August 2024 edition of Veritasium (*These Illusions Fool Almost Everyone*), the YouTube science channel, begins with two sounds, and the host/producer Derek Mueller says, "To me, sound A is clearly higher." The comments section is full of people who heard B as higher.

[6] See *Inharmonic Partials and Banjo Ring* and *Supplementary Recordings of Banjo and Resonator Guitar* at
shttps://www.its.caltech.edu/∼politzer/banjo-ring/inharmonic-partials-banjo-ring.pdf and
https://www.its.caltech.edu/∼politzer/more-partials-mp3s/more-partials-mp3s.pdf

[7] Fingerstyle selections in order as labeled: I:3ply,II:8mm,III:cherry,andIV:11ply. Frailed selections I:cherry,II:3ply,III:11ply,andIV:8mm.

[8] e.g., T. Rossing, *Science of Percussion Instruments*, World Scientific Publishing (2000)