Inharmonic Partials and Banjo Ring

David Politzer*

California Institute of Technology

(Dated: March 9, 2024)

Banjo sound is rich in inharmonic partials, i.e., strong frequency components of a plucked note that are not integer multiples of the pitch frequency. Their origins are identified here by experiments that reveal how their amplitudes can be reduced. And the results are as simple as possible: inharmonic partials overwhelmingly come from the vibration modes of the head and the other strings. The sudden bridge disturbance at the onset of a pluck produces the sound of a gentle head tap and a soft version of the sound of the other unplucked strings. Rim vibrations are also identified but do not play a significant roll in this aspect of the sound. It is suggested that inharmonic partials are the long-conjectured source of the ring of the banjo. With the origin of those partials established, the connection to “ring” becomes a question of psychacoustics.

---

* politzer@theory.caltech.edu; http://www.its.caltech.edu/∼politzer; 452-48 Caltech, Pasadena CA 91125

Inharmonic Partials and Banjo Ring

I. BACKGROUND

Woodhouse et al.[1] applied the simplest physics of the basic banjo parts (strings, bridge, head, and air) to give an account of basic banjo sound: compared to guitar, say, banjo plucks are relatively loud and short and have substantial frequency content. Much of the focus was on how string vibration is converted into sound *on average,* i.e., over intervals containing many individual resonances. One subtlety observed in actual sound recordings but not extensively addressed was the presence of inharmonic partials, which are rather stronger and longer-lived than on guitar. It had been suggested that these are what makes a banjo "ring," but their physical origin remained elusive.[2]

"Ring, Ring the Banjo!" was Stephen Foster's imperative — the title and lyric of a song he published in 1851.[3] And banjos have been ringing ever since.

Years ago, my ear picked up an extreme example of ring that can be heard sometimes in three-finger picking. After much careful listening, I identified the source. Occasionally, the 1st string is used as a drone, in addition to the 5th, and melody and harmony are played on the three strings between. With three fingers plucking eight notes per measure, there is a natural syncopation, putting the drones and melody notes in different places in different measures. In some people's playing, that 1st string drone can really stand out. As a drone, repeated again and again, it sounded like a cold chisel, struck by a ball-peen hammer. I dubbed it "clang."

Here are the sounds of three actual cold chisels and, in FIG. 1, spectrograms of those sounds:

http://www.its.caltech.edu/∼politzer/banjo-ring/cold-chisels-three-singles.mp3

A. the physical origins

The physical origins of the inharmonic partials are systematically identified by investigating ways to suppress or at least reduce them. The answers turn out to be as simple as could be. The longer-lived ones are harmonics of other strings that do not match the harmonics of the plucked string. (The other strings' harmonics that do match contribute to

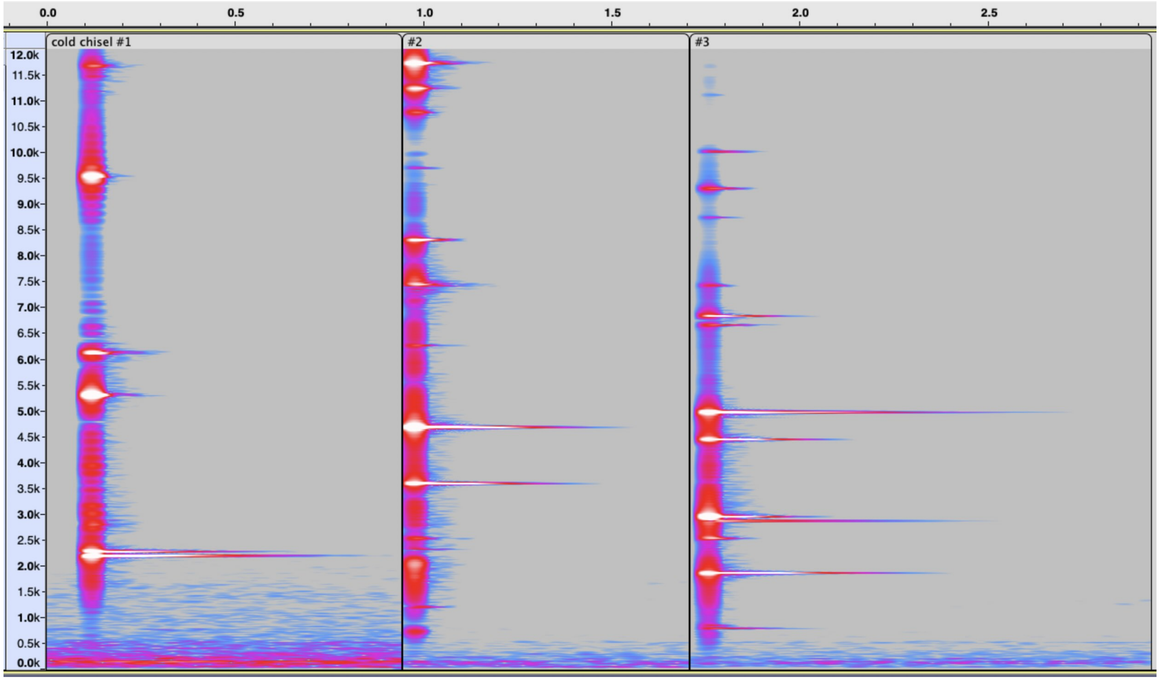

FIG. 1: cold-chisels-three-singles

the phenomenon of sympathetic string vibration.) There is inevitably some dissonance, with the severity depending on the pitches at that time of those strings as fretted or open. The short-lived ones are resonances of the head itself. That's somewhere between the "ping" of a taut mylar head and the "boing" of a looser skin head. In retrospect, this is pretty obvious. The builder and player has some control over the size of these effects, and, naturally, people differ in their preferences.

B. metallic sound

In general, a metallic sound need not have been produced by some vibrating metal. Rather, as identified by John Chowning[4], there must be substantial, long-lived frequency components (at least longer than a thud or clonk) that are not integer multiples. We learn to identify objects by their sounds, and metal objects produce an enormous variety. But it need not be metal to sound like it.

C. Banjo Ring!

In addition to canonical integer-multiple string harmonics, plucked banjo strings produce accompanying sounds much like those analyzed in FIG. 1. FIG. 2 shows representative spectrograms contrasted with a typical flat-top acoustic guitar. On the left is the Deering Vega White Oak 12″; on the right is the Hartel Ashborn replica; and in the center is a Simon & Patrick (Godin) flat-top acoustic guitar. The guitar is tuned CGCGCD — to approximate the string harmonics of the double-C banjo tunng. The Hartel/Ashborn is tuned similarly but lower. Overall amplitudes are adjusted to be of comparable volume, while microphone placement is a matter of apples & oranges — but all at 20″. The instruments are quite different.

And the sounds in the same order are

http://www.its.caltech.edu/∼politzer/banjo-ring/Vega55percent-SP100percent-mic-moved-Hartel-Ashborn-1st-singles.mp3

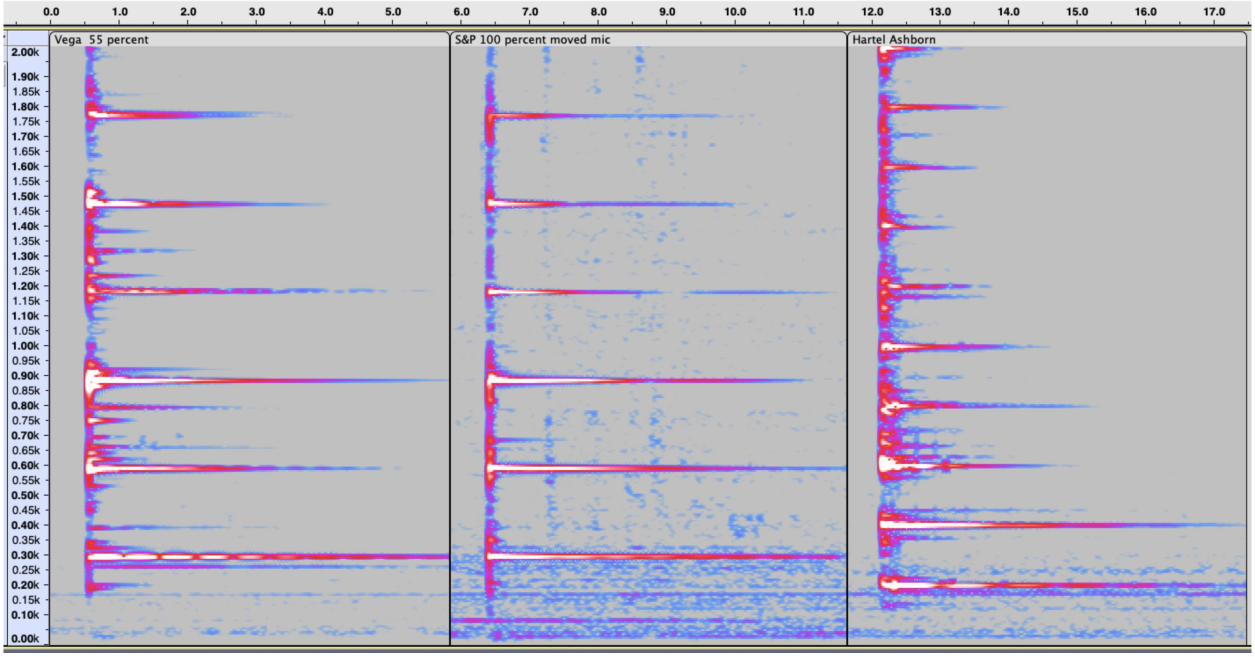

FIG. 2: 1st string plucks on a Deering Vega White Oak 12″, a guitar, and a Hartel Ashborn 1850's reproduction (a played selection is linked in ref. [3])

The *harmonic* content of the sounds are the strong, equally spaced components, all integer multiples of the pitch frequency, e.g., the plucked 294 Hz D string. Banjo ring or clang is presumably everything else.

II. INHARMONIC PARTIALS FROM THE STRINGS

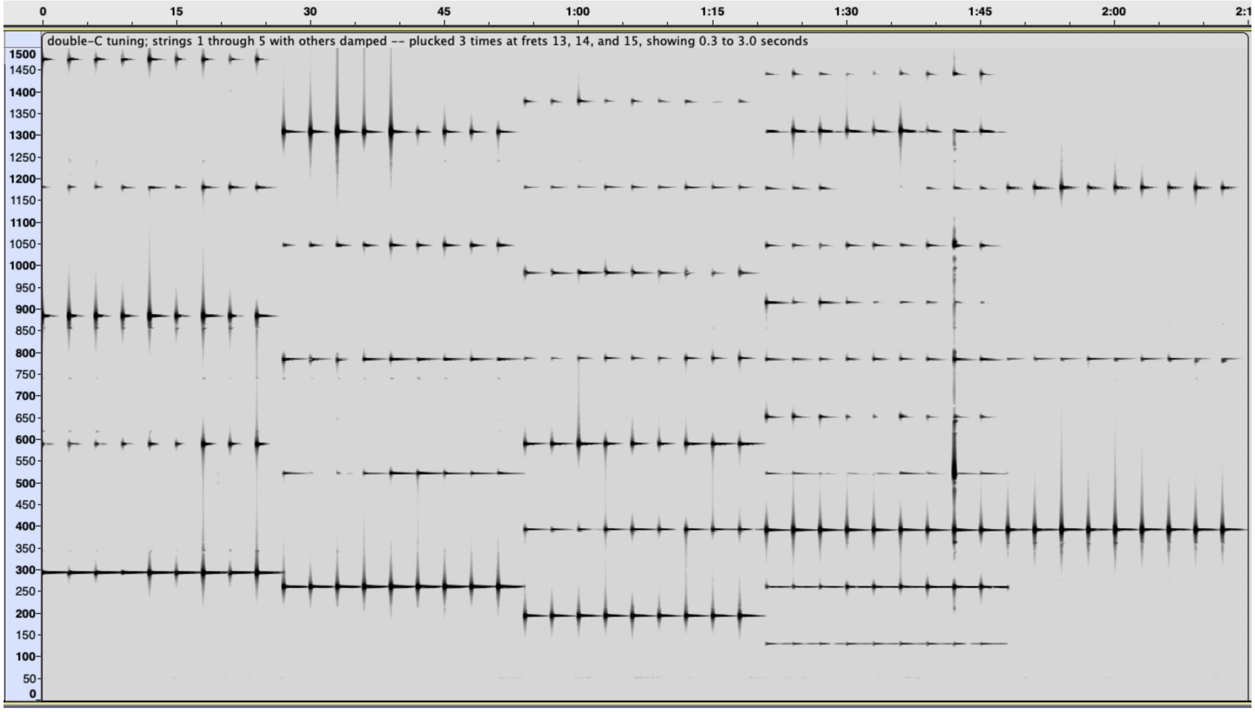

FIG. 3: Spectra of each open string in double-C tuning with other strings damped, each analyzed from 0.3 to 3.0 seconds; left to right: 1st through 5th string, i.e., D, C, G, C, g

FIG. 3 is a graphic display of the five individual strings' harmonics in double-C tuning. In the spectrograms that follow, it is the 1st string that is plucked. Among the other strings, there are harmonics that match the 1st string's and ones that don't.

When the 1st string is plucked, the sudden bridge motion excites all the other strings, albeit with much smaller amplitude than the plucked string. The inharmonic motions (relative to the plucked string) are "free" decays with Q's (i.e., roughly the number of oscillations before decaying to negligible) that are common to any string. However, because of the small initial amplitudes, in practice they die away much sooner than those of the plucked string.

The string motions that are harmonic relative to the plucked string are also driven by the vibrating plucked string. The frequency matching modes actually exchange energy back and forth. These contributions continue as long as the plucked string vibrates. This is a more general phenomenon than "sympathetic" strings, as on the lute, sarod, sitar, etc., where the extra strings are tuned to the same pitch as the played strings.

This mechanism is clearly demonstrated by comparing the pluck sound with all strings open to that with all other strings damped — as shown in FIG. 4.

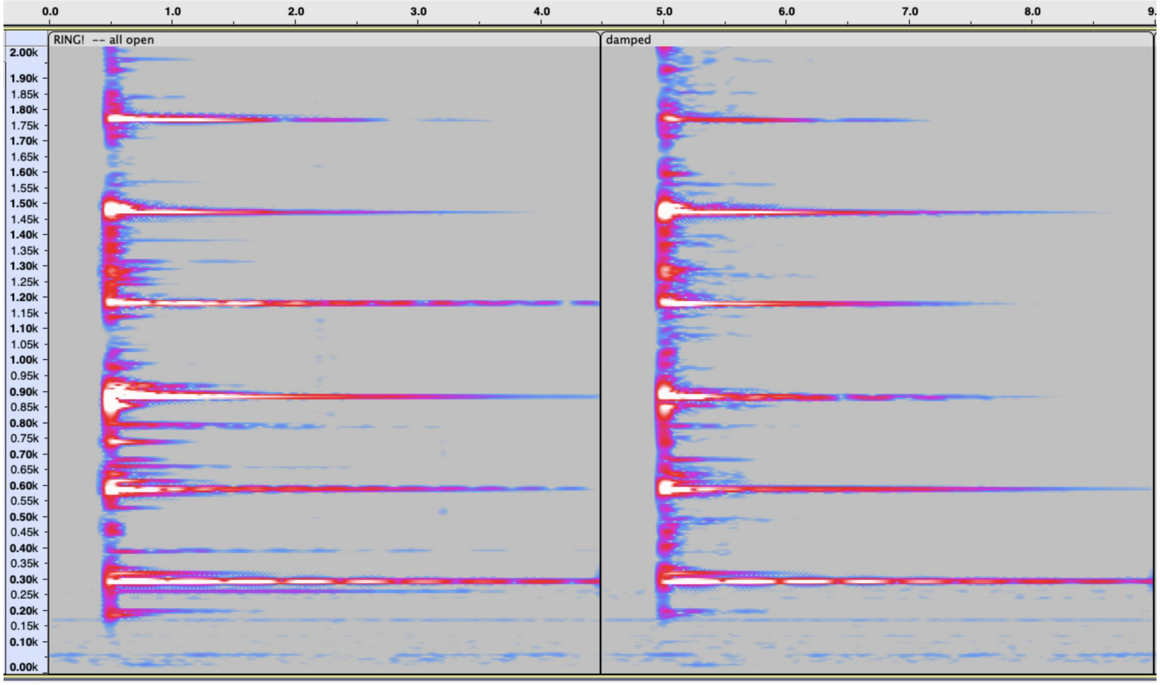

FIG. 4: 1st string pluck with others open *vs.* damped

This is a general phenomenon. It will occur to some extent for any tuning and for any fretting of the unplucked strings.

IMPORTANT NOTE ON SPECTROGRAMS & SOUND FILES:

In FIG. 4 and all subsequent spectrograms not labeled otherwise, in a given spectrogram, the same banjo is plucked on the 1st string at the 14th fret. The pluck is performed by looping #42 gauge magnet wire around the string and pulling sideways until it breaks. The wire breaking tension determines the force of the pluck. The differences from pluck to pluck with a given configuration are negligible compared to the differences highlighted in the spectrograms and sound files.

III. INHARMONIC PARTIALS FROM THE HEAD

Two strategies help establish that the short-lived, strong, inharmonic partials are normal modes of the head. One approach is to vary the head tension and observe the impact on the partials' frequencies. A Nechville banjo, with its shallow pitch, helical tensioning mechanism facilitated the operation. The wrenches in FIG. 5 are used to turn one side of the helical thread against the other and tighten down the head. Head mode frequencies increase monotonically with tension.

FIG. 5: Nechville pot with wrenches

The second strategy involves partial damping of the head modes. The method was inspired by an insanely clever device invented, manufactured, and sold by Ric and Deb Hollander. They call it the Banjo Bolster™ (http://www.banjobolster.com). Essentially, it's a fiberfill sausage in a thin cotton poplin casing. About 3″ in diameter and half-the pot circumference in length, it lies along the inside of the rim, tucked between the two ends of the dowel stick or co-rods. Importantly, it *does not* touch the head. It is light enough and stiff enough that it just sits there firmly, without needing to be secured further.

The Banjo Bolster™ has an enthusiastic following of people who say that it dramatically reduces "unwanted" overtones without significant reduction in anything else. Close examination of its performance revealed that the reduced overtones are overwhelmingly the short-lived inharmonic partials. Furthermore, the effect of varying head tension confirmed the connection to head resonances. The magic of how the Banjo Bolster™ damps head mode sounds and not the more musical ones and how it differs significantly from any traditional form of pot stuffing is discussed in section §VII.

The Nechville does not have co-rods. So, I opted for my own creation: a pancake-shaped poplin shell. A further bit of the Banjo Bolster™ magic is its efficiency. Installing a second one in the other half of the pot yields only a minor increase in the damping efficiency. And filling the whole pot with a fiberfill pancake, only does a little more.

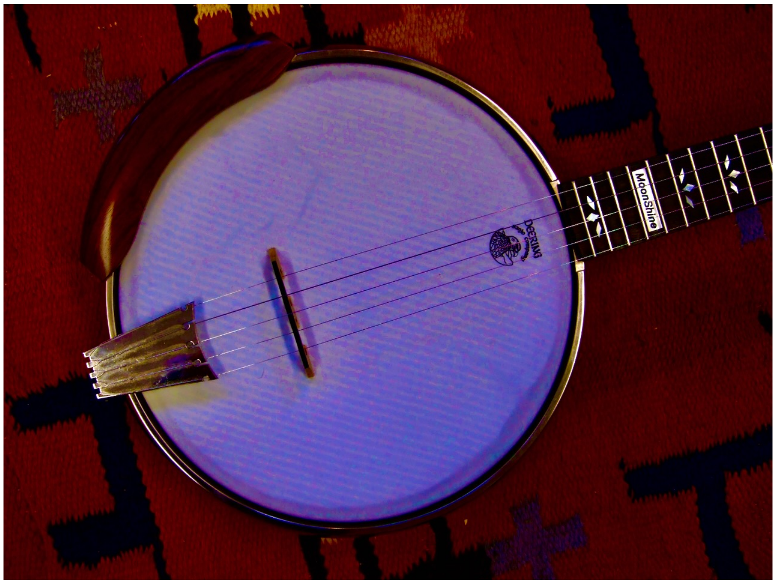

FIG. 6: stripes,visible with the naked eye, reveal damper contact with head — to be avoided when recording sounds; photo is color-intensified for print version

The fiberfill pancake shell had stripes, which became visible when the fabric touched the head. That is shown in FIG. 6. The desired installation features no contact. So, stripe appearance was a useful diagnostic.

FIG. 7 shows the effect of increasing the head tension in fourteen steps from a DrumDial reading of 80 to 91. These are 1st string plucks with no form of damping. The first and last columns are the sound of bridge taps with all strings damped at 80 DD on the left and 91.5 DD on the right. The fourteen string plucks are all identical wire-breaks. The two bridge taps are not particularly normalized.

Although not always unambiguous, clearly some partials increase steadily with head tension and others are independent.

In FIG. 8, half of the tension settings with no damping are paired to their immediate

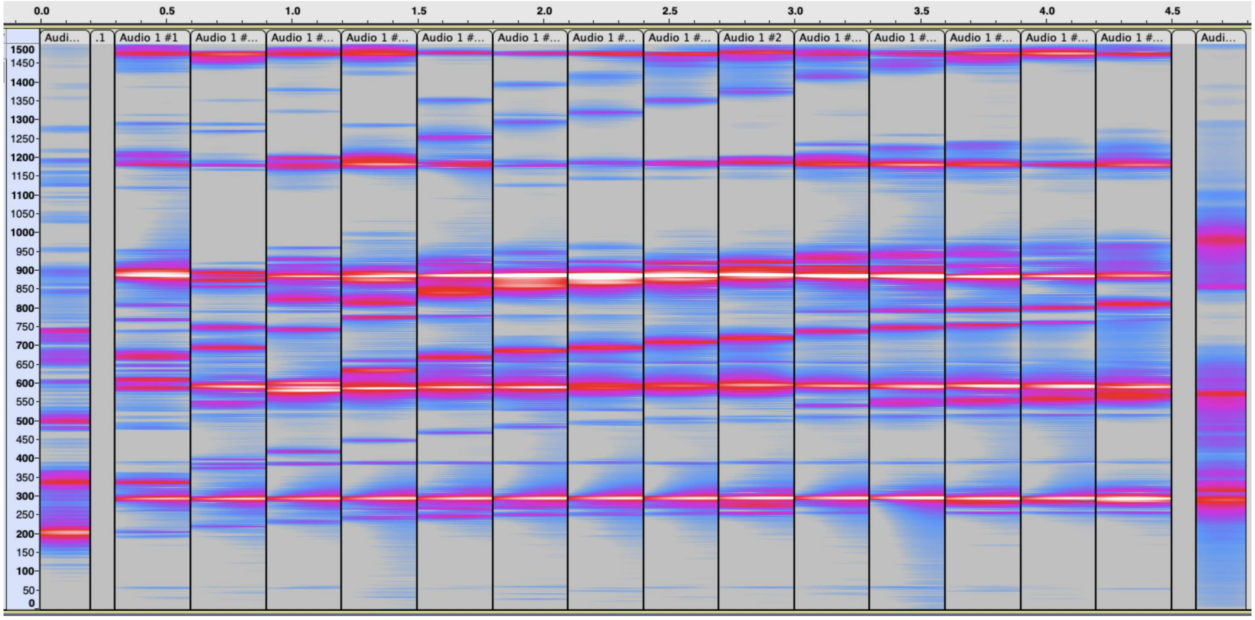

FIG. 7: left- and right-most columns are bridge taps at 80 and 91 on the DrumDial with all strings damped; the 14 columns in between are 1st string plucks with no damping for tension readings between 80 and 91 DD.

right with the sound of the same tension with the fiberfill pancake installed. The pancaked columns show the reduction in the strength of the partials that increase with head tension. To be sure, there are some other reductions as well.

IV. THE SOUND OF THE INHARMONIC PARTIALS

FIG. 9 summarizes the investigation so far. Plucking the 1st string on a Deering Vega White Oak 12″, the first column has no damping, the second column is with damping of all other strings, the third column is with the Banjo Bolster™ installed but all strings open, and the fourth column has both Bolster and damped strings. The two forms of damping virtually eliminate all inharmonic partials with any perceptible sustain.

And this is the sound of those plucks in the same order:

http://www.its.caltech.edu/∼politzer/banjo-ring/open-damped-bolstered-bolstered-and-damped.mp3

All are unmistakably banjo plucks. As mentioned previously, the Bolster has an enthusiastic, faithful following. Plucking strings while the others are damped is not really a feasible style of playing.[5]

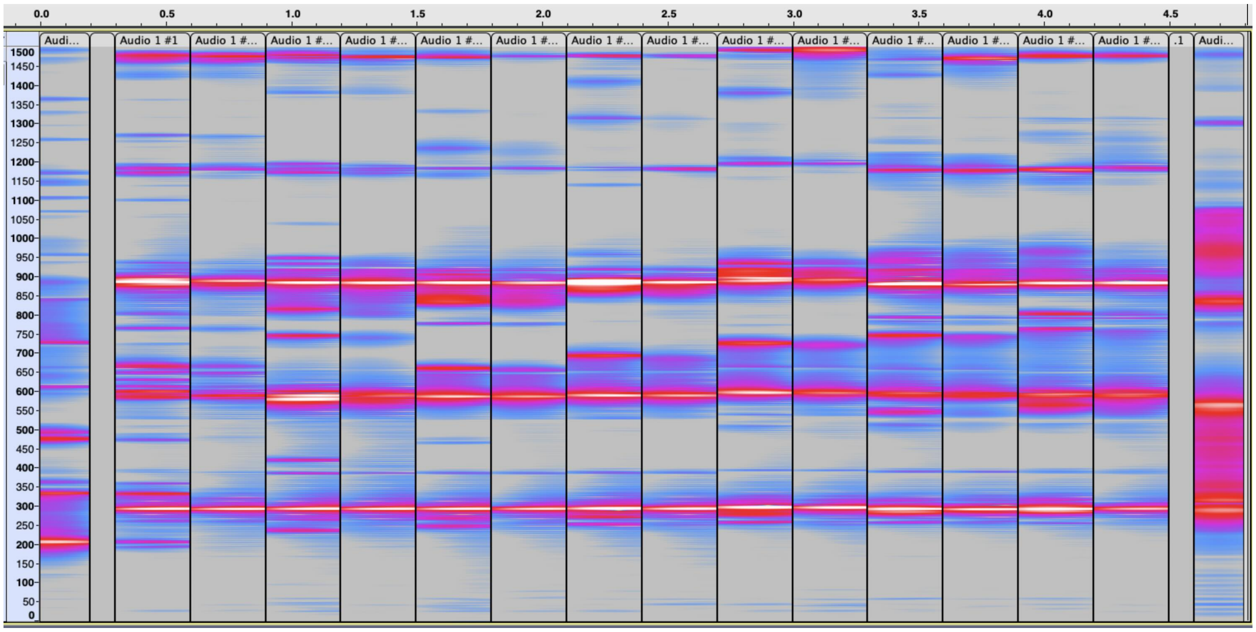

FIG. 8: half of the tension setting sounds are paired to their right with the same tension with the fiberfill pancake installed.

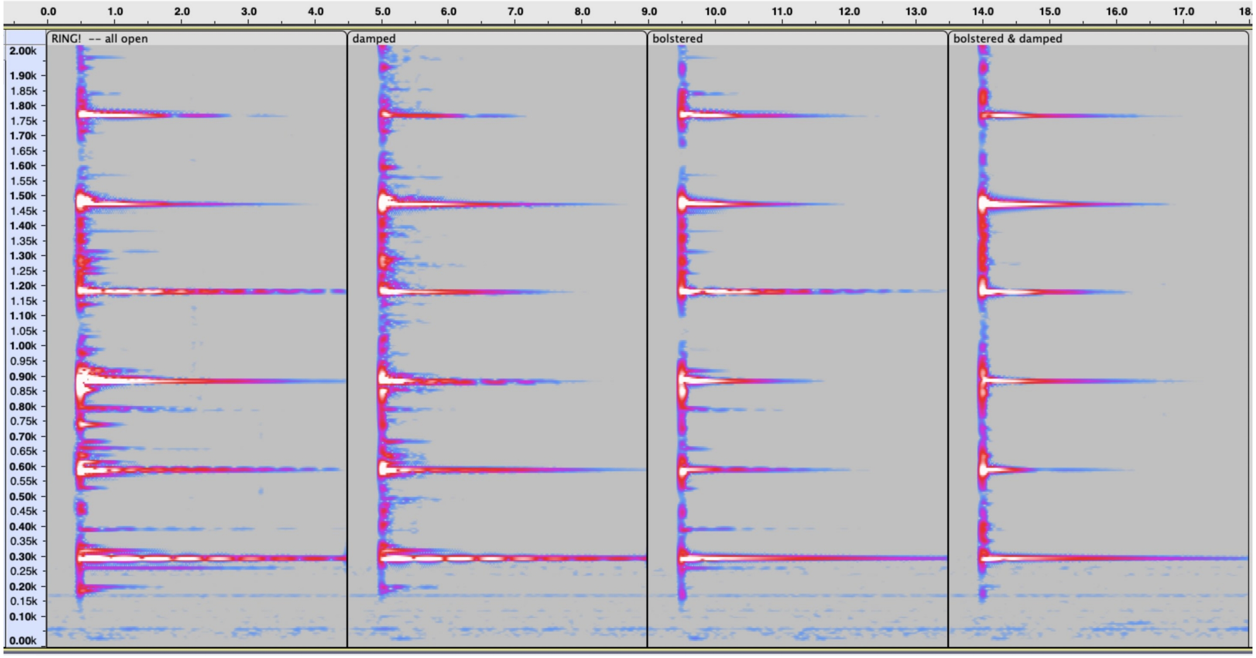

FIG. 9: 1st string plucks with no damping, with other strings damped, with a Banjo Bolster™, and with Bolster and strings damped

V. MATHEMATICA SYNTHESIS OF CLANG PARTIALS

In a somewhat related endeavor concerning resonator guitars[6], an attempt to synthesize the sound of the inharmonic partials proved to be an enlightening surprise. The idea is to match the form of the partials of an actual instrument with the sounds of functions constructed on a computer. I used Mathematica. Sine waves of a given frequency and amplitude are multiplied by an appropriate rise function and a decay envelope. Frequency, amplitude, and decay time are the crucial parameters. My method is crude and cumbersome, and making a closer match would require iterations. But a precise match is not the point. In real playing, the details will differ. The goal is to capture qualitative aspects of the partials. The inharmonic partials of resonator guitars are *much* stronger than those of flat-top acoustic guitars, and, on their own, they sound like gongs.

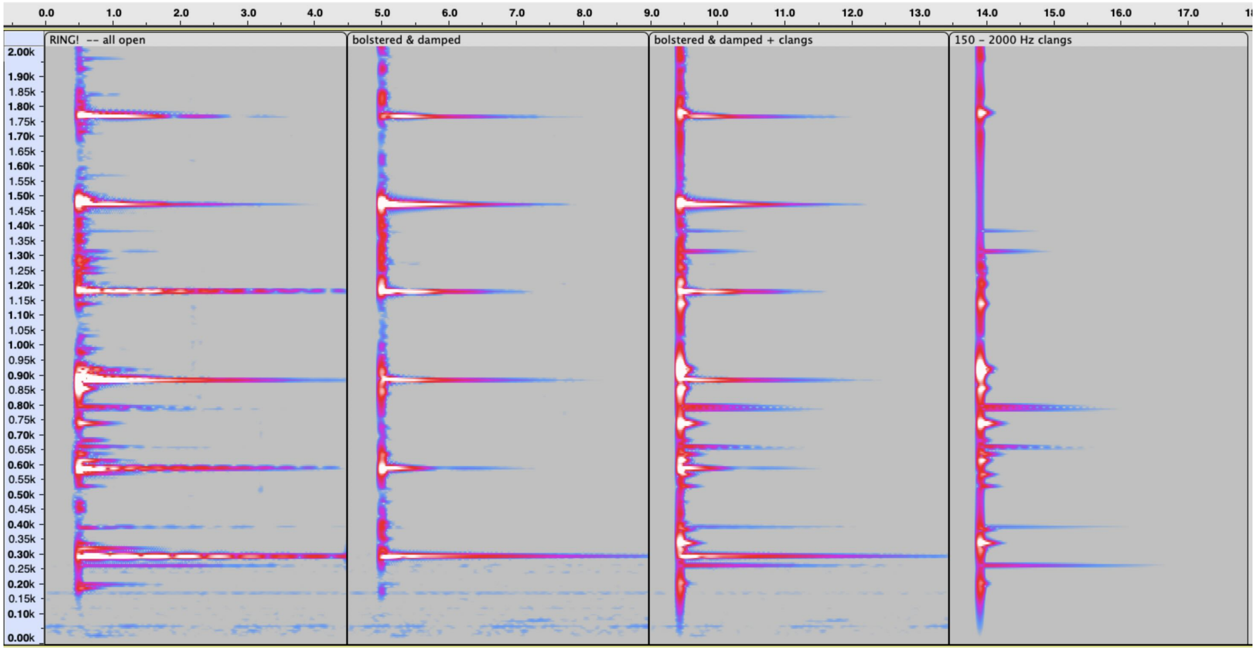

FIG. 10: 1st string plucks with no damping, with other strings damped and a Banjo Bolster™, bolstered and damped *and* synthesized inharmonic partials, and the synthesized sounds by themselves.

The result is displayed in FIG. 10. And here are the sounds of those four: click here or type: http://www.its.caltech.edu/∼politzer/banjo-ring/open-bolstered&damped-bolstered&damped+clangs-clangs-alone.mp3

The synthesized sounds do not include the *harmonic* partials of the unplucked strings.

Hence, adding only inharmonmic synthesized sounds to the bolstered and damped version lacks much of the "fullness" of the all-open sound that is provided by beats and other aspects of minor mistunings. The longer-lived components of the synthesized sounds come from the damped string partials that do not match the harmonic partials of the plucked string, which is tuned to D. Taken together, it is not surprising that they sound like a C because those four strings are tuned gCGC.

As mentioned previously, for any tuning and for any fretting of the strings, the unplucked strings will produce string-like, high Q partials that do not match those of the plucked string.

VI. RIM VIBRATION

Players and builders know that the nature of the rim has an impact on the instrument's sound. So, it is reasonable to ask whether rim flexing has an effect on the specific features observed so far, perhaps as a connection between the head and the air in the pot. A method was devised to add substantial mass to a rim so that the sound could be compared with and without that extra mass on the same banjo. In particular, a Deering "Basic"[7] has a steel rim, and magnets will adhere without rattling or buzzing when played. About 14 lbs worth fit around the rim — as shown in FIG. 11.

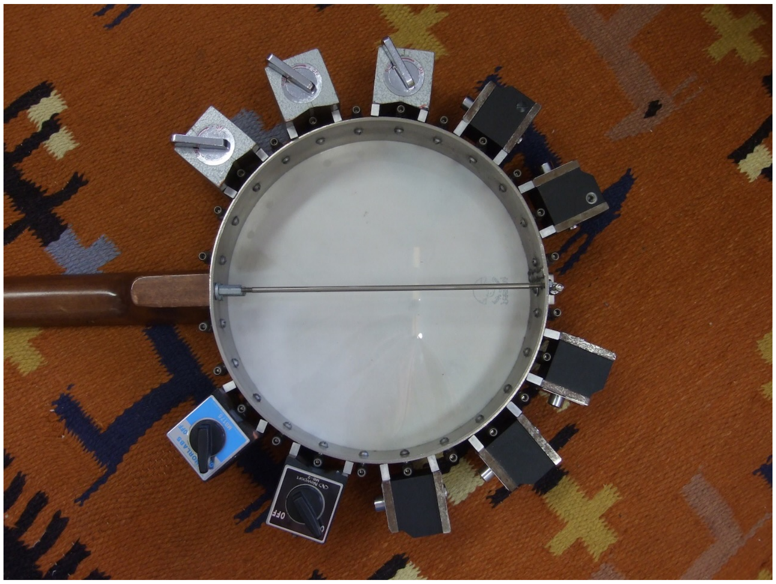

FIG. 11: an extra 14 lbs of magnets attached to the steel rim

The procedure was to tap on the bridge with the strings damped and record the signal from a piezo disk attached to the outside of the rim with double-sticky tape. FIG. 12 shows spectrograms of four taps with rim unweighted and weighted for the frequency range under investigation. Note that the overall amplitude of one set compared to the other was modified to highlight the frequency dependence of the amplitude rather than its absolute value, which was hard to control. Naturally, the weighted rim moved less overall.

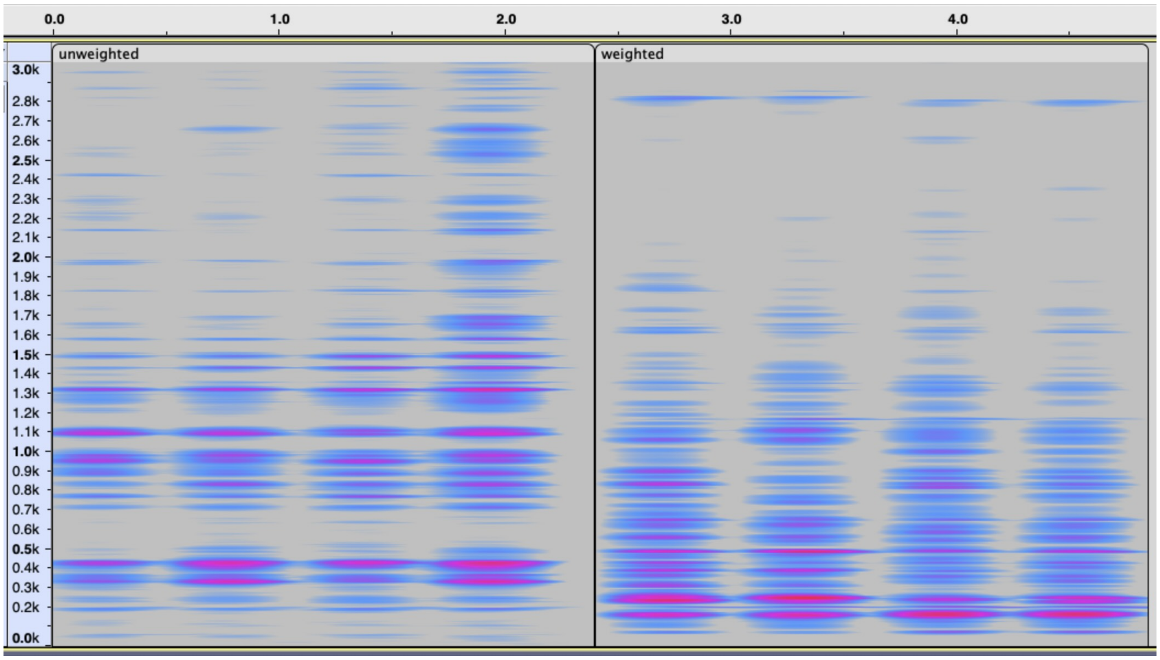

FIG. 12: spectrograms for 0 to 3000 Hz of the surface vibration of the rim, unweighted and weighted, for taps on the bridge with strings damped

In each case, there is a pair of lowest resonances separated by about 25%, with the weighted pair about a factor of 2 lower than the unweighted one. These are likely flexing modes in the radial direction with four nodes, and the rotational degeneracy is split by the co-rod.

4th string plucks were first chosen to investigate the possible impact of this flexing because the 4th string has the most modes in the region of interest. However, no difference due to the weighting was observed between 0 and 2000 Hz in the type of analyses presented in the previous discussion of inharmonic partials. Likewise, 1st string plucks revealed no particular difference either.

If, instead, one looks over a wide range of the banjo spectrum, e.g., 0 to 12,000 Hz, the piezo on the rim picks up broad frequency regions where the rims differ, shown in FIG. 13.

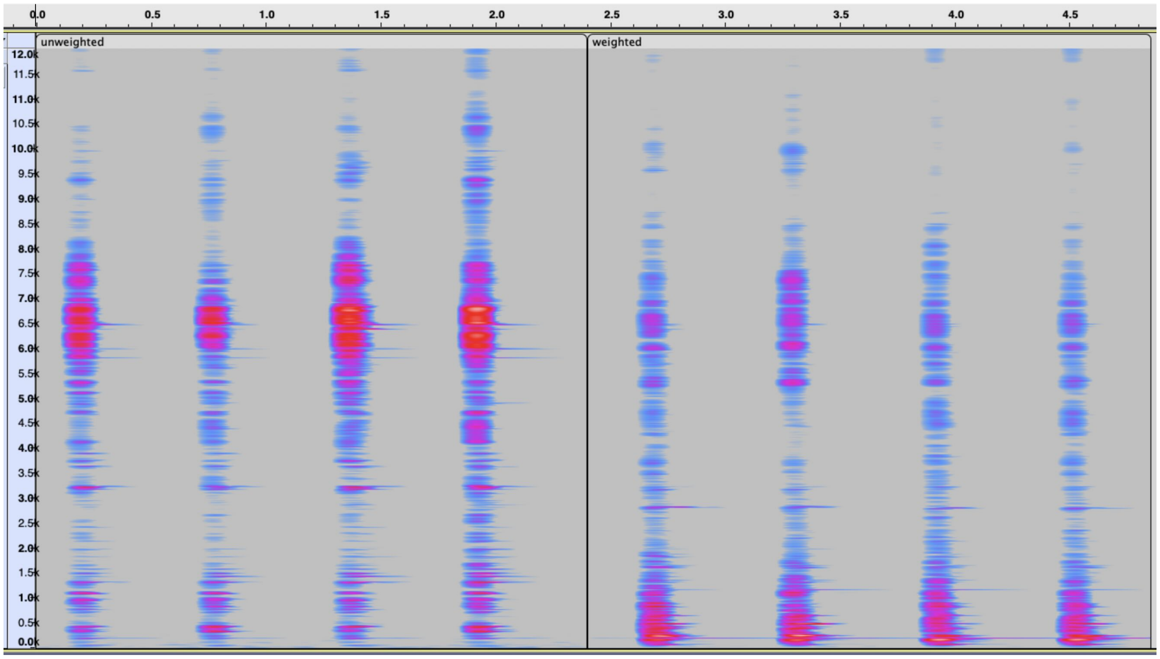

FIG. 13: spectrograms for 0 to 12000 Hz of the surface vibration of the rim, unweighted and weighted, for taps on the bridge with strings damped

However, there is no simple, overall, frequency dependence to those differences. Rather, where which banjo sounds stronger switches between unweighted and weighted.

If one looks at spectrograms over a wide frequency range of well-controlled plucks, there do seem to be some very slight differences. Again, these are broad regions where which sounds louder varies from region to region. However, the connection to FIG. 13 was not clear, leaving rim physics as a subject for future inquiry.

Played music selections[8] on the two banjos sounded very similar, although you might pick up a few small differences:

http://www.its.caltech.edu/∼politzer/banjo-ring/why-unweighted.mp3

http://www.its.caltech.edu/∼politzer/banjo-ring/why-weighted.mp3

However, the small differences appearing in numerical analyses of the played samples did not match those of well-controlled plucks. An obvious issue is the ability to play similarly on the two instruments without extensive practice. The weighted version was awkward to hold and nearly four times as heavy as unweighted.

VII. HOW FIBERFILL DAMPS HEAD MODES WITHOUT TOUCHING

Polyester fiberfill is long-known as a fine absorber of sound — if installed at a Goldylocks density. So, the fiberfill is absorbing sound inside the pot. Significantly, it does not touch the head. On the whole, the BanjoBolster™ and other polyester-filled shapes substantially reduce the amplitudes of head modes in the sound of the instrument without substantially reducing string mode sound. The following is an attempt to explain that selectivity by considering an analogous two-oscillator system. It will do double-duty, explicating the interactions of pot air to head and those of head to strings.

As a preliminary, consider the strings and head of the instrument. Uncoupled, they each would have their own characteristic modes of vibration. If we ignore the internal dynamics of the bridge (which turn out only to be important at higher frequencies), the bridge simply serves to couple the strings to the head. In general, individual modes of the coupled system are each combinations of string modes and head modes. However, to succeed as a musical instrument, the coupling must be weak enough that there are clear, predominantly string modes (i.e., with high Q's).

To a first approximation, the decays of vibrations in a banjo are dominated by sound radiation from the head.[1] Frictional losses are far less important. The impedance mismatch at the bridge allows the string to keep vibrating through many cycles. However, the mismatch is small enough that the string energy transfers to the head before appreciable dissipation on the string. And the head is a very efficient transducer of vibrations into sound — in contrast to thin sheets of wood, which produce more heat than sound.

Pure head modes would be the resonances of the head in vacuum, while pure pot air modes exist if the head is rigid.[9] The actual coupling between the two systems is obviously strong in some sense. It extends over the whole surface of the head. However, to the extent that individual frequencies of the uncoupled systems are substantially different for vaguely similar mode shapes, the coupled system will have some modes that are mostly head and some that are mostly air.

In both the head–string system and the pot air–head system, we have weakly coupled, complicated systems, where the damping is primarily associated with just one of the systems. Thinking of the interaction between the coupled systems as a double sum of pairwise interactions between the modes, we arrive at systems like the one pictured in FIG. 14. In ref.[10], the m's and k's were chosen to be equal in the representative modes from each system to facilitate the algebra. We should now picture the more general possibilities.

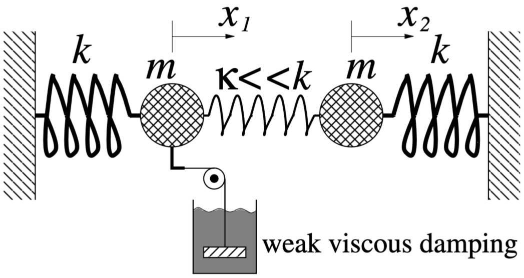

FIG. 14: Ref. [10]'s FIG. 1 for the simplest arithmetic. The case of different frequencies in the uncoupled ($\kappa \to 0$) limit is treated in ref. [10] §VI.

The modes of the weakly coupled combined system are of the form of mostly one or mostly the other. However, all combined modes are damped, including the ones that are mostly the uncoupled undamped system. But the damping has a much greater effect on the combined modes that are mostly the uncoupled damped system.

The experiments suggest that the BanjoBolster™ provides dramatic damping of acoustic air vibration in the pot. The evidence is that filling the pot with fiberfill (i.e., the pancake) does little more. The pot air coupling to the head endows the mostly head modes with some fiberfill frictional damping. This adds to the head damping due to sound radiation. Both are in the regime termed "weak" damping, in that the frequencies of individual mostly head modes are clearly discernible in the final sound. (That requires Q's $\gg 1$, even if they are not as large as string mode Q's.)

Thinking now about the string-head system, in the uncoupled limit (i.e., $\kappa \to 0$), the oscillator representing a head mode is damped, and the oscillator representing a string mode vibrates forever. If the smallest parameter is their coupling, then the combined two modes remain mostly head and mostly string, and their decays times are only slightly altered, i.e., the mostly-head mode lives a bit longer because of its string component and the mostly-string mode develops a long but finite decay from energy loss through its head component.

The magnitudes of the parameters are essential. But for a banjo-like object to be a musical instrument, i.e., to have the appropriate and observed Q's, the result of the fiberfill pot damping is to give small extra damping to the mostly-string modes and more damping to the mostly-head modes.

VIII. CONTRAST WITH OTHER "STUFFING"

People have long stuffed extra stuff inside their pots to alter the sound. The traditional methods involve placing something that is somewhere between firm to rigid up against the inside of the head, wedged against the dowel stick or co-rod. FIG. 15 shows the type of sponge often used, while socks or dish towels do as well. The rigid devices are similarly placed very near the edge of the pot at the joint with the neck. (Serious reduction in overall volume can be achieved with a rigid post right under the bridge, which obviously reduces its motion.)

Sponge and socks will do some air motion damping, but they are not nearly as effective as the appropriate density of fiberfill. (Woofer cabinets are not filled with old socks.) However, the direct contact with the head gives a mechanical damping of head motion. The location of that damping is very relevant to its impact on the sound. In contrast, damping the pot air modes without contacting the head, damps head motion over its whole area.

Damping head motion specifically near the edge reduces high frequency sound in two ways. 1) High frequency head modes have relatively more amplitude near the edge than lower frequency ones. And 2) although the high frequency head modes involve up-and-down motion all across the head, their sound is radiated most effectively from the motion nearest the edge.[1]

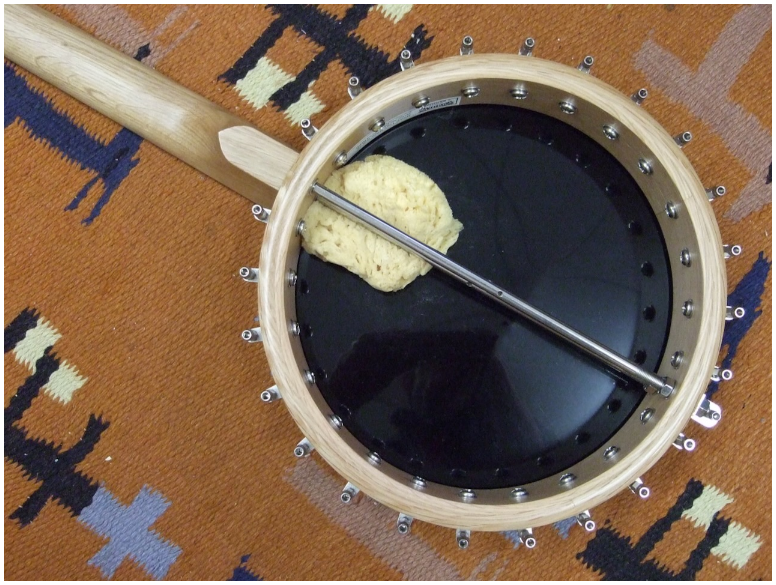

FIG. 15: sea sponge installed

As shown in FIG. 16, there's not much difference between the sponge and bolster below 1500 Hz. But, looking over a much larger range, e.g., up to 15,000 Hz (FIG. 17), dramatic differences become apparent.

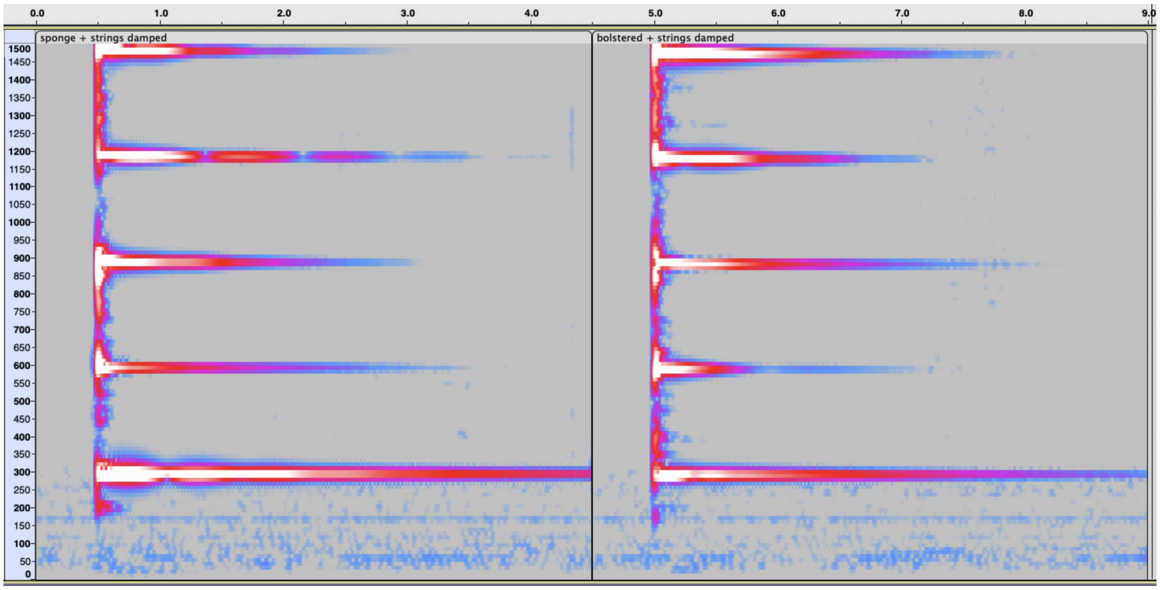

FIG. 16: 1st string pluck with other strings damped, comparing sponge stuffing to the BanjoBolster™; 0 to 1500 Hz

The important test is the sound. Listen to the two cases, first with sponge and then with bolster:

http://www.its.caltech.edu/∼politzer/banjo-ring/sponge-vs-bolster-strings-damped.mp3

There are interesting details in FIG. 17 that serve as reminders that frictional dissipation is a complex subject. The most obvious is the failure of the fiberfill, at least in the form a BanjoBolster™, above 3500 Hz. That's no problem for the traditional fiberfill application of damping the sound coming off the back of a woofer inside a speaker cabinet.

IX. CORRECTING: STRING STRETCHING, BREAK ANGLES, & "RING"

Ref. [2] was the first suggestion that the "Ring of the Banjo!" was produced by inharmonic partials. The original hint and motivation came from combining two threads: questions about the impact of the bridge's moving up and down while the string vibrates and Chowning's identification of audio range frequency modulation as producing a "metallic" sound.[4]

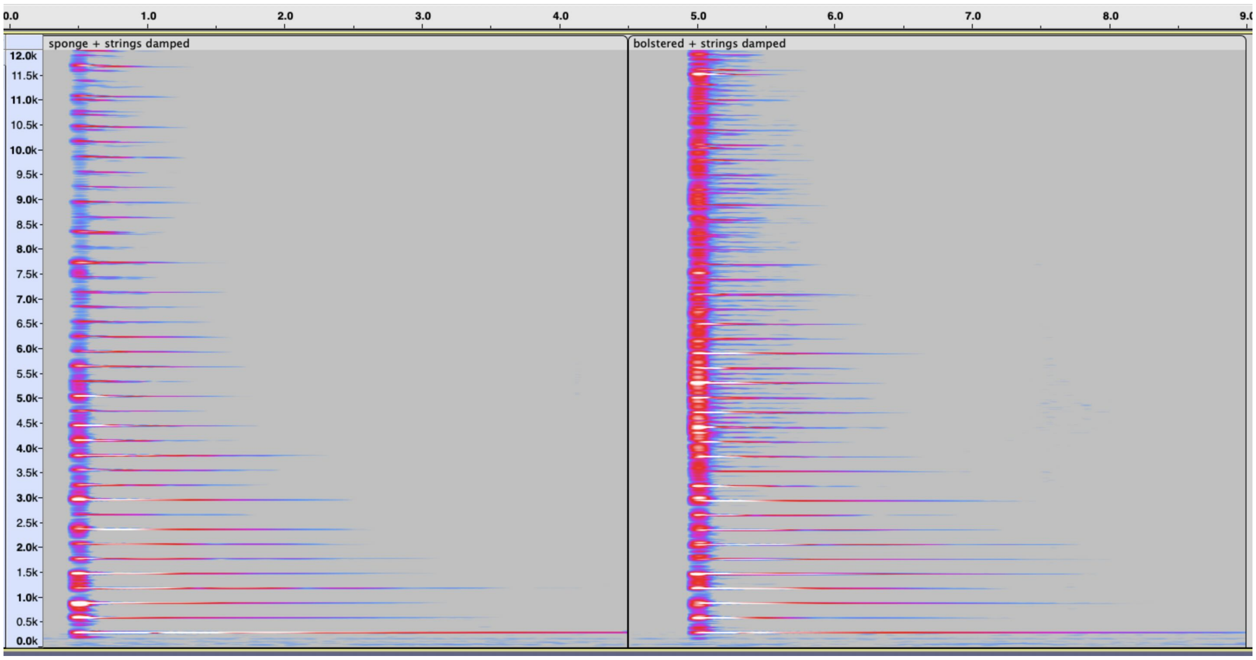

FIG. 17: 1st string pluck with other strings damped, comparing sponge stuffing to the BanjoBolster™; 0 to 12,000 Hz

Variations in break angle of the strings passing over the bridge were well-known to players and builders as producing sounds ranging from mellow to sharp, when going from smaller to greater angles. One consequence of bridge motion is that it produces stretching of the string, and that is greater for larger break angle. However, a convincing connection of that stretching phenomenon to the produced sound was missing. Perhaps it was of insufficient magnitude to matter. Perhaps it ultimately disappears into the harmonic partials, leaving “ring” unexplained.

1. *errors & setting things right*

One correction is to the simple math. If the string tension is constant while the string vibrates, then the frequencies of all its harmonic partials are proportional to the square root of the tension. However, a simple analysis showed that sinusoidal variation in tension does not lead to sinusoidal variation in frequencies unless the frequency of the variation is negligible. Rather, the result is what is known in physics as “parametric oscillation.”[11] The everyday example is “pumping” with one’s legs on a playground swing. In general, sinusoidally varying tension produces partials at sums and differences of the tension modulation and original string's frequencies. This poses a challenge to unequivocal identification in actual sounds.

The work with Jim Woodhouse produced a very different and very convincing description of the impact of bridge motion on the sound[1]. Of course, break angle is a crucial element. The basic bridge motion produces an enhanced region of frequency response of the head to the strings, quite analogous to Woodhouse's work long ago on the violin.[12] (Such regions are called formants in the context of speaking and singing.) Bridge flexing and other motions produce yet higher frequency enhanced regions.

A unique design of bridge and tailpiece allowed a comparison of 0° and 13° break angles on what was otherwise the exact same banjo.[13] This yielded a substantial expansion of the evidence from actual banjo sounds that was originally presented in ref. [2]. However, close examination of the sound differences produced by switching the tailpiece from 13° to 0° break angle reveal that no new inharmonic partials are produced in the region studied closely in this note. Rather, the difference is what is predicted by ref. [1]. The high frequency enhancement produced by 13° disappears with 0°. Instead, the bridge motion only produces a low-pass filter. The sound is a disappointingly dull banjo.

X. FUTURE EFFORTS...

Barring a future challenge to the picture presented, the Ring of the Banjo can rest — or be taken up by some other researcher.

As mentioned in section §V, the difference in sound between flat-top acoustic guitars and resonator guitars is also the presence of significant inharmonic partials.[6] The computer synthesized sounds produced to characterize the difference, analogous to what was done in section §V, on their own, sound like a gong. However, it is already clear that their physical origin is different from banjos. Tracking down that origin is an interesting challenge.

---

[1] J. Woodhouse, D. Politzer, H. Mansour: a technical exposition is available as open-access as *Acoustics of the Banjo: measurements and sound synthesis & theoretical and numerical modeling*, Acta Acustica, **5**, 15 and 16 (2021) https://doi.org/10.1051/aacus/2021009 and

https://doi.org/10.1051/aacus/2021008); *Pickers' Guide to* Acoustics of the Banjo, HDP: 21 – 01, http://www.its.caltech.edu/∼politzer – APRIL 2021 is an informal account of some of the salient results.

[2] *String Stretching, Frequency Modulation, and Banjo Clang*, HDP:14 - 02, https://www.its.caltech.edu/∼politzer/FM.pdf; also Acta Acustica (united with Acustica), AAuA 101(1) 1, January 2015. This paper was published in a respectable journal and gained some notoriety, principally on its man-bites-dog aspect. What is right and what is quite wrong is discussed in section §IX of this note.

[3] The lyrics are a stark reminder of the banjo's history in the Americas. Leaving that for the time being in the present context, it is worth considering what sort of banjo Forster thought to be ringing. To that end, here is the basic melody of that tune played on an exquisite reproduction of an 1850's banjo:
https://www.its.caltech.edu/∼politzer/banjo-ring/ashborn-foster-ring-ring-123-faster.mp3
The original was made by James Ashborn. He made more and better guitars than C.F. Martin, is often credited as a pioneer of factory-scale production, but went into politics and closed down his business. Jim Hartel (http://www.minstrelbanjo.com/) built the reproduction as well as several other copies of surviving 19th Century treasures. The only metal parts are tension hooks and nuts and a thin tension ring. As shown in section §I C, the Ashborn is rich in inharmonic partials and, to my mind, has a sweet ring.

[4] J. Chowning, *The Synthesis of Complex Audio Spectra by a Means of Frequency Modulation*, J. Audio Eng. Soc. 21 (7) 526 (1973). Chowning applied frequency modulation to a single frequency sound. When the modulation frequency was raised and entered the audio region, the modulated sound no longer seemed to go up and down in pitch. Rather, according to Chowning, it developed a "metallic" timbre. The math was familiar from the basis of FM radio. FM produces side bands.

[5] Actually, I made an attempt to do just that, i.e., hear the sound of a banjo with only one string vibrating at a time. See *String Sing-Along (Sympathetic Vibration) Is Not the Key to Banjo Sound*, http://www.its.caltech.edu/∼politzer/turkey/turkey.pdf. A library of pluck sounds was recorded with all strings open and with other strings damped. A short music sample was synthesized from this library. There are several sound files and waveforms, but the essence is in the synthesized tunes: https://www.its.caltech.edu/∼politzer/turkey/2-

measures-synth.mp3 and https://www.its.caltech.edu/∼politzer/turkey/2-measures-damped-synth.mp3. That write-up includes another 1850's-style banjo, ring and all: https://www.its.caltech.edu/∼politzer/turkey/ark.mp3

[6] *Resonator Guitar Synthesis*, HDP: 23 - 04, https://www.its.caltech.edu/∼politzer/pluck-synthesis/pluck-synthesis.pdf

[7] The banjo in FIG. 10 and on page 1 was purchased second-hand at McCabe's Guitar Shop, Santa Monica, California, several years ago. Greg Deering confirmed that the model was the first that the Deering Banjo Company brought to market in 1975, and they had poplar necks. He called it the "Basic." Somewhat bemused, he noted that some form of similar steel rims have remained in the Deering line ever since. "Some people really like them." They certainly can be played loud.

[8] Adam and David Moss are twin singer-songwriters, who teamed up to form The Brother Brothers (https://www.thebrotherbrothersmusic.com/. The Basic Deering music samples are the melody of their *Why Don't You Play the Banjo Anymore?* It's one of the saddest songs I know.

[9] *Banjo Rim Height and Sound in the Pot*, HDP:16 - 03, https://www.its.caltech.edu/∼politzer/rim-height.pdf

[10] *The plucked string: an example of non-normal dynamics*, American Journal of Physics, Am. J. Phys. **83** (2015) 395 and/or HDP: 14 – 04, https://www.its.caltech.edu/∼politzer/plucked-string-final-411.pdf, *Zany strings and finicky banjo bridges*, HDP: 14 – 05, https://www.its.caltech.edu/∼politzer/zany.pdf

[11] *Banjo Break Angle Tension Modulation as Parametric Oscillation*, HDP:20 - 01, https://www.its.caltech.edu/∼politzer/parametric.pdf

[12] J. Woodhouse, *On the "Bridge Hill" of the Violin*, AAuA 91 (2005) 155–165, https://euphonics.org/wp-content/uploads/2022/03/BridgeHill.pdf

[13] *Banjo Ring from Stretching String: A Zero Break Angle Demo*, HDP: 19 - 01, https://www.its.caltech.edu/∼politzer/zero-break/zero-break.pdf

[14] *Resonator Guitar Synthesis*, HDP: 23 - 04, http://www.its.caltech.edu/∼politzer/pluck-synthesis/pluck-synthesis.pdf.