The Principal Formant of the Banjo – Adjustable Tailpiece and Numerical Evaluations

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A simple, homemade tailpiece allows a playable range of string break angles on a Goodtime banjo of $2\frac{1}{2}$ to $18\frac{1}{2}$ degrees. This illustrates a range of banjo admittance, the body response to string vibration. The theoretical admittance prediction is explored further over a wide range of both break angles and bridge mass to illustrate its qualitative behavior.

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I. INTRODUCTION

FIG. 1. A simple adjustable tailpiece allows a maximum break angle of $18\frac{1}{2}^{\circ}$ (left). The lowest playable angle without a special bridge is $2\frac{1}{2}^{\circ}$ (right).

The interaction of head, bridge, strings, and tailpiece determine the banjo's sound, which is quite distinct from other string instruments. One important characteristic is the frequency-smoothed bridge admittance. This can be modeled from first principles[1] and evaluated using the physical parameters of the individual parts[2]. The consequence is one giant formant, dominating the entire frequency range, with a particular central frequency and rates of fall-off above and below.

II. BACKGROUND

A technical measure of an instrument's response to the force of the strings at the bridge is the point admittance. We imagine a sinusoidal force of amplitude F_o is applied at a point on the bridge, and it produces a sinusoidal velocity of the same frequency of amplitude v_o. The amplitude of the admittance is v_o/F_o. (The phase of the velocity relative to the force can be encoded as the phase of the complex admittance.)

The upper panel of Fig. 2 shows careful measurements of admittance from ref. [1]. The

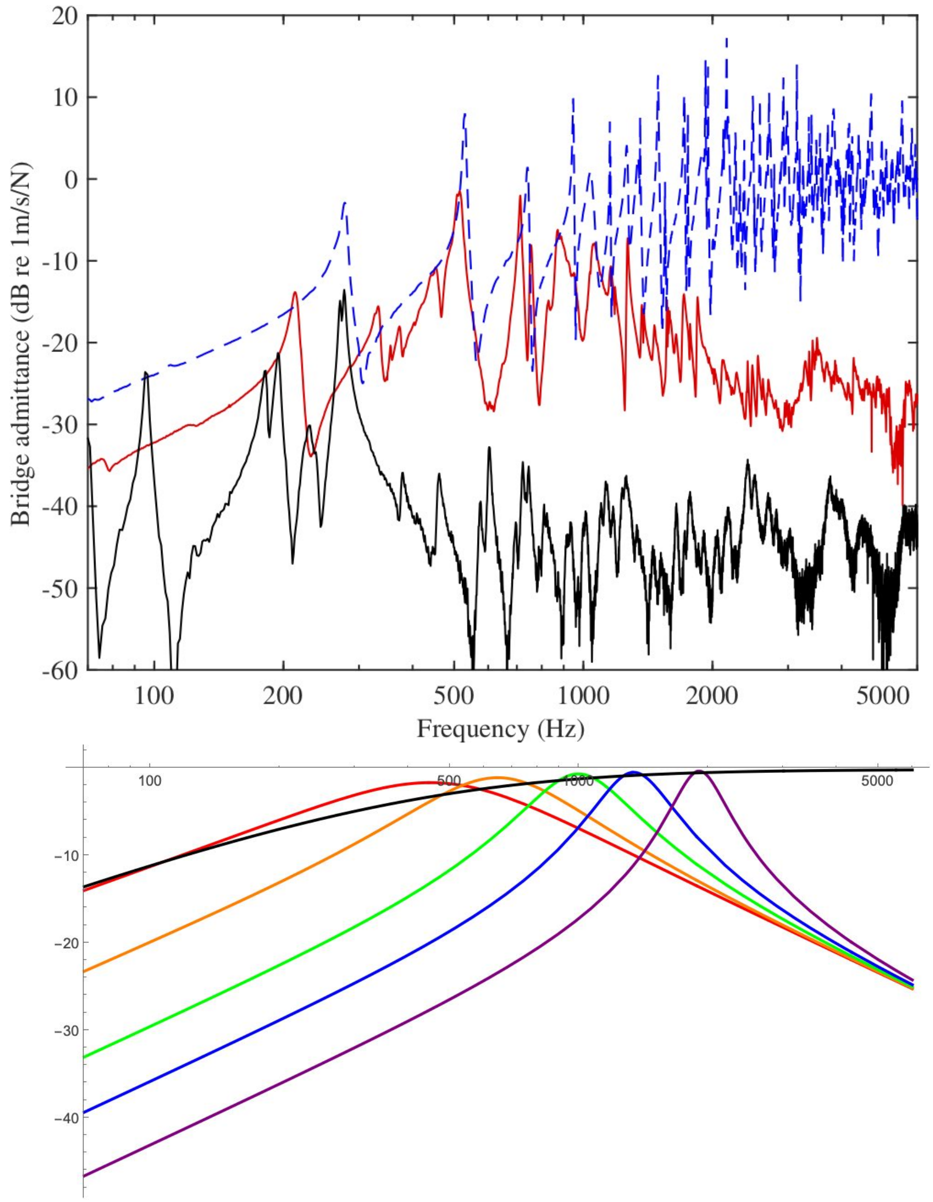

FIG. 2. Upper figure from ref. [1]: measured bridge admittance at the location of the 1st string slot: blue is the head without bridge & strings, red is with bridge and damped strings, and black is guitar with damped strings. Lower figure: calculated, averaged admittance: head alone in black, colors include bridge and damped strings; orange uses parameters from the actual measured instrument shown above[2], while red, green, blue, and purple are for smaller and greater string-break angles with the same bridge.

blue curve is for the head without strings at the location of the bridge. The red curve is with damped strings at normal tension of the bridge at the location of the 1$^{\text{st}}$ string. The black curve is from a typical flat-top steel-string guitar, measured at the bridge with damped strings.

The orange curve uses the physical parameters[2] of the banjo measured above in a simple model to give the frequency-smoothed admittance. The relevant parameters are the string and head tensions and Young's moduli, the string break angle, the bridge mass, and the head density. While the quantitative agreement was very satisfying, a surprise was the fact that the head parameters are quantitatively more significant than those of the strings in locating the peak of the formant.

An outstanding source for clear, concise explanations of virtually all background theory and technical details related to this study can be found in Jim Woodhouse's on-line reference https://euphonics.org. A guide to locating a few of the most relevant sections is provided in endnote [3] below.

A few features of the upper panel of Fig. 2 deserve note. Smoothing the (black) guitar admittance yields a constant independent of frequency while the (blue) drumhead admittance is an increasing function. Both behaviors are consequences of the ideal "density of states" (resonances per unit frequency. A reference for these properties is given in endnote [3]. Adding strings and bridge change the banjo completely. A strong region is selected, and the admittance falls off in both directions. Players experience the location of the strong region as a consequence of some combination of bridge mass and string break angle. The much smaller overall value of guitar admittance compared to banjo reflects the much larger mass density of the guitar top compared to the banjo head.

III. PREDICTION FOR A RANGE OF BREAK ANGLES

The simple part of the model of banjo admittance is the oscillator composed of the bridge with a restoring force given by an imbalance between the downward force of the strings and the upward force of the head when the bridge is away from equilibrium. For the relevant small oscillations, that force can be approximated as linear with a "spring" constant. That spring constant divided by the bridge mass (and immediate surrounding head) gives the square of the frequency of the bridge oscillator. The oscillator decays via coupling to outgoing waves in the head. The brilliant insight of ref. [4] is that the frequency-smoothed admittance can be calculated for outgoing waves into an infinite medium. (The peaks and valleys of the actual admittance come from waves bouncing off the actual rim and returning to the bridge in or out of phase with the outgoing wave.)

Again, endnote [3] points to a concise description of the result for string vibration driving a soundboard through a resonance. The outgoing waves on a banjo head are described mathematically by Bessel functions. The qualitative features of the resulting formulae might not be immediately apparent. So, I have done the calculation numerically and plotted the results for an assortment of spring constants. In the lower panel of Fig. 2, the inputs for the orange curve were measured physical parameters of the banjo represented in the admittance measurement directly above. Green, blue, and purple are for increasingly higher spring constants or break angles. Red represents a smaller spring constant. I did not attempt to convert the chosen spring constants to break angles on the particular banjo because those are elaborate calculations[2]. In fact, the blue and purple versions are not practically attainable but are chosen here to illustrate the trends.

One feature that surprised me was the behavior for values below that of the orange curve. The admittance soon becomes roughly independent of that spring constant value and matches the result for zero spring constant or zero break angle. The smoothed-admittance peak sits very near to the value shown by the red curve. The fall-off of the bare head admittance for decreasing values of frequency dominates over the below-resonance fall-off of the oscillator as if it had a single damping constant.

FIG. 3. `tailpiece-bridge combo yielding` 13° `and` 0° `break angles`

The practical consequence is that all low break angles sound pretty much the same. Recordings from an earlier tailpiece adventure can be interpreted as illustrating that fact.

That particular tailpiece is shown in Fig. 3. Rather than being continuously adjustable, it had only two positions. Flipping it over allowed break angles of 13º and 0º. The 0º was playable because of special features of the bridge. It was glued and screwed to the head, and the strings passed through channels that were angled, rising from the tail side, so that they pressed down on the bridge as they exited on the pluck side. Nevertheless, the tailstrings between bridge and tailpiece were parallel to the strings on the pluck side. So, the strings exerted no net downward force on the head. However, the plucked string vibration was transferred to the head without buzzing or any other problem.

I still have played-music sound samples, although no admittance measurements were made at the time. Here they are, two short selections, each played on the two tailpiece configurations. It's left to the listener to tell which is which.

http://www.its.caltech.edu/∼politzer/zero-break/A.mp3
http://www.its.caltech.edu/∼politzer/zero-break/B.mp3
http://www.its.caltech.edu/∼politzer/zero-break/C.mp3
http://www.its.caltech.edu/∼politzer/zero-break/D.mp3

The point in the present context is that 0º doesn't sound extreme. It still sounds like a banjo, just one with a relatively low break angle. The downward break-angle force on the bridge is not essential to the sound. Of course, it is essential for mechanical reasons if a normal bridge is being used.

IV. WIRE-BREAK NEAR-SOUND

As ref. [3] §10.4 shows, e.g., in its Fig. 6, the downward sudden force of a wire-break applied at the bridge with all strings damped can be used to measure impedance. The technically challenging part is measuring the bridge motion in response. Jim Woodhouse uses a laser Doppler vibrometer to record the motion right next to the applied force, be it wire-break or impulse hammer. Ideally, a tiny microphone placed right above the point of interest would record the motion imparted by the surface to the air. My own crude implementation uses a small (not tiny) USB microphone going into a Mac using Audacity. All parts of the system are designed for voice and music recording. The only means readily available in the end to reduce amplitude enough to avoid distortion was to provide sufficient distance of the mic from the bridge. The spectra shown below are for the mic located 1″

above the bridge, but that is $1\frac{5}{8}''$ above the head, which is the source of almost all of the sound picked up the the mic, whose diameter was $\frac{3}{8}''$. It is therefore no surprise that the relation of those sound recordings to admittance is not direct.

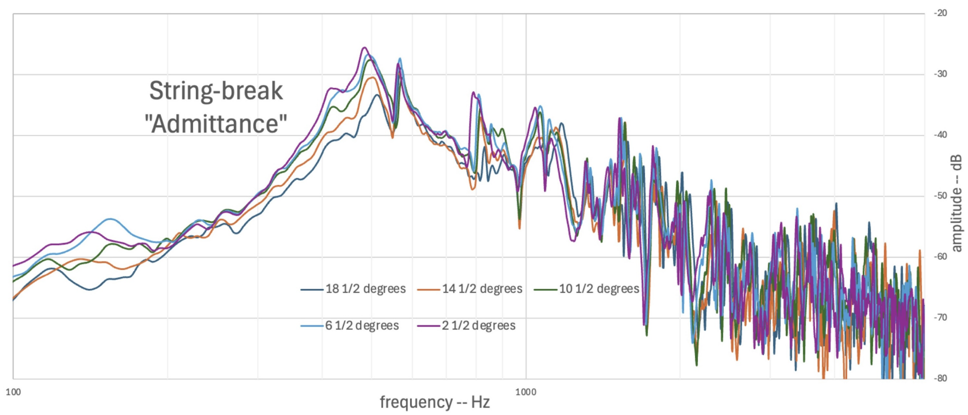

FIG. 4. `wire-break near sound, displayed with the lowest FFT frequency resolution that still shows all distinct peaks`

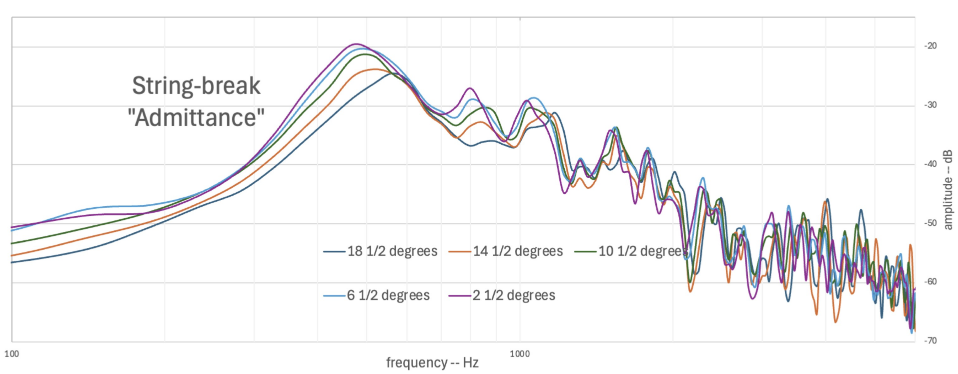

FIG. 5. `wire-break near sound, displayed with the FFT frequency resolution` $8\times$ `lower than that used in Fig. 4.`

Fig.s 4 & 5 are spectra of the same sounds, with the break angles identified by color. The FFT used for Fig. 4 produces the lowest frequency resolution that still shows the location of all of the significant peaks in the data. Fig. 5 employs a resolution that is $8\times$ lower. That makes the trends a bit clearer as they would appear with peak-to-peak smoothing.

The general shapes are as expected – with the exception of a weak formant centered around 3500 Hz that was first identified by Jim Rae [5] and explained in ref. [1] as due to the lowest bending excitation of the bridge. The range of main peak frequencies, ∼480 to ∼580 Hz, may seem small when presented this way, but it is quite apparent to most players (even if not to typical audiences). Furthermore, essentially all real banjos are set-up within the range of the middle three angles.

Even though Fig.s 4 & 5 represent sound recordings, the shapes and trends of the ideal impedances displayed in Fig. 2 are more relevant to real banjo sound, the reason being that we don't listen to the instrument with our ears 1″ above the bridge.

V. SHORT PLAYED MUSIC EXAMPLES

I recorded the same short sample, playing and set-up as identically as I could manage, with the break angles at $18\frac{1}{2}^{\mathrm{o}}$, $10\frac{1}{2}^{\mathrm{o}}$, and $2\frac{1}{2}^{\mathrm{o}}$. The selection is just bits from a great tune, *Serenity Song*, by Benny Bleu. I'm sure it's the lyrics that make it an audience favorite.

https://www.its.caltech.edu/∼politzer/adjustable-tailpiece/18.5.mp3

https://www.its.caltech.edu/∼politzer/adjustable-tailpiece/10.5.mp3

https://www.its.caltech.edu/∼politzer/adjustable-tailpiece/2.5.mp3

VI. VARYING BRIDGE MASS AT FIXED BREAK ANGLE

The same Mathematica code that did a numerical evaluation of the smoothed admittance formula could be used to generate the behavior for various bridge masses with a given string break angle. The results are displayed in Fig. 6. The orange curve uses the parameters of the banjo most featured in ref. [1]. That had m_{bridge}= 2.2 gm. Dark red is m_{bridge}= 0 gm, and red is m_{bridge}= 1.1 gm. Green is m_{bridge}= 4.4 gm, and blue is m_{bridge}= 11 gm. The dark red m_{bridge}= 0 gm curve differs from the black head-only curve because the strings and head still provide a restoring force for the vertical motion at the bridge location. (The simple model assumes that the string vibrations are terminated at that point.) Almost all banjo playing is done with bridges in the region between red and green. The trend for increasing bridge mass shows how weighting the bridge can serve as a mute but also modifies the timbre and sustain.

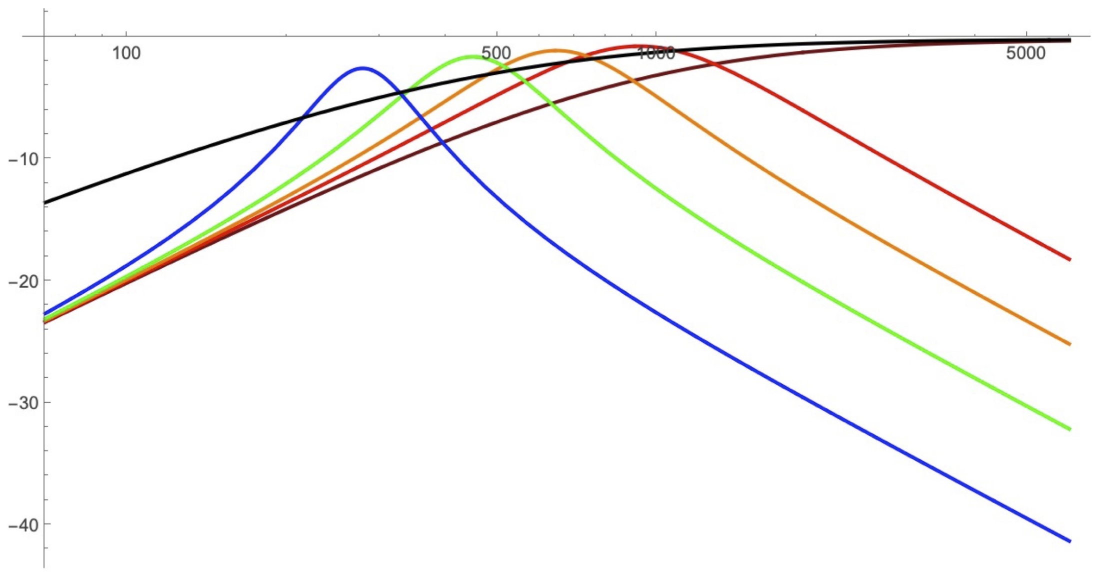

FIG. 6. smoothed admittance calculations: black is the head alone, dark red is bridge mass $m_{\mathtt{bridge}}$= 0, red is $m_{\mathtt{bridge}}$ =1.1 gm, orange is $m_{\mathtt{bridge}}$= 2.2 gm, green is $m_{\mathtt{bridge}}$= 4.4 gm, blue is $m_{\mathtt{bridge}}$= 11 gm.

VII. ALTERNATIVE PERSPECTIVE ON THE PRINCIPAL FORMANT

I have termed the bridge-tailpiece shaping of the frequency spectrum the "principal formant" because it dominates the whole frequency spectrum. One may ask whether there isn't something analogous in the physics of the acoustic guitar. The answer is yes, but it's a negligibly small effect. For the violin, there is a very small effect, which is only perceptible because of how the bridge sits between the f-holes. The holes give that region the chance to flex, somewhat independent of the surrounding top, with some fundamental resonant frequency, usually around 2500 Hz, producing what is known as the bridge hill.

It may be helpful to think of the banjo principle formant as arising from a combination of two separate effects. The dark red m_{bridge}= 0 curve of Fig. 6 illustrates that the action of the spring of the break angles serves as a high-pass filter that limits what string vibration frequencies are transferred to the head. The red curve of Fig. 2 is the smoothed admittance for negligible values of the break-angle induced return force. So, the bridge alone acts as a low-pass filter. A peak appears around 400 Hz simply because the head modes don't readily support vibrations below that. For bridge and break angle values in between, both filters are operative.

[1] J. Woodhouse, D. Politzer, and H. Mansour, *Acoustics of the Banjo: measurements and sound synthesis & theoretical and numerical modeling*, Acta Acustica, **5**, 15 and 16 (2021) https://doi.org/10.1051/aacus/2021009 and https://doi.org/10.1051/aacus/2021008); *Pickers' Guide to* Acoustics of the Banjo, D. Politzer, J. Woodhouse, and H. Mansour, HDP: 21 – 01, http://www.its.caltech.edu/∼politzer – APRIL 2021 is an informal account of some of the salient results.

[2] D. Politzer, *Arithmetic of Spring Forces on the Banjo Bridge*, HDP: 21-04, http://www.its.caltech.edu/∼politzer/head-stiffness/head-stiffness.pdf

[3] Jim Woodhouse has a still-growing resource on the science of musical instruments: https://euphonics.org. Mostly, it is about the physics, but Chapter 10 has a lot of practical information about doing measurements. There's plenty of good advice, with explanations of pros and cons. §10.4 describes the wire-break method of measuring admittance. §5.3 is about formants and admittance. §5.3.1 deals with driving strings through a resonance, which is applied to the violin bridge hill, but it is just as applicable to the principal formant of the banjo. §5.3.2 explains the idea of a frequency-smoothed admittance, as derived by E. Skudrzyk. §10.4 derives the "density of states," i.e., the number of resonances per unit interval in frequency, of different media. For a thin plate (whose restoring force is stiffness), such as the soundboard of an acoustic guitar, the density is independent of frequency. In contrast, the density for a membrane (whose restoring force is tension), like the head of a banjo, grows linearly with frequency. This difference accounts for the dramatic difference between the blue and black curves of Fig. 1.

[4] E. Skudrzyk, *The mean-value method of predicting the dynamic-response of complex vibrators*, Journal of the Acoustical Society of America, **67** 4 (1980) 1105-1135.

[5] J. Rae, *Banjo*, Rossing, T. (eds) *The Science of String Instruments* (2010) Springer, New York, NY, Chapter 5 – https://doi.org/10.1007/978-1-4419-7110-4_5