A Dynamic Level-k Model in Games

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Introduction



Deviation from Backward Induction

Basic Assumptions

- Doubts that others applying backward induction
- Subjective expected utility maximizers
- History of plays is a signal for beliefs of opponents strategy



- Violations of Backward Induction
 - Limited Induction: $G' \subset G \Rightarrow \delta(L^1, ..., L^I, G) \ge \delta(L^1, ..., L^I, G')$

• Time Unraveling: $\lim_{t\to\infty} \delta(L^1(t),...L^I(t),G) = 0$

The Model

• Belief of the probability that the opponent plays L_k at t+1

$$B_{k}^{i}(t) = N_{k}^{i}(t) \left[\sum_{k'=0}^{S} N_{k'}^{i}(t) \right]^{-1}$$
$$N_{k}^{i}(t) = N_{k}^{i}(t-1) + I(k,t), \forall k$$

• The Optimal Rule

$$k^{*} = \arg \max_{k=1,...,S} \sum_{s=1}^{S} \left[\sum_{k'=1}^{S} B_{k}^{i}(t) \pi_{i}(a_{ks}, a_{k's}) \right]$$

• The Proportion of Players who hold Initial Belief k

$$\Phi(k) = e^{-\lambda} \lambda^{k} (k!)^{-1}, k = 0, ..., S.$$

Limitations of the Model (I)

- Problems with the Belief Function
 - Players are restricted to play with a single rule for each round in a repeated game. No belief updating within a single round is allowed.
 - The stopping time is mutually determined by both players. But the belief suggests that an always-corporative player adopts an increasing limited backward-selection rule Lⁱ_k.
 - Such belief, when becoming common knowledge, can only sustain the backward induction equilibrium at t+1.
 - Even if beliefs are not part of the game, i.e., players do not strategically determine beliefs, for any random error of defection occur at any stage, the beliefs lead to faster convergence to zero deviation.

Limitations of the Model (II)

Lack of Intuition

- The model assumes players' willingness to corporate (Lⁱ_k), regardless the objectives, in order to explain the corporative behavior.
- The model assumes a monotonic updating mechanism to explain the convergence to backward induction.
- The model still fail to explain why players may move back from defection to corporation, such as in a repeated prisoner dilemma game.

Estimation

• No intuitive interpretations for the resulting parameters.

Empirical Evidence and Estimation

Caltech v.s. Pasadena Community College

- C take one stage earlier than P
- 4-Stage v.s. 6-Stage Centipede game

Outcome	4	3	2	1	0
Caltech $(N=100)$	0.06	0.43	0.28	0.14	0.09
PCC (N=181)	0.08	0.31	0.42	0.16	0.03

Outcome	6	5	4	3	2	1	0
Caltech $(N=100)$	0.02	0.09	0.39	0.28	0.20	0.01	0.01
PCC $(N=181)$	0	0.05	0.09	0.44	0.28	0.12	0.02

Estimation

$$P(O) = \sum_{k} \sum_{k'} P_{t}^{A} (k|\beta,\tau) P_{t}^{B} (k'|\beta,\tau) I\{\max(k,k'),O\}$$
$$L = \prod_{t} \left[(1-\varepsilon) P(O) + \varepsilon \frac{1}{S+1} \right]$$

•	Result	Parameter	Backward Induction	Naive Belief	Static Level \boldsymbol{k}	Full Model
		λ	∞	0	1.76	1.32
	Caltech	β	∞	0.08	∞	0.25
		ϵ	1.00	0.97	0.21	0.29
		Log Likelihood	-357.1	-355.0	-312.3	-305.8

Take Away

• Why is the Dynamic Level-k Model Interesting?



- A simple theory applying to a dynamic setting;
- Predicts and explains the characteristics of systematic violations between backward induction and lab evidence;
- The underlying premise of the theory has an intuitive appeal on beliefs of corporative behavior;
- The results conceptualize a tracing procedure for backward induction.