Neural Random Utility and Measured Value

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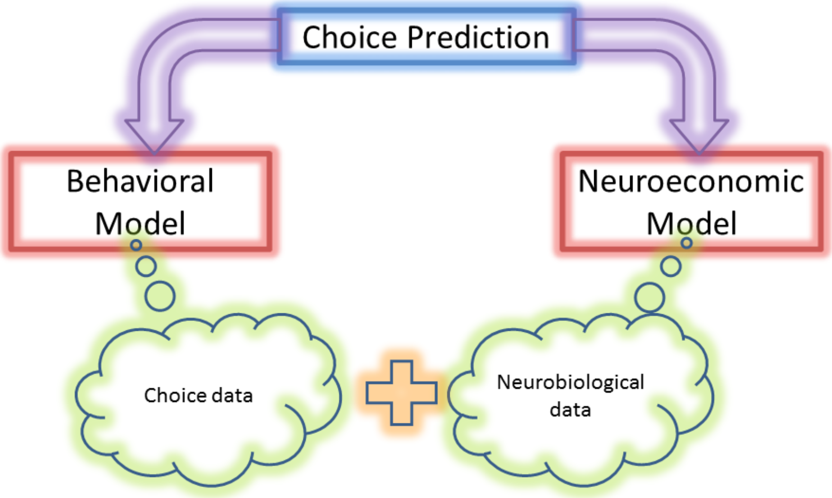
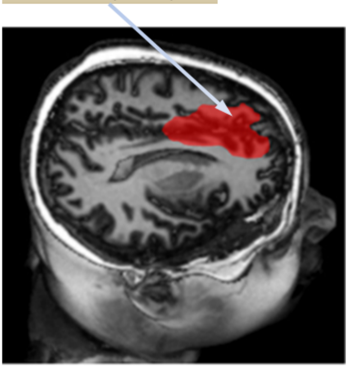

Choice Prediction
Behavioral Model
Neuroeconomic Model
Choice data
Neurobiological data

Neurobiological dataset

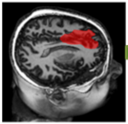
Medial Prefrontal Cortex (mPFC)



Functional Magnetic Resonance Imaging (fMRI) scanner

Levy and Glimcher (2012), Bartra et al (2013): meta-studies indicating that ***activity in mPFC is tightly correlated with the values subjects place on choice objects***

Neural Random Utility Model



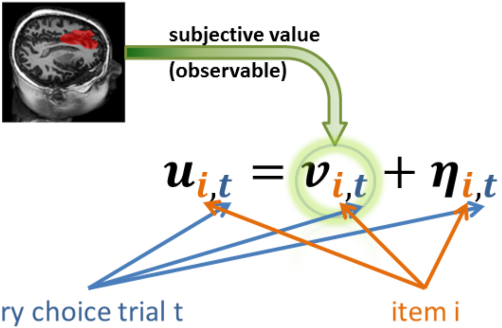
subjective value

(observable)

$$u_{i,t} = v_{i,t} + \eta_{i,t}$$

Note: subjective value can be measured even in the absence of the choice set

Neural Random Utility Model



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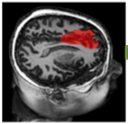
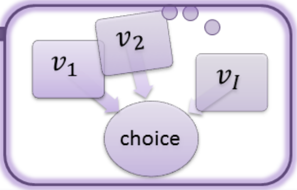
Neural Random Utility Model



subjective value
(observable)

choice
stochasticity

Choice Mechanism



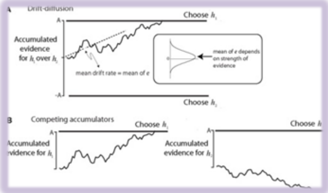
$$u_{i,t} = v_{i,t} + \eta_{i,t}$$

binary choice trial t

item i

stochasticity in
subjective value due to
perception (eg: temperature)

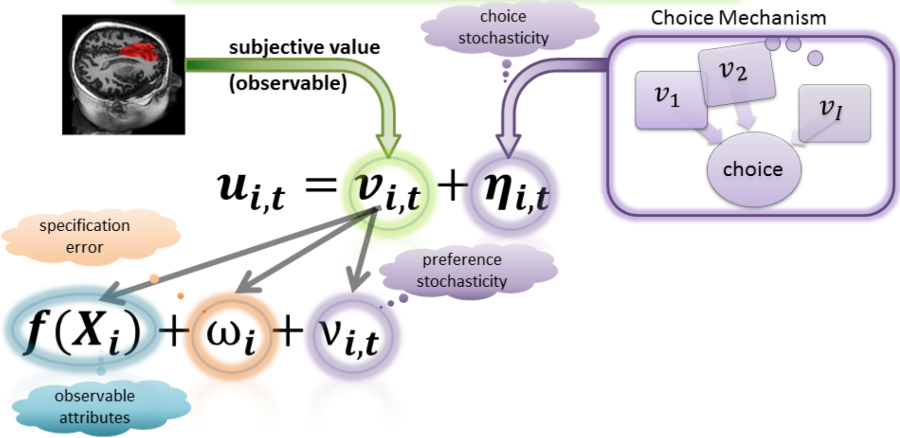
preference
stochasticity

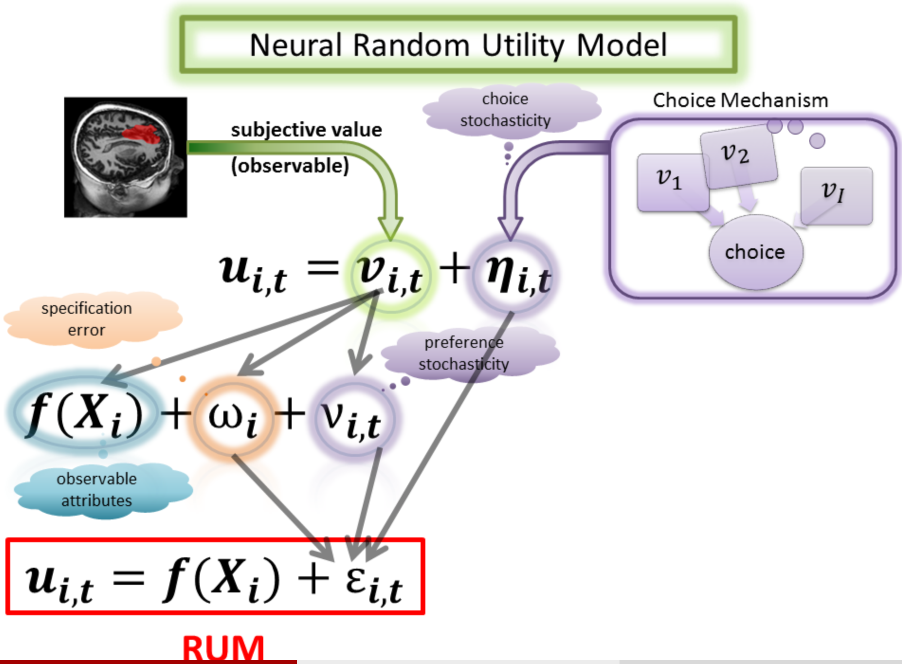


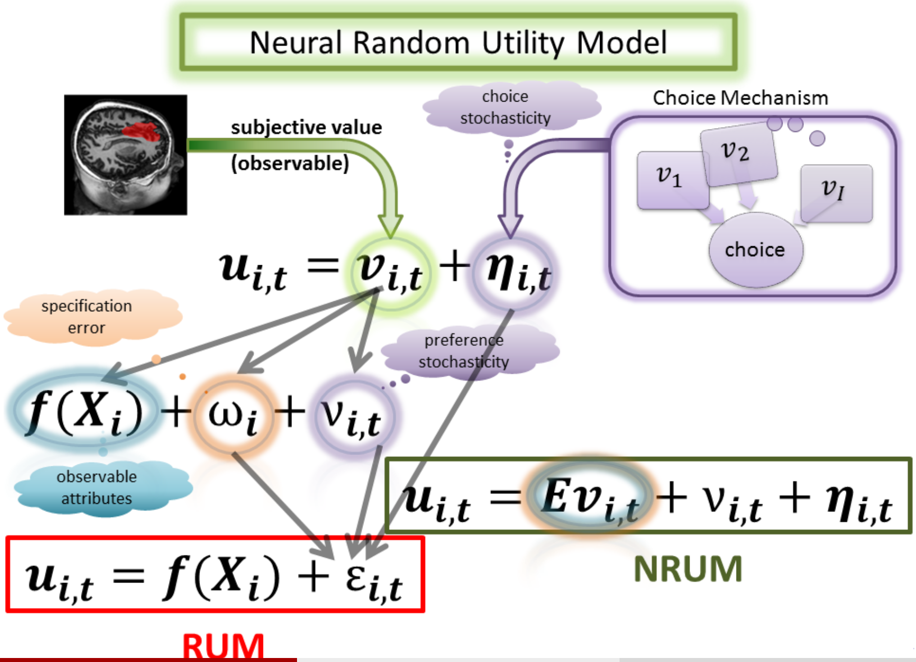
Ryan Webb (2013):
Drift Diffusion implies Random Utility

Note: subjective value can be measured
even in the absence of the choice set

Neural Random Utility Model




Neural Random Utility Model
subjective value
(observable)
choice
stochasticity
Choice Mechanism
v_1
v_2
v_I
choice
$u_{i,t} = v_{i,t} + \eta_{i,t}$
specification
error
preference
stochasticity
$f(X_i) + \omega_i + \nu_{i,t}$
observable
attributes
$u_{i,t} = f(X_i) + \varepsilon_{i,t}$
RUM


Neural Random Utility Model
subjective value
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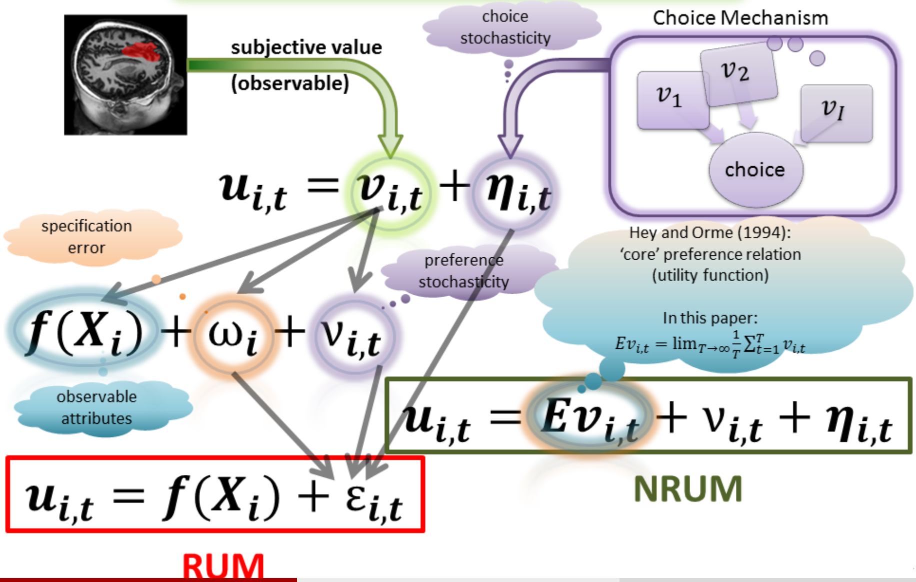
Neural Random Utility Model



- $u_{i,t} = v_{i,t} + \eta_{i,t}$
- DM chooses i vs. j on trial t if $u_{i,t} > u_{j,t}$ (consider only binary choices) $\Rightarrow y_{ij,t} = \mathbf{1}\left(u_{i,t} > u_{j,t}\right)$
- $\mathrm{P}\left[y_{ij,t} = 1 \mid v_{i,t}, v_{j,t}\right] = \mathrm{P}\left[\tilde{v}_{ij,t} > \tilde{\eta}_{ji,t} \mid v_{i,t}, v_{j,t}\right]$, where $\tilde{v}_{ij,t} \equiv v_{i,t} - v_{j,t}$, $\tilde{\eta}_{ji,t} \equiv \eta_{j,t} - \eta_{i,t}$
- assume[4] $\boxed{\tilde{\eta}_{ji,t} \sim \text{iid } \mathcal{N}\left(0, \sigma^2_{\tilde{\eta}}\right)} \leftarrow$ A1

$\Rightarrow \mathrm{P}\left[y_{ij,t} = 1 \mid v_{i,t}, v_{j,t}\right] = \Phi\left(\frac{\tilde{v}_{ij,t}}{\sigma_{\tilde{\eta}}}\right)$

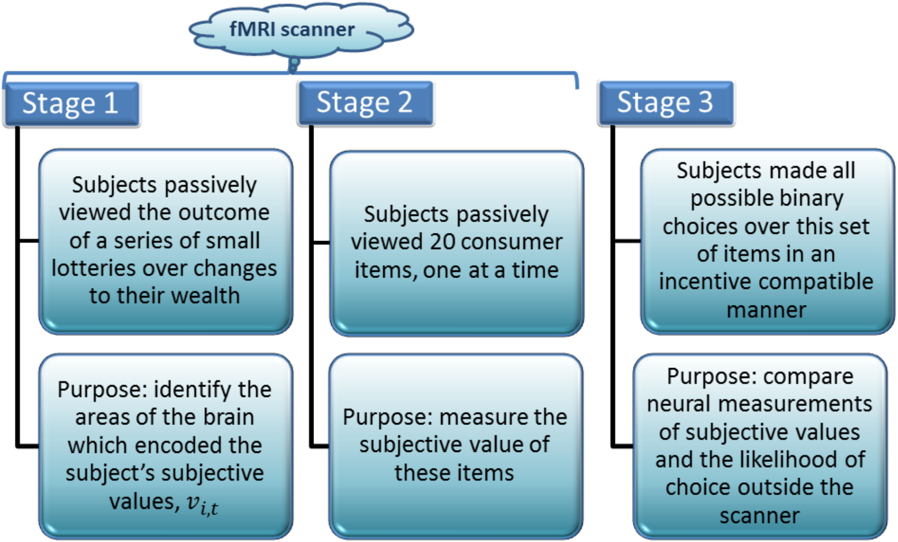
- assume $\boxed{v_t = \left(v_{1,t}, \ldots, v_{I,t}\right) \text{ is independent over trials}} \leftarrow$ A2
- $\nu_{i,t} \equiv v_{i,t} - \mathbb{E}\left[v_{i,t}\right]$ (mean over trials)

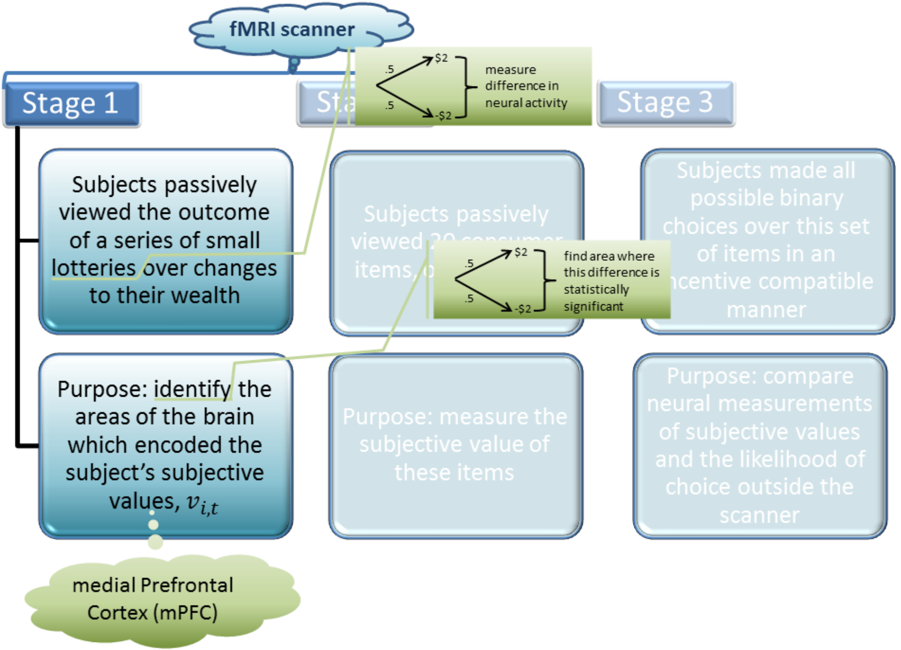
$\Rightarrow \mathrm{P}\left[y_{ij,t} = 1 \mid \mathbb{E}\left[v_{i,t}\right], \mathbb{E}\left[v_{j,t}\right]\right] = \mathrm{P}\left[\mathbb{E}\left[\tilde{v}_{ij,t}\right] > \tilde{\nu}_{ij,t} + \tilde{\eta}_{ji,t} \mid \mathbb{E}\left[v_{i,t}\right], \mathbb{E}\left[v_{j,t}\right]\right]$

- assume $\boxed{\tilde{\nu}_{ij,t} \equiv \nu_{i,t} - \nu_{j,t} \sim \text{iid } \mathcal{N}\left(0, \sigma^2_{\tilde{\nu}}\right)} \leftarrow$ A3

$\Rightarrow \mathrm{P}\left[y_{ij,t} = 1 \mid \mathbb{E}\left[v_{i,t}\right], \mathbb{E}\left[v_{j,t}\right]\right] = \Phi\left(\frac{\mathbb{E}\left[\tilde{v}_{ij,t}\right]}{\sigma_{\tilde{\eta}+\tilde{\nu}}}\right)$, where $\sigma^2_{\tilde{\eta}+\tilde{\nu}} = \sigma^2_{\tilde{\eta}} + \sigma^2_{\tilde{\nu}}$

[4] Item-pair independence follows from the binary choice setup: realizations for different item-pairs must occur on different trials


fMRI scanner
Stage 1
Subjects passively viewed the outcome of a series of small lotteries over changes to their wealth
Purpose: identify the areas of the brain which encoded the subject's subjective values, $v_{i,t}$
Stage 2
Subjects passively viewed 20 consumer items, one at a time
Purpose: measure the subjective value of these items
Stage 3
Subjects made all possible binary choices over this set of items in an incentive compatible manner
Purpose: compare neural measurements of subjective values and the likelihood of choice outside the scanner


fMRI scanner
Stage 1
St
Stage 3
.5
$2
.5
-$2
measure difference in neural activity
Subjects passively viewed the outcome of a series of small lotteries over changes to their wealth
Subjects passively
viewed
items,
Subjects made all possible binary choices over this set of items in an
ncentive compatible manner
.5
$2
.5
-$2
find area where this difference is statistically significant
Purpose: identify the areas of the brain which encoded the subject's subjective values, $v_{i,t}$
Purpose: measure the subjective value of these items
Purpose: compare neural measurements of subjective values and the likelihood of choice outside the scanner
medial Prefrontal Cortex (mPFC)

fMRI scanner

four DVD movies, two books, four art posters, three music CDs, two pieces of stationery, and five monetary lotteries

Stage 2

Subjects passively viewed the outcome of a series of small lotteries over changes to their wealth

Subjects passively viewed 20 consumer items, one at a time

All items were presented 12 times in random order to each subject. On 20 randomly selected trials (which were excluded from analysis), subjects were asked whether they preferred the item they had just seen or a randomly selected amount of money (from \$1 to \$10). Subjects were told that one of these question trials would be randomly realized at the end.

of items in an

Subjects were thinking about the value of the items they were watching

by observing the neural activity in mPFC

Purpose: identify areas of the brain which encoded the subject's subjective values, $v_{i,t}$

Purpose: measure the subjective value of these items

Purpose: compare neural measurements of subjective values and the likelihood of choice outside the scanner

$v_{i,m}, i = 1, \ldots, 20, m = 1, \ldots 11$
for each subject

fMRI scanner

Stage 1

Subjects passively viewed the outcome of a series of small lotteries over changes to their wealth

Purpose: identify the areas of the brain which encoded the subject's subjective values, $v_{i,t}$

Each possible binary comparison was presented **twice** (switching the left-right location on each repetition).

The result of one of these choices was randomly selected for realization.

items, one at a time

The choices of subjects were largely consistent (mostly transitive and non-random).

Choices were highly idiosyncratic across subjects.

subjective value of

The goal of this experiment is to determine **whether subjective value measured in the** ***absence*** **of choice can be used to predict** ***later*** **choices**

Stage 3

Subjects made all possible binary choices over this set of items in an incentive compatible manner

Purpose: compare neural measurements of subjective values and the likelihood of choice outside the scanner

- Stage 2 $\rightarrow v_{im}$, $i = 1, \ldots, 20$, $m = 1, \ldots, 11$ for each subject

$\rightarrow$ rank $\bar{v}_i = \frac{1}{11} \sum_{m=1}^{11} v_{im}$ to order the items

- Compare to Stage 3: prediction rate is $59 \pm 1\%$ (i.e., in $59 \pm 1\%$ of trials subjects chose according to this ordering) $\rightarrow$ not much!

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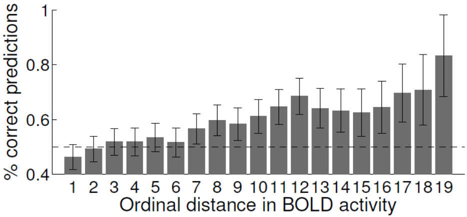
- Can do better!
  - segregate prediction accuracy according to the rank-distance in neural activity between two items



$\Rightarrow$ ordering of subjective values can predict choice outcomes

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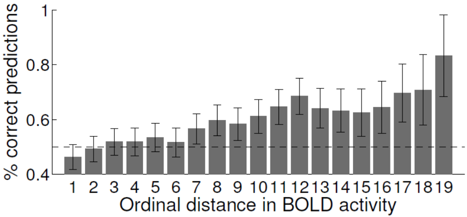

$\Rightarrow$ ordering of subjective values can predict choice outcomes

Q: Is subjective value a cardinal quantity? $\Rightarrow$ NRUM

$$\mathrm{P}\left[y_{ij,t} = 1 \mid v_{i,t}, v_{j,t}\right] = \Phi\left(\frac{\tilde{v}_{ij,t}}{\sigma_{\tilde{\eta}}}\right) \quad \text{vs}$$

$$\boxed{\mathrm{P}\left[y_{ij,t} = 1 \mid \mathbb{E}\left[v_{i,t}\right], \mathbb{E}\left[v_{j,t}\right]\right] = \Phi\left(\frac{\mathbb{E}\left[\tilde{v}_{ij,t}\right]}{\sigma_{\tilde{\eta}+\tilde{\nu}}}\right)}$$

Do not observe $v_{i,t}$ on the trial t in which choice was made

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Do not observe $v_{i,t}$ on the trial t in which choice was made

To get $\mathbb{E}\left[\tilde{v}_{ij,t}\right]$:

Blood-Oxygenation Level Dependent (BOLD) signal

$$B_{i,m}=a+\gamma v_{i,m}+\mu_{i,m}, \quad \mu_{i,m} \sim \text{iid } \mathcal{N}\left(0, \sigma_\mu^2\right)$$

measurement error

$$\bar{B}_i=a+\gamma \bar{v}_i+\bar{\mu}_i \quad \text{(average over } m\text{)}$$

$$\tilde{\bar{B}}_{ij}=\gamma \tilde{\bar{v}}_{ij}+\tilde{\bar{\mu}}_{ij} \quad \text{(take difference)}$$

Note: Orderings of $B_{i,m}$ and $v_{i,m}$ coincide

Ignoring Measurement Error

$$P\left[y_{ij,t}=1 \mid \mathbb{E}\left[v_{i,t}\right], \mathbb{E}\left[v_{j,t}\right]\right]=\Phi\left(\frac{\mathbb{E}\left[\tilde{v}_{ij,t}\right]}{\sigma_{\tilde{\eta}+\tilde{\nu}}}\right)$$

$$\mathbb{E}\left[\tilde{v}_{ij,t}\right]: \quad \tilde{\bar{B}}_{ij}=\gamma \tilde{\bar{v}}_{ij}+\tilde{\bar{\mu}}_{ij}$$

$$P\left[y_{ij,t}=1 \mid \tilde{\bar{B}}_{ij}\right]=\Phi\left(\frac{\gamma^{-1}}{\sigma_{\tilde{\eta}+\tilde{\nu}}}\tilde{\bar{B}}_{ij}\right)$$

Probit model:

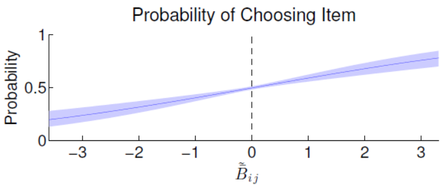
$$\boxed{P\left(y_{ij,t}=1\right)=\Phi\left(c+\frac{\gamma^{-1}}{\sigma_{\tilde{\eta}+\tilde{\nu}}}\tilde{\bar{B}}_{ij}\right)}$$

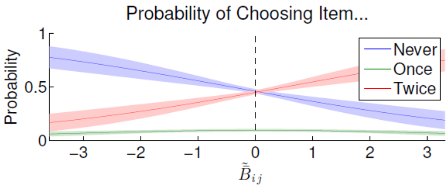
Coefficient	No Constant	Constant
$\gamma^{-1}/\sigma_{\tilde{\nu}+\tilde{\eta}}$	0.24 (0.10)	0.24 (0.10)
c		-0.01 (0.08)

$$y_{ij} = \begin{cases} 0, & y_{ij,1} = y_{ij,2} = 0 \\ 1, & y_{ij,1} + y_{ij,2} = 1 \\ 2, & y_{ij,1} = y_{ij,2} = 1 \end{cases} \qquad \begin{aligned} \mathrm{P}(y_{ij}=0) &= \left(1 - \Phi\left(\frac{\gamma^{-1}}{\sigma_{\tilde{\eta}+\tilde{\nu}}}\tilde{\bar{B}}_{ij}\right)\right)^2 \\ \mathrm{P}(y_{ij}=1) &= 2\left(1 - \Phi\left(\frac{\gamma^{-1}}{\sigma_{\tilde{\eta}+\tilde{\nu}}}\tilde{\bar{B}}_{ij}\right)\right)\Phi\left(\frac{\gamma^{-1}}{\sigma_{\tilde{\eta}+\tilde{\nu}}}\tilde{\bar{B}}_{ij}\right) \\ \mathrm{P}(y_{ij}=2) &= \Phi^2\left(\frac{\gamma^{-1}}{\sigma_{\tilde{\eta}+\tilde{\nu}}}\tilde{\bar{B}}_{ij}\right) \end{aligned}$$

$$\Rightarrow \quad \mathrm{P}(y_{ij}=0) < \mathrm{P}(y_{ij}=2) < \mathrm{P}(y_{ij}=1) \quad \text{for small positive } \tilde{\bar{B}}_{ij}$$

Data: too few *once* choices when $\tilde{\bar{B}}_{ij}$ is small (ordered Probit model)





$\Rightarrow$ need to account for measurement error: $\tilde{\bar{B}}_{ij} = \gamma\tilde{\bar{v}}_{ij} + \tilde{\bar{\mu}}_{ij}$

Intuition: small $\tilde{\bar{v}}_{ij}$ for which *once* is most likely might correspond to large $\tilde{\bar{B}}_{ij}$ due to measurement error

Accounting for Measurement Error

$$P\left[y_{ij,t}=1 \mid \mathbb{E}\left[v_{i,t}\right], \mathbb{E}\left[v_{j,t}\right]\right]=\Phi\left(\frac{\mathbb{E}\left[\tilde{v}_{ij,t}\right]}{\sigma_{\tilde{\eta}+\tilde{\nu}}}\right) \qquad \mathbb{E}\left[\tilde{v}_{ij,t}\right] : \tilde{\tilde{B}}_{ij}=\gamma\tilde{\tilde{v}}_{ij}+\tilde{\tilde{\mu}}_{ij}$$

$$\boxed{P\left[y_{ij,t}=1 \mid \tilde{\tilde{B}}_{ij}, \tilde{\tilde{\mu}}_{ij}\right]=\Phi\left(\frac{\gamma^{-1}(\tilde{\tilde{B}}_{ij}-\tilde{\tilde{\mu}}_{ij})}{\sigma_{\tilde{\eta}+\tilde{\nu}}}\right)}$$

$$\mu_{i,m} \sim \text{iid } \mathcal{N}\left(0,\sigma_{\mu}^{2}\right) \quad \Rightarrow \quad \tilde{\tilde{\mu}}_{ij} \sim \mathcal{N}\left(0,\sigma_{\tilde{\tilde{\mu}}}^{2} \equiv \frac{2}{11}\sigma_{\mu}^{2}\right)$$

Random-effects Probit model:

$$\boxed{P\left[y_{ij,1}, y_{ij,2} \mid \tilde{\tilde{B}}_{ij}\right]=\int_{-\infty}^{+\infty}\frac{e^{-\tilde{\tilde{\mu}}_{ij}^{2}/2\sigma_{\tilde{\tilde{\mu}}}^{2}}}{\sqrt{2\pi}\sigma_{\tilde{\tilde{\mu}}}}\left[\prod_{t=1}^{2}P\left[y_{ij,t} \mid \tilde{\tilde{B}}_{ij}, \tilde{\tilde{\mu}}_{ij}\right]\right] d\tilde{\tilde{\mu}}_{ij}}$$

Accounting for Measurement Error

Random-effects Probit model:

$$P\left[y_{ij,1}, y_{ij,2} \mid \tilde{\bar{B}}_{ij}\right] = \int_{-\infty}^{+\infty} \frac{e^{-\tilde{\bar{\mu}}_{ij}^{2}/2\sigma_{\tilde{\bar{\mu}}}^{2}}}{\sqrt{2\pi}\sigma_{\tilde{\bar{\mu}}}} \left[\prod_{t=1}^{2} P\left[y_{ij,t} \mid \tilde{\bar{B}}_{ij}, \tilde{\bar{\mu}}_{ij}\right]\right] d\tilde{\bar{\mu}}_{ij}$$

$$P\left[y_{ij,t} = 1 \mid \tilde{\bar{B}}_{ij}, \tilde{\bar{\mu}}_{ij}\right] = \Phi\left(\frac{\gamma^{-1}(\tilde{\bar{B}}_{ij} - \tilde{\bar{\mu}}_{ij})}{\sigma_{\tilde{\eta}+\tilde{\nu}}}\right)$$

Caveats:

1. $\tilde{\bar{B}}_{ij}$ and $\tilde{\bar{\mu}}_{ij}$ are not independent: $\mathbb{C}\mathrm{ov}\left(\tilde{\bar{B}}_{ij}, \tilde{\bar{\mu}}_{ij}\right) = 2\mathbb{V}\mathrm{ar}\left[\bar{\mu}_i\right] = \frac{2}{11}\sigma_\mu^2$

$\Rightarrow$ RE Probit estimate of $\frac{\gamma^{-1}}{\sigma_{\tilde{\eta}+\tilde{\nu}}}$ will be biased towards zero

2. $\tilde{\bar{\mu}}_{ij}$ are not independent over choice pairs: $\mathbb{C}\mathrm{ov}\left(\tilde{\bar{\mu}}_{ij}, \tilde{\bar{\mu}}_{ij'}\right) = \mathbb{V}\mathrm{ar}\left[\bar{\mu}_i\right]$

$\Rightarrow$ RE Probit estimate of standard errors will be biased towards zero

$\Rightarrow$ use multi-way clustering techniques (Cameron et al., 2011)

Coefficient	Probit		RE Probit	
	No Constant	Constant	No Constant	Constant
$\frac{\gamma^{-1}}{\sigma_{\tilde{\eta}+\tilde{\nu}}}$	0.24 (0.10)	0.24 (0.10)	1.16 (0.52)	1.16 (0.51)
c		-0.01 (0.08)		-0.06 (0.37)
$\frac{\sigma^2_{\tilde{\tilde{\mu}}}}{\gamma^2\sigma^2_{\tilde{\eta}+\tilde{\nu}}}$			22.36 (3.49)	22.36 (3.50)

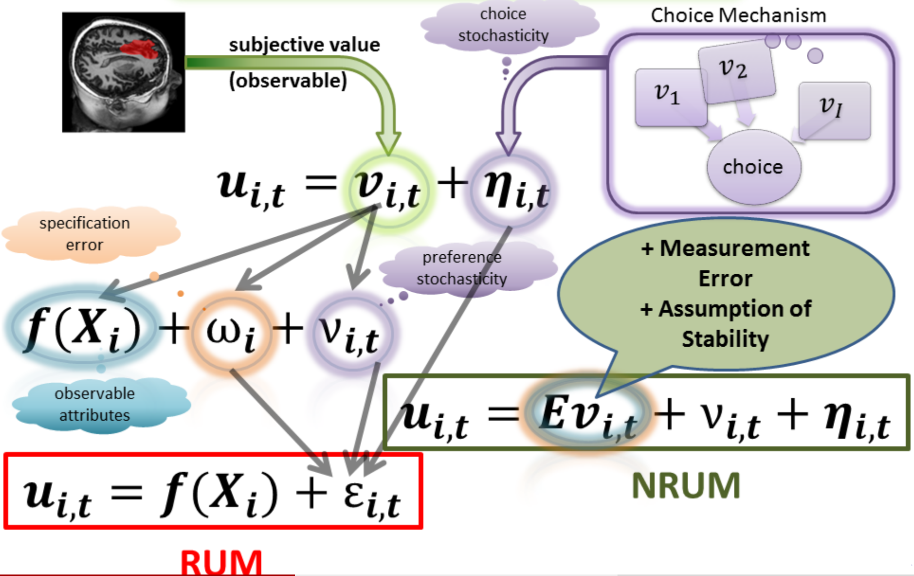
Subject specific RE Probit

Coeff	Est.	Std. Err.	P-Val	Coeff	Est.	Std. Err.	P-Val
c_1	0.03	1.14	0.98	γ_1^{-1}	-1.17	1.07	0.27
c_2	-0.15	1.25	0.91	γ_2^{-1}	0.66	2.89	0.82
c_3	-0.07	1.27	0.95	γ_3^{-1}	-3.25	2.36	0.17
c_4	-0.34	1.17	0.77	γ_4^{-1}	10.14	2.90	0.00
c_5	0.08	1.22	0.95	γ_5^{-1}	1.39	0.57	0.02
c_6	-0.07	1.22	0.95	γ_6^{-1}	-3.23	2.50	0.20
c_7	-0.14	1.30	0.91	γ_7^{-1}	2.78	3.30	0.40
c_8	0.41	1.22	0.73	γ_8^{-1}	10.39	3.53	0.00
c_9	-0.18	1.18	0.88	γ_9^{-1}	4.98	2.38	0.04
c_{10}	0.69	1.24	0.58	γ_{10}^{-1}	5.01	1.39	0.00
c_{11}	0.07	1.23	0.95	γ_{11}^{-1}	2.61	3.18	0.41
c_{12}	-0.44	1.14	0.70	γ_{12}^{-1}	13.04	3.80	0.00
$\sigma^2_{\tilde{\mu}}$	20.49	3.46					

Note: $\sigma_{\tilde{\eta}+\tilde{\nu}} = 1$

- significant reduction of observations
- six γ_s^{-1} are significant and positive / six γ_s^{-1} are not significantly different from zero

Neural Random Utility Model



- The prediction based on NRUM:
  - Simulate $y^*_{s,ij,1}$, $y^*_{s,ij,2}$ using

$$P\left[y_{s,ij,t} = 1 \mid \tilde{\tilde{B}}_{s,ij}\right] = \Phi\left(\frac{\gamma_s^{-1}}{\sigma_{\tilde{\eta}+\tilde{\nu},s}}\tilde{\tilde{B}}_{s,ij}\right)$$

subject RE Probit estimate

  - If $y^*_{s,ij,1} + y^*_{s,ij,2} = y_{s,ij,1} + y_{s,ij,2}$, then *success*
- Compare to the prediction at chance:
  - Data: the frequency of $y_{s,ij,1} + y_{s,ij,2} = 0$ is 46%, $y_{s,ij,1} + y_{s,ij,2} = 1$ is 9%, $y_{s,ij,1} + y_{s,ij,2} = 2$ is 45%
  - Percent of correct predictions: $\frac{1}{4} \times 46 + \frac{1}{2} \times 9 + \frac{1}{4} \times 45 \approx 27\%$
- Compare to RUM:

$$P\left[y_{s,ij,t} = 1 \mid X_i, X_j\right] = \Phi\left((X_i - X_j)\beta_s\right)$$

'Amazon star' rating & price

	BOLD			Amazon*		Price		A+P*	P+B	A+P+B*
	RE		$\frac{\gamma^{-1}}{\sigma_{\tilde{\eta}+\tilde{\nu}}} = 10$	RE		RE		RE	RE	RE
	Pop	Sub	Pop	Pop	Sub	Pop	Sub	Sub	Sub	Sub
chance	27	27	27	27	27	27	27	27	27	27
pop	31	43	46	47	46	53	52	52	57	60
sub_1	29	36	36	55	60	60	63	62	62	62
sub_2	30	28	47	38	26	54	55	27	55	47
sub_3	24	49	29	33	35	46	40	44	51	45
sub_4	32	53	53	46	45	62	66	56	71	65
sub_5	45	48	59	65	72	54	54	79	57	77
sub_6	26	40	35	65	70	59	61	71	64	75
sub_7	28	33	45	44	39	41	29	65	33	50
sub_8	30	49	49	47	45	50	47	45	56	70
sub_9	35	50	53	41	35	59	62	47	64	59
sub_{10}	33	47	51	48	48	45	42	48	52	54
sub_{11}	30	33	41	43	33	57	59	46	60	46
sub_{12}	32	51	49	42	37	48	47	38	56	62

Table IV: Choice prediction rates (%) resulting from 1000 simulated samples generated by our estimates. Prediction rates are calculated for both (Pop)ulation and (Sub)ject-based estimates, and prediction rates are shown for the (pop)ulation as a whole and for each (sub)ject. Prediction rates are also calculated using both (A)mazon and (P)rice observables, (P)rice and the (B)OLD measure, and all three predictors. *Amazon ratings were not available for the five lotteries, so choice pairs with the lotteries were excluded for these sets of predictions.

	BOLD			Amazon*		Price		A+P*	P+B	A+P+B*
	RE		$\frac{\gamma^{-1}}{\sigma_{\tilde{\eta}+\tilde{\nu}}}=10$	RE		RE		RE	RE	RE
	Pop	Sub	Pop	Pop	Sub	Pop	Sub	Sub	Sub	Sub
chance	27	27	27	27	27	27	27	27	27	27
pop	31	43	46	47	46	53	52	52	57	60
sub_1	29	36	36	55	60	60	63	62	62	62
sub_2	30	28	47	38	26	54	55	27	55	47
sub_3	24	49	29	33	35	46	40	44	51	45
sub_4	32	53	53	46	45	[illegible]	[illegible]	[illegible]	[illegible]	65
[illegible]	45	48	59	65	72	[illegible]	[illegible]	[illegible]	[illegible]	77
[illegible]	26	40	35	65	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	75
[illegible]	28	33	45	44	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
sub_8	30	49	49	47	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	70
sub_9	35	[illegible]	[illegible]	41	35	[illegible]	[illegible]	[illegible]	[illegible]	59
sub_{10}	33	[illegible]	[illegible]	48	48	45	42	[illegible]	52	54
sub_{11}	30	[illegible]	[illegible]	43	33	57	59	[illegible]	[illegible]	46
sub_{12}	32	[illegible]	[illegible]	42	37	48	[illegible]	[illegible]	[illegible]	[illegible]

biased estimate

calibrated estimate

Evidence of stability of $Ev_{i,t}$ in this choice experiment

no context effects

Table IV: Choice prediction rates (%) resulting from 1000 simulated samples generated by our estimates. Prediction rates are calculated for both (Pop)ulation and (Sub)ject-based estimates, and prediction rates are shown for the (pop)ulation as a whole and for each (sub)ject. Prediction rates are also calculated using both (A)mazon and (P)rice observables, (P)rice and the (B)OLD measure, and all three predictors. *Amazon ratings were not available for the five lotteries, so choice pairs with the lotteries were excluded for these sets of predictions

	BOLD RE Pop	BOLD RE Sub	BOLD $\frac{\gamma^{-1}}{\sigma_{\tilde{\eta}+\tilde{\nu}}} = 10$ Pop	Amazon* RE Pop	Amazon* RE Sub	Price RE Pop	Price RE Sub	A+P* RE Sub	P+B RE Sub	A+P+B* RE Sub
chance	27	27	27	27	27	27	27	27	27	27
pop	31	43	46	47	46	53	52	52	57	60
sub_1	29	36	36	55	60	60	63	62	62	62
sub_2	30	28	47	38	26	54	55	27	55	47
sub_3	24	49	29	[illegible]	35	46	40	44	51	45
sub_4	32	53	53	46	45	[illegible]	[illegible]	56	71	65
sub_5	45	48	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	57	77
sub_6	26	40	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	64	75
sub_7	28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	33	50
sub_8	30	49	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	70
sub_9	35	50	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	47	64	59
sub_{10}	33	47	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	48	52	54
sub_{11}	30	33	41	43	[illegible]	57	59	46	[illegible]	[illegible]
sub_{12}	32	51	49	42	37	48	[illegible]	[illegible]	[illegible]	[illegible]

NRUM just matches the performance of a coarse behavioral model

Neural value measure can add predictive power to behavioral model

Table IV: Choice prediction rates (%) resu[illegible] ples generated by our estimates. Predicti[illegible] (Pop)ulation and (Sub)ject-based estimates, and pr[illegible] the (pop)ulation as a whole and for each (sub)ject. Prediction rates are also calculated using both (A)mazon and (P)rice observables, (P)rice and the (B)OLD measure, and all three predictors. *Amazon ratings were not available for the five lotteries, so choice pairs with the lotteries were excluded for these sets of predictions.

Main Contribution:

1. An econometric framework for relating neural measurements to choice prediction, the Neural Random Utility Model, was introduced.
2. The comparison of the predictive power of NRUM with established techniques was done based on data from a laboratory experiment:
    - the measured neural activity cardinally encodes valuations and predict choice behavior
    - accounting for measurement error and combining neural data with standard observables improves predictive performance