Estimating a Dynamic Oligopolistic Game with Serially Correlated Unobserved Production Costs

SS223B-Empirical IO

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- However, incorporating unobserved (*to the researcher*) state variables that are **serially correlated and endogenous** remains prohibitively difficult.
- In this paper the authors propose a likelihood based method relying on sequential importance sampling to estimate dynamic discrete games of complete information with **serially correlated unobserved endogenous state variables.**

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- This application is interesting because the firm specific production costs are serially correlated unobserved state variables that are endogenous to past entry decisions.
- It is worth to note that the proposed method is applicable to similar games that have a Markovian representation of the latent dynamics and an algorithm to solve the game.

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- In order to evaluate the effects of current experience on future market performance as measured by future costs and entry, they formulate and estimate a dynamic game theoretic model of oligopolistic competition.
- In a dynamic setting, current entry can have a potential spillover effect on future entry.

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- It allows for serially correlated firm specific costs that evolve endogenously based on past entry decisions.
- Furthermore, endogeneity of costs to past entry decisions induces heterogeneity among firms even if they are identical ex ante, which they need not be.
- They estimate the model parameters using **Bayesian MCMC methods.**

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- A market opening is defined to be an entry opportunity that becomes available to generic manufacturers each time a branded product goes off patent.
- The actions available to firm i when market t opens are to enter or not, which is denoted as

$$A_{i,t} = \begin{cases} 1, & \text{If firm } i \text{ enter;} \\ 0, & \text{otherwise.} \end{cases}$$

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- The equation governing the log cost of firm i at market t is

$$c_{it} = \mu_c + \rho_c(c_{i,t-1} - \mu_c) - \kappa_c A_{i,t-1} + \sigma_c e_{it}, \tag{5}$$

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- μ_c is a location parameter that represents the overall average of the log cost over a long period of time.
- The autoregressive parameter ρ_c represents the degree of persistence between the current cost and its long run stationary level

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- The log cost can be decomposed into a sum of two components, a **known component** (or observable to the researcher based on past actions), $c_{k,i,t}$ and a component **unobservable to the researcher,** $c_{u,i,t}$ as follows:

The model

$$c_{i,t} = c_{u,i,t} + c_{k,i,t} \tag{6}$$

$$c_{u,i,t} = \mu_c + \rho_c(c_{u,i,t-1} - \mu_c) + \sigma_c e_{it} \tag{7}$$

$$c_{k,i,t} = \rho_c c_{k,i,t-1} - \kappa_c A_{i,t-1} \tag{8}$$

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$$c_{k,i,t} = \rho_c c_{k,i,t-1} - \kappa_c A_{i,t-1} \quad (11)$$

- The total (lump sum) revenue to be divided among firms who enter a market at time t is $R_t = exp(r_t)$, which is realized from the following independent and identical distribution,

$$r_t = \mu_r + \sigma_r e_{I+1,t}, \quad (12)$$

where $e_{I+1,t}$ is normally distributed with mean zero and unit variance.

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$$\Pi_{it} = A_{i,t} \times \left\{ \frac{R_t^{\gamma}}{N_t} - C_{it} \right\}, \tag{13}$$

where $\gamma \in (0.908, 1)$.

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where $\gamma \in (0.908, 1)$.

- The firms total discounted profit at time t is

$$\sum_{j=0}^{\infty} \beta^j \Pi_{it+j}, \quad 0 < \beta < 1. \tag{15}$$

Solving the Model

- The Bellman equation for the choice specific value function for firm i's dynamic problem at time t is given by

$$V_i(A_{i,t}, A_{-i,t}, C_{i,t}, C_{-i,t}, R_t) = \Pi_{i,t} + \mathbb{E}_{|\Omega_t}(V_i(A_{i,t}, A_{-i,t}, C_{i,t}, C_{-i,t}, R_t)), \tag{16}$$

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  where $\Omega_t = (A_{i,t}, A_{-i,t}, C_{i,t}, C_{-i,t}, R_t)$
- The solution concept is given by "em Pure Strategy Perfect Markov Equilibrium".
- The numerical scheme is as follows:

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2. For that parameter value, generate values for the latent variable over the sample period by means of the importance sampler.
3. Solve the dynamic game to compute the equilibrium outcome as function of the observed and unobserved state variables and the parameter value.
4. Use the equilibrium outcome generated from the solution to compute a likelihood that depends on the observed data and latent state variables (at the given parameter value).

5. Integrate out the latent state variables by averaging the log likelihood over repetitions of the importance sampler to obtain a log likelihood that depends only observed variables (at the given parameter value).

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- **Cycling through steps (1) to (6) generates an MCMC chain that is a sample from the posterior distribution of the parameters from which the posterior mean, mode, standard deviation, etc. can be computed.**

Results

Table 2. Posterior Distribution

Parameter	Number of Potential Entrants (excluding "other" firms) 3 firms	4 firms
μ_c	10.04 (0.030)	10.05 (0.0053)
ρ_c	0.9845 (0.0021)	0.9872 (0.00015)
σ_c	0.3714 (0.015)	0.3695 (0.0013)
κ_c	0.06139 (0.0043)	0.07090 (0.00027)
μ_r	9.921 (0.11)	10.01 (0.0088)
σ_r	1.515 (0.080)	1.677 (0.0049)
γ	0.9375	0.9375
β	0.96875	0.96875
p_a	0.9375	0.9375
CER firm 1	0.09	0.11
CER firm 2	0.09	0.09
CER firm 3	0.10	0.10
CER firm 4		0.16
CER all firms	0.10	0.11
MCMC Reps	3000000	3000000
stride	375	375

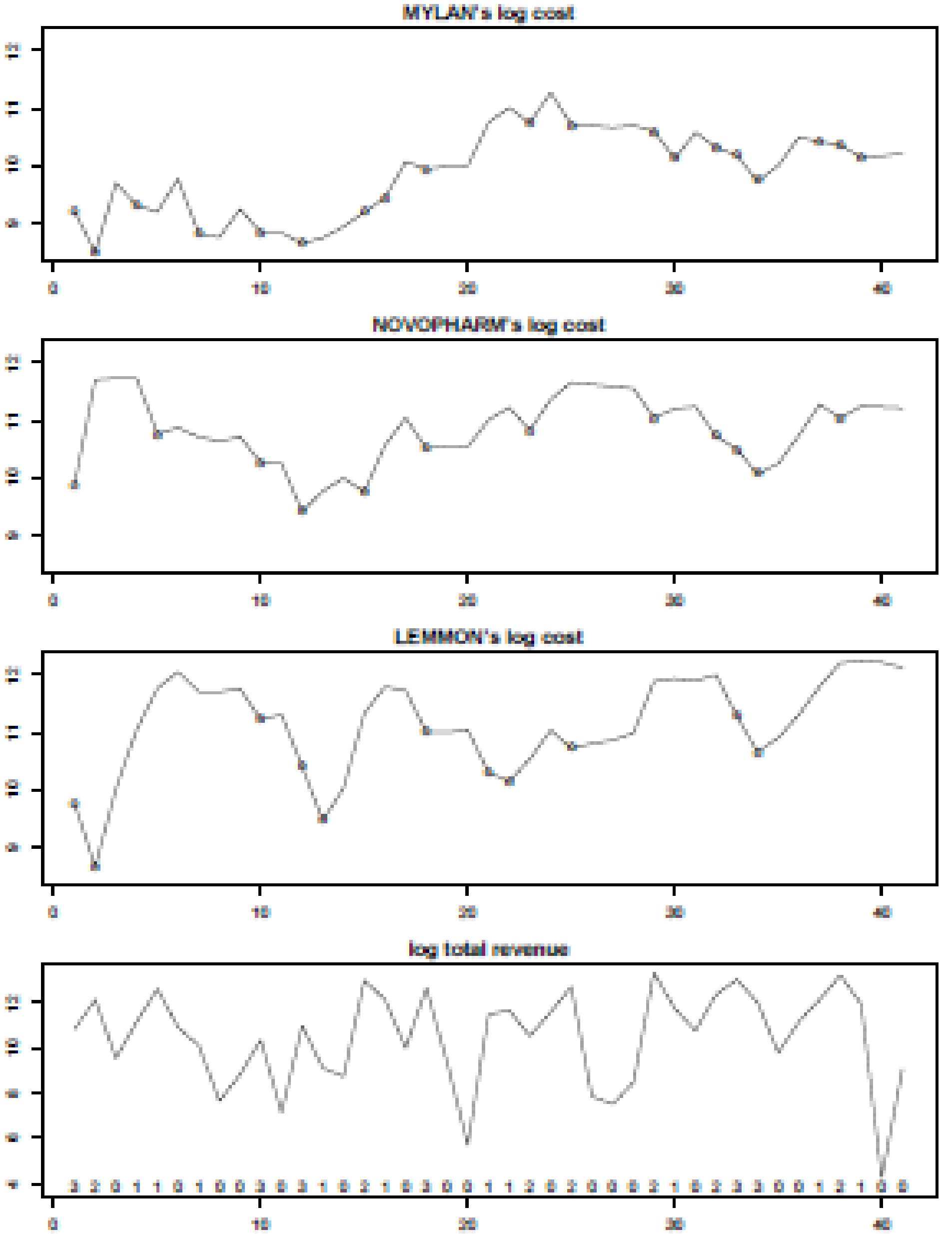

Figure 7. Cost, Revenue, and Entry Decisions. Plotted as a solid line in the first three panels is the logarithm of cost for the three dominant firms in the three firm model. The logarithm of cost is computed by averaging at Step 2c of the importance sampler at the maximum likelihood estimate. The circles in these plots indicate that the firm entered the market at that time point. The bottom panel shows the logarithm of total revenue. The numbers at the bottom are the count of the number of dominant firms who entered the market at that time point.

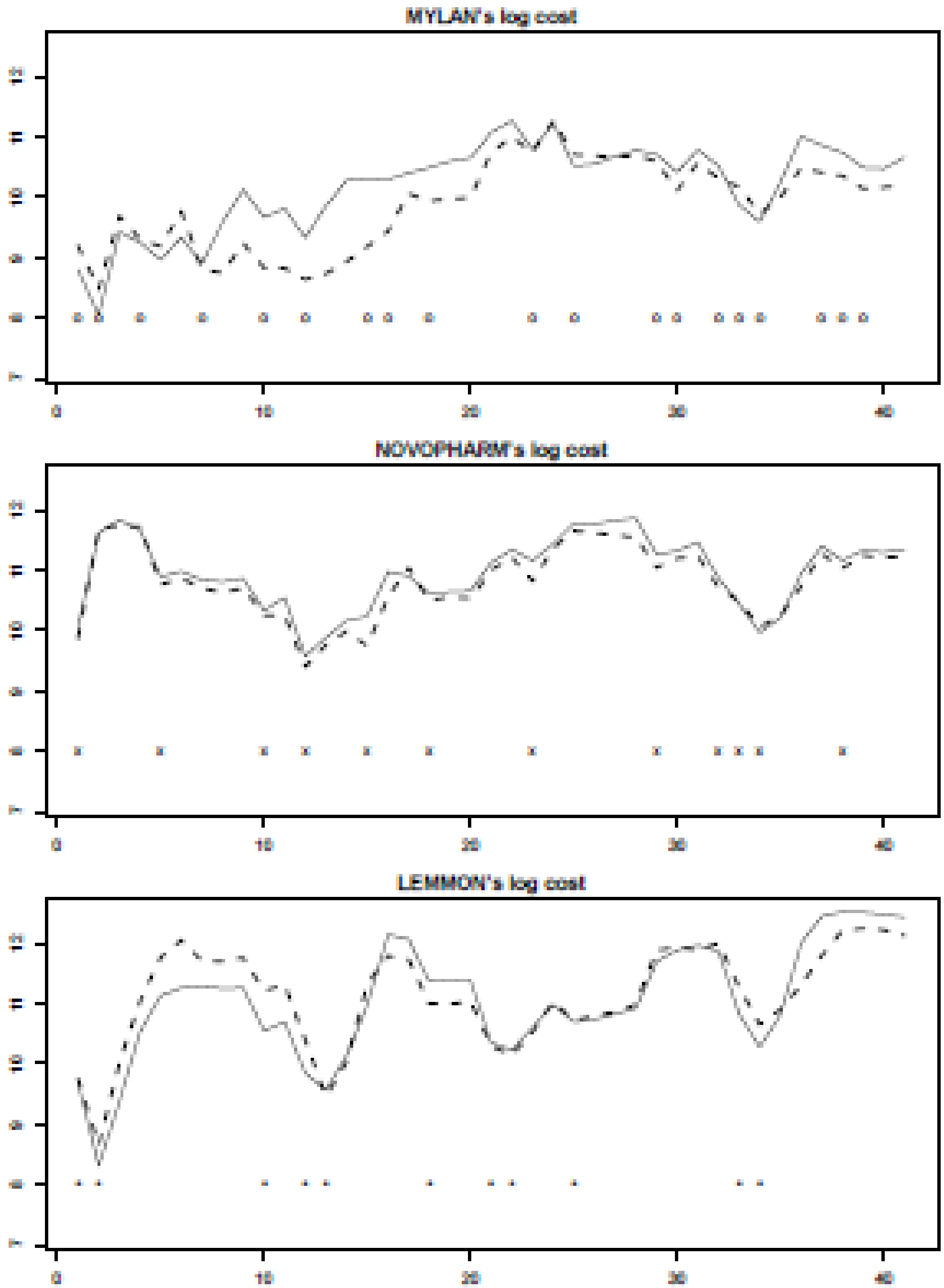

Figure 8. Cost and Entry Decisions of the Dominant Firms. Plotted is the logarithm of cost for the three dominant firms. The dashed line is under the three firm model, and the solid under the four firm model. The circles indicate the markets that Mylan entered, crosses the same for Novopharm, and the asterisks for Lemmon. The logarithm of cost as described in the legend of Figure 7.

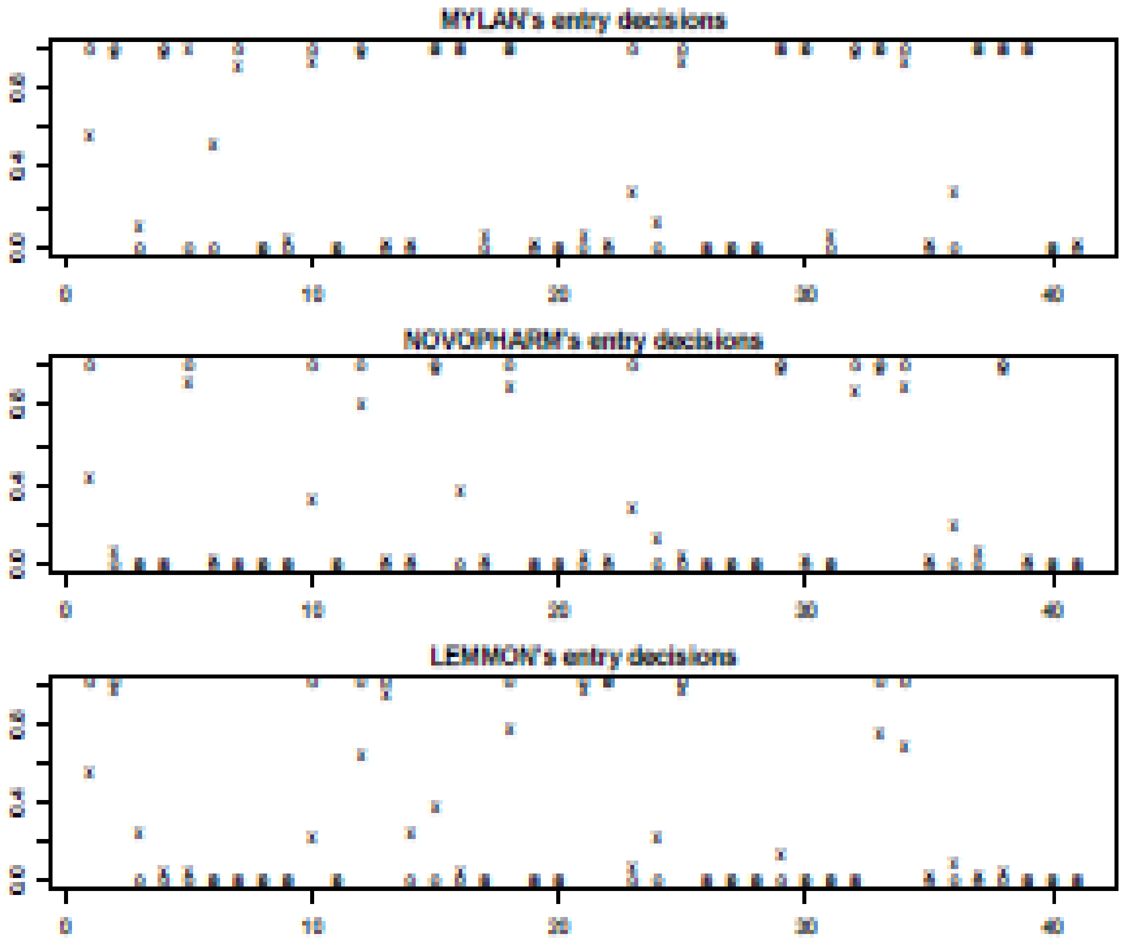

Figure 9. Actual and Predicted Entry Decisions. Plotted as circles are the entry decisions of the three dominant firms in the three firm model. The crosses are the average predictions of the three firm model computed by averaging game solutions at Step 2c of the importance sampler at the maximum likelihood estimate.