Estimating Dynamic Discrete Choice Models with Hyperbolic Discounting, with an Application to Mammography Decisions*

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Abstract

This paper extends the semi-parametric estimation method for dynamic discrete choice models using Hotz and Miller's (1993) conditional choice probability (CCP) approach to the setting where individuals may have hyperbolic discounting time preferences and may be naive about their time inconsistency. We implement the proposed estimation method to adult women's decisions to undertake mammography to evaluate the importance of present bias and naivety in the under-utilization of preventive health care. Preliminary results show evidence for both present bias and naivety.

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*Preliminary. All comments are welcome. Please do not circulate without the authors' permission.

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1 Introduction

Dynamic discrete choice models have been used to understand a wide range of economic behavior. The early dynamic discrete choice models that are empirically implemented tend to be parametric;[1] but recently, a growing list of authors have addressed the non- or semi-parametric identification of dynamic discrete choice models. The earliest attempt in this regard is Hotz and Miller (1993) which pioneered the approach of using conditional choice probabilities to infer about choice-specific continuation values. Rust (1994a, 1994b) showed that the discount factor in standard dynamic discrete choice models are generically not identified; Magnac and Thesmar (2002) expanded Rust's non-identification results, and proposed exclusive restrictions that lead to the identification of the standard discount factor (see more below).

All of the above-mentioned literature model the impatience of the decision makers by assuming that agents discount future streams of utility or profits *exponentially* over time. As is now well known, Strotz (1956, p.172) showed that exponential discounting is not just an analytically convenient assumption; without this assumption, intertemporal marginal rates of substitution will change as time passes, and preferences will be time-inconsistent. A recent theoretical literature has built on the work of Strotz (1956) and others to explore the consequences of relaxing the standard assumption of exponential discounting. Drawing both on experimental research and on common intuition, economists have built models of quasi-hyperbolic discounting to capture the tendency of decision makers to seize short term rewards at the expense of long-term preferences.[2] This literature studies the implications of time-inconsistent preferences, and their associated problems of

[1] The earliest formulation and estimation of parametric dynamic discrete choice models include Wolpin (1984) for fertility choice, Miller (1984) for occupational choice, Pakes (1986) for patent renewal, and Rust (1987) for bus engine replacement.

[2] A body of experimental research, reviewed in Ainslie (1992) and Loewenstein and Elster (1992), indicates that hyperbolic time discounting may parsimoniously explain some basic features of the intertemporal decision making that are inconsistent with simple models with exponential discounting. Specifically, standard decision models with exponential discounting are not easy to reconcile with commonly observed preference reversals: subjects choose the larger and later of two prizes when both are distant in time, but prefer the smaller but earlier one as both prizes draw nearer to the present (see Rubinstein 2003, however, for an alternative explanation of preference reversals).

self-control, for a variety of economic choices and environments.[3]

A small list of empirical papers that attempted to estimate dynamic models with hyperbolic discounting time preferences have followed the parametric approach (Fang and Silverman 2007, Laibson, Repetto and Tobacman 2007 and Paserman 2007).[4] Fang and Silverman (2007) empirically implement a dynamic structural model of labor supply and welfare program participation for never-married mothers with potentially time-inconsistent preferences. Using panel data on the choices of single women with children from the National Longitudinal Survey of Youth (NLSY 1979), they provide estimates of the degree of time-inconsistency, and of its influence on the welfare take-up decision. For the particular population of single mothers with dependent children, they estimate the present bias factor and the standard discount factor to be 0.338 and 0.88 respectively, implying a one-year ahead discount rate of 238%. Laibson, Repetto and Tobacman (2007) use Method of Simulated Moments (MSM) to estimate time preferences - both short and long run discount rates – from a structural buffer stock consumption model that includes many realistic features such as stochastic labor income, liquidity constraints, child and adult dependents, liquid and illiquid assets, revolving credit and retirement. Under parametric assumptions on the model, the model is identified from matching the model's predictions of retirement wealth accumulation, credit card borrowing and consumption-income co-movement with those observed in the data. Their benchmark estimates imply a 48.5% short-term annualized discount rate and a 4.3% long-term annualized discount rate. Paserman (2007) estimates the structural parameters of a model of job search with hyperbolic discounting and endogenous search effort, using data on duration of unemployment spells and accepted wages from the NLSY 1979. Under parametric assumptions of the model, identification

[3] For example, models of time-inconsistent preferences have been applied by Laibson (1997) and O'Donoghue and Rabin (1999a,b) to consumption and savings; by Barro (1999) to growth; by Gruber and Koszegi (2001) to smoking decisions; by Krusell, Kuruşçu, and Smith (2002) to optimal tax policy; by Carrillo and Mariotti (2000) to belief formation; and by Della Vigna and Paserman (2001) to job search.

[4] Also related, Arcidiacono, Sieg and Sloan (2007) estimate a parametric forward-looking dynamic discrete choice model of smoking and heavy drinking for late-middle age men in the Health and Retirement Studey (HRS) and find that a forward-looking model fits the data (mainly the age profile for heavy-drinking and smoking) better than a myopic model. Their model assumes exponential discounting and thus does not incorporate the possibility that time inconsistent preferences may play a role in the consumption of alcohol and cigaretts.

of the hyperbolic discounting parameters comes from the variation in the relative magnitude of unemployment duration and accepted wages. Indeed he finds that the results are sensitive to the specific structure of the model and on the functional form assumption for the distribution of offered wages. For low-wage workers, he rejects the exponential discounting model and estimates a one-year discount rate of about 149%.[5]

None of the above papers allow for the possibility that a hyperbolic discounting decision-maker may also be naive. Most importantly, the identification of the present bias and standard discount factors in these papers are often based on parametric assumptions imposed on the model. To the best of our knowledge, it is not known whether dynamic discrete choice models with hyperbolic discounting preferences can be semi-parametrically identified using standard short-panel data that are typically used in these papers.[6]

In this paper, we examine the conditions under which dynamic discrete choice models with hyperbolic discounting time preference can be partially identified using short-panel (two periods) data. We show that, if there exist *exclusion variables* that affect the transition probabilities of states over time but do not affect the decision-makers' static payoff functions, a condition that is similar to that in Magnac and Thesmar (2002) necessary for the identification of dynamic discrete choice models with standard exponential discounting, then we can potentially identify all three discount factors $\beta, \tilde{\beta}$ and δ.

The intuition for why exclusion variables that affect the transition of state variables but not static payoffs can provide source of identification for the discount factors can be simply described as follows. Consider two decision-makers who share the same period-payoff relevant state variables but differ only in the exclusion variables. Because their exclusion variables only affect the transition of the payoff-relevant state variables, their effects on the choices in the current period will inform us about the degree to which the agents discount the future. The intuition for why $\beta, \tilde{\beta}$ and δ can

[5]There are other inferential studies about discount rates that exploit specific clear-cut intertemporal trade-offs. For example, Hausman (1979), and Warner and Pleeter (2001) estimate discount rates ranging from 0 to 89% depending on the characteristics of the individual and intertemporal trade-offs at stake.

[6]The exception is Fang and Silverman (2006), which argued that exponential discounting and hyperbolic discounting models are distinguishable, using an argument based on observed choice probabilities.

be separately identified will be provided later in Section 2.4.

We provide two estimation approaches that are intimately related to our identification arguments. One approach is based on maximizing a pseudo-likelihood function and the other is based on minimizing the estimated variance of the utility functions. Monte Carlo experiments show that both estimators perform well in large samples, but in relatively small samples, the maximum pseudo-likelihood based estimator performs better. We thus use maximim pseudo-likelihood based estimator in our empirical application.

Our paper also represents an interesting intermediate case between the literature on estimating dynamic discrete choice single-agent decision problems (see Miller 1984, Wolpin 1984, Pakes 1986, Rust 1987, Hotz and Miller 1993 for early contributions and Rust 1994a, 1994b for surveys) and the more recent literature on estimating dynamic games (Pakes and McGuire 1994, Pakes, Ostrovsky and Berry 2007, Bajari, Benkard and Levin 2007). As is well-known, if an agent has hyperbolic discounting time preferences, the outcome of her decision process can be considered as the equilibrium outcome of an *intra-personal game* with the players being *the selves at different periods.* There are two crucial differences, however, between the intra-personal games we analyze for agents with time-inconsistent time preferences and those in the existing dynamic games literature. The first key difference is that in our case, we *do not observe* the actions of all the players. More specifically, the outcomes – choices and the evolutions of the state variables – we observe in the data are affected only by the current selves, even though the current selves' choices are impacted by their perception of future selves' actions. Secondly, the dynamic games literature (e.g. Bajari, Benkard and Levin 2007) may allow for players to have different period payoff functions, in our setting, however, the payoffs for the players – the current self and the future selves – differ only in time preferences; moreover, under hyperbolic discounting, we are assuming a rather restricted form of time preference differences between the players.

We also apply our identification and estimation method to investigate the role of time inconsistent preferences in women's mammography decisions. We consider a simple model where mammography can potentially lower the probability of death in the next two years, and it may lower the probability of bad health conditional on

surviving in two years, but undertaking mammography may involve immediate costs (most of which we would like to interpret as psychological and physical costs instead of financial costs). In particular, we use the indicator for either the woman's mother is still alive or she died at age greater than 70 as the exclusive variable that does not enter the instantaneous utility but affects the transition probability of other state variables that enter the instantaneous utility function. Our preliminary estimates indicate that individuals exhibit both present bias and naivety as $\tilde{\beta}/\beta$ **is estimated to be about 1.80, suggesting both** $\beta < 1$ **(present bias) and** $\tilde{\beta} > \beta$ **(naivety). We also estimate** $\tilde{\beta}\delta$ **to be about 0.86. These suggest that both present bias and naivety might have played an important role in the fact that nearly 25% of the women do not undertake mammography as adviced by American Cancer Association, which is universally regarded as a very cost effective way for early detection of breast cancer. [need to be updated]**

The remainder of the paper is structured as follows. In Section 2 we describe a general dynamic discrete choice model with hyperbolic discounting time preferences and provide detailed analysis for identification and estimation of the model, particularly the discount factors; we also evaluate the performance of our proposed estimation methods from Monte Carlo experiments. In Section 3 we provide the background information for mammography, which is the decision we examine in our empirical application; we also describe the data set used in our study and provide some basic descriptive statistics of the samples; we then provide details about the empirical specification of our model of the decision for undertaking mammography and present the preliminary main estimation results. In Section 4 we discuss a few important issues abstracted away in our analysis. Finally, Section 5 concludes.

2 Dynamic Discrete Choice Model with Hyperbolic Discounting Time Preferences

2.1 Basic Model Setup

Consider a decision maker whose intertemporal utility is additively time separable. The agent's instantaneous preferences are defined over the action she chooses from a discrete set of alternatives $i \in \mathcal{I} = \{0, 1, ..., I\}$, and a list of state variables denoted by $h \equiv (x, \boldsymbol{\varepsilon})$ where x, which for notational simplicity includes time t, are observed by the researcher, and $\boldsymbol{\varepsilon} \equiv (\varepsilon_1, ..., \varepsilon_I)$ are the vector of random preference shocks for each of the I alternatives. We make the following assumption about the instant utility from taking action i, $u_i^*(h) \equiv u_i^*(x, \boldsymbol{\varepsilon})$:

Assumption 1 ***(Additive Separability)*** *The instantaneous utilities are given by, for each $i \in \mathcal{I}$,*

$$u_i^*(x, \boldsymbol{\varepsilon}) = u_i(x) + \varepsilon_i,$$

where $(\varepsilon_1, ..., \varepsilon_I)$ has a joint distribution G, which is absolutely continuous with respect to the Lebesgue measure in R^I.

We assume that the *time horizon is infinite* with time denoted by $t = 1, 2, ...$ The decision-maker's intertemporal preferences are represented by a simple and now commonly used formulation of agents' potentially time-inconsistent preferences: (β, δ)-preferences (Phelps and Pollak 1968, Laibson 1997, and O'Donoghue and Rabin 1999a):

Definition 1 *(β, δ)-preferences are intertemporal preferences represented by*

$$U_t(u_t, u_{t+1}, ...) \equiv u_t + \beta \sum_{k=t+1}^{\infty} \delta^{k-t} u_k$$

where $\beta \in (0, 1], \delta \in (0, 1]$.

Following the terminology of O'Donoghue and Rabin (1999a), the parameter δ is called the *standard discount factor* and captures long-run, time-consistent discounting; the parameter β is called the *present-bias factor* and captures short-term impatience. The standard model is nested as a special case of (β, δ)-preferences when $\beta = 1$. When $\beta \in (0, 1)$, (β, δ)-preferences capture "quasi-hyperbolic" time discounting (Laibson, 1997). We say that an agent's preferences are time-consistent if $\beta = 1$, and are present-biased if $\beta \in (0, 1)$.

The literature on time-inconsistent preferences distinguishes between *naive* and *sophisticated* agents (Strotz 1956, Pollak 1968, O'Donoghue and Rabin 1999a,b). An agent is *partially naive* if the self in every period t underestimates the present-bias of her future selves, believing that her future selves' present bias is $\tilde{\beta} \in (\beta, 1)$; in the extreme, if the present self believes that her future selves are time-consistent, i.e. $\tilde{\beta} = 1$, she is said to be *completely naive.* On the other hand, an agent is *sophisticated* if the self in every period t correctly knows her future selves' present-bias β and anticipates their behavior when making her period-t decision.

Following previous studies of time-inconsistent preferences, we will analyze the behavior of an agent by thinking of the single individual as consisting of many autonomous *selves,* one for each period. Each period-t self chooses her current behavior to maximize her current utility $U_t(u_t, u_{t+1}, ...)$, while her future selves control her subsequent decisions.

More specifically, let the observable state variable in period t be $x_t \in \mathcal{X}$ where $\mathcal{X}$ denotes the support of the state variables and the unobservable choice-specific shock $\varepsilon_{it} \in \Im$, and $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, ..., \varepsilon_{It})$. We assume that $\mathcal{X}$ is a finite set and denote $X = \#\mathcal{X}$ to be the size of the state space.

A *strategy profile* for all selves is $\boldsymbol{\sigma} \equiv \{\sigma_t\}_{t=1}^{\infty}$ where $\sigma_t : \mathcal{X} \times \Im^I \to \mathcal{I}$ for all t. It specifies for each self her action in all possible states and under all possible realizations of shock vectors. For any strategy profile $\boldsymbol{\sigma}$, write $\boldsymbol{\sigma}_t^+ \equiv \{\sigma_k\}_{k=t}^{\infty}$ as the *continuation strategy profile* from period t on.

To define and characterize the equilibrium of the intra-personal game of an agent with potentially time-inconsistent preferences, we first introduce a useful concept: write $V_t\left(x_t, \boldsymbol{\varepsilon}_t; \boldsymbol{\sigma}_t^+\right)$ as the agent's period-t expected continuation utility when the state variable is x_t and the shock vector is $\boldsymbol{\varepsilon}_t$ under her *long-run* time preference for a given continuation strategy profile $\boldsymbol{\sigma}_t^+$. We can think of $V_t\left(x_t, \boldsymbol{\varepsilon}_t; \boldsymbol{\sigma}_t^+\right)$ as representing (hypothetically) her intertemporal preferences from some prior perspective when her own present-bias is irrelevant. Specifically, $V_t\left(x_t, \boldsymbol{\varepsilon}_t; \boldsymbol{\sigma}_t^+\right)$ must satisfy:

$$V_t\left(x_t, \boldsymbol{\varepsilon}_t; \boldsymbol{\sigma}_t^+\right) = u^*_{\sigma_t(x_t, \boldsymbol{\varepsilon}_t)}\left(x_t, \varepsilon_{\sigma_t(x_t, \boldsymbol{\varepsilon}_t)t}\right) + \delta \mathrm{E}\left[V_{t+1}\left(x_{t+1}, \boldsymbol{\varepsilon}_{t+1}; \boldsymbol{\sigma}_{t+1}^+\right) | x_t, \sigma_t\left(x_t, \boldsymbol{\varepsilon}_t\right)\right], \qquad (1)$$

where $\sigma_t(x_t, \boldsymbol{\varepsilon}_t) \in \mathcal{I}$ is the choice specified by strategy σ_t, and the expectation is taken over both the future state x_{t+1} and $\boldsymbol{\varepsilon}_{t+1}$.

We will define the equilibrium for a partially naive agent whose period-t self believes that, beginning next period, her future selves will behave optimally with a present-bias factor of $\tilde{\beta} \in [\beta, 1]$. Following O'Donoghue and Rabin (1999b, 2001), we first define the concept of an agent's *perceived continuation strategy profile* by her future selves.

Definition 2 *The perceived continuation strategy profile for a partially naive agent is a strategy profile* $\tilde{\boldsymbol{\sigma}} \equiv \{\tilde{\sigma}_t\}_{t=1}^{\infty}$ *such that for all* $t = 1, 2, ...,$ *all* $x_t \in \mathcal{X}$, *and all* $\boldsymbol{\varepsilon}_t \in \Im^I$,

$$\tilde{\sigma}_t(x_t, \boldsymbol{\varepsilon}_t) = \arg\max_{i \in \mathcal{I}} \left\{ u_i^*(x_t, \epsilon_{it}) + \tilde{\beta}\delta \mathrm{E}\left[V_{t+1}\left(x_{t+1}, \boldsymbol{\varepsilon}_{t+1}; \tilde{\boldsymbol{\sigma}}_{t+1}^{+}\right) | x_t, i \right] \right\}.$$

That is, if an agent is partially naive with perceived present-bias by future selves at $\tilde{\beta}$, then her period-t self will anticipate that her future selves will follow strategies $\tilde{\boldsymbol{\sigma}}_{t+1}^{+} \equiv \{\tilde{\sigma}_k\}_{k=t+1}^{\infty}$. Given this perception, the period-t self's best response is called *perception-perfect strategy profile.*

Note, importantly, what the strategy profile $\tilde{\boldsymbol{\sigma}} \equiv \{\tilde{\sigma}_t\}_{t=1}^{\infty}$ describes is the *perception of the partially naive agent regarding what her future selves will play.* It is *not* what will generate the *actual play* that we observe in the data. What we actually observe is generated from the perception-perfect strategy profile that we define below.

Definition 3 *A perception-perfect strategy profile for a partially naive agent is a strategy profile* $\boldsymbol{\sigma}^* \equiv \{\sigma_t^*\}_{t=1}^{\infty}$ *such that, for all* $t = 1, 2, ...,$ *all* $x_t \in \mathcal{X}$, *and all* $\boldsymbol{\varepsilon}_t \in \Im^I$,

$$\sigma_t^*(x_t, \boldsymbol{\varepsilon}_t) = \arg\max_{i \in \mathcal{I}} \left\{ u_i^*(x_t, \varepsilon_{it}) + \beta\delta \mathrm{E}\left[V_{t+1}\left(x_{t+1}, \boldsymbol{\varepsilon}_{t+1}; \tilde{\boldsymbol{\sigma}}_{t+1}^{+}\right) | x_t, i \right] \right\}.$$

It is key to note the *difference and connection* between $\tilde{\boldsymbol{\sigma}}$ and $\boldsymbol{\sigma}^*$. $\tilde{\boldsymbol{\sigma}}$ is the unobserved *perception* of the partially naive agent regarding what her future selves will do, under the partially naive assumption that her future selves does not suffer from the present bias as described by the parameter β, but instead is governed by present bias parameter $\tilde{\beta}$ that may differ from β. $\boldsymbol{\sigma}^*$ is what the self in each period will optimally choose to do and that is what will be observed in the data. Note also that when β and $\tilde{\beta}$ coincide, that is, when the agent is sophisticated, then $\boldsymbol{\sigma}^* = \tilde{\boldsymbol{\sigma}}$.

Assumption 2 ***(Stationarity)*** *We assume that the observed choices are generated under the* ***stationary*** *perception-perfect strategy profile of the infinite horizon dynamic game played among different selves of the decision makers.*

2.2 Decision Process

Now we describe the decision process of the decision maker. First, define the current choice-specific value function (deterministic component), $W_i(x)$, as follows:

$$W_i(x) = u_i(x) + \beta\delta \int V(x')\pi(x'|x,i)dx', \tag{2}$$

where $\pi(x'|x,i)$ denotes the transition probabilities for state variables x when action i is taken; and $V(\cdot)$ is the *perceived long-run* value function defined according to (1) under perception perfect strategy profile for a partially naive agent $\tilde{\boldsymbol{\sigma}}$ as defined in Definition 3).

We can also use $V(\cdot)$ as defined in (1) to define the *choice-specific value function of the next-period self as perceived by the current self*, $Z_i(x)$, as follows:

$$Z_i(x) = u_i(x) + \tilde{\beta}\delta \int V(x')\pi(x'|x,i)dx'. \tag{3}$$

Note that there are two key difference between $W_i(x)$ and $Z_i(x)$. The first difference is in how they discount the future streams of payoffs: in $W_i(x)$ the payoff t periods from the current period is discounted by $\beta\delta^t$, while in $Z_i(x)$ the payoff t periods from now is discounted by $\tilde{\beta}\delta^t$. The second difference is interpretational: $W_i(x)$ represents how the current-period self evaluates the deterministic component of the payoff from choosing alternative i, while $Z_i(x)$ is how the current-period self perceives how her future self would evaluate the deterministic component of the payoff from choosing alternative i. it is obvious but important to note that $W_i(x)$ will regulate the current self's optimal choice, but $Z_i(x)$ will regulate the perception of the current self regarding the choices of her future selves.

Given $Z_i(x)$, we know that the current self's perception of her future self's choice, i.e., $\tilde{\sigma}$ as defined in Definition 2 is simply

$$\begin{aligned}\tilde{\sigma}(x,\boldsymbol{\varepsilon}) &= \max_{i\in\mathcal{I}}\left[u_i(x) + \varepsilon_i + \tilde{\beta}\delta\int V(x')\pi(x'|x,i)dx'\right] \\ &= \max_{i\in\mathcal{I}}\left[Z_i(x) + \varepsilon_i\right]. \end{aligned} \tag{4}$$

Let us define the probability of choosing alternative j by the the next period self as perceived by the current period self, $\tilde{P}_j(x)$ when next period state is x :

$$\begin{aligned} \tilde{P}_j(x) &= \Pr\left[\tilde{\sigma}(x,\boldsymbol{\varepsilon}) = j\right] \\ &= \Pr\left[Z_j(x) + \varepsilon_j \geq Z_{j'} + \varepsilon_{j'} \text{ for all } j' \neq j\right]. \end{aligned} \tag{5}$$

With the characterization of $\tilde{\sigma}(x,\boldsymbol{\varepsilon})$, now we can provide a characterization of $V(\cdot)$. For this purpose, further denote

$$V_i(x) = u_i(x) + \delta \int V(x')\pi(x'|x,i)dx'. \tag{6}$$

According to the definition of $V(\cdot)$ as given by (1), $V(x)$ is simply the expected value of $[V_i(x) + \varepsilon_i]$ where i is the chosen alternative according to $\tilde{\sigma}(x,\boldsymbol{\varepsilon})$. Thus it must satisfy the following relationship:

$$V(x) = \mathrm{E}_{\boldsymbol{\varepsilon}}\left[V_{\tilde{\sigma}(x,\boldsymbol{\varepsilon})}(x) + \varepsilon_{\tilde{\sigma}(x,\boldsymbol{\varepsilon})}\right]. \tag{7}$$

Now note from (3) and (6), we have

$$V_i(x) = Z_i(x) + \left(1 - \tilde{\beta}\right)\delta \int V(x')\pi(x'|x,i)dx'. \tag{8}$$

The relationship (8) is crucial as it allows us to rewrite (7) as:

$$\begin{aligned} V(x) &= \mathrm{E}_{\boldsymbol{\varepsilon}}\left[V_{\tilde{\sigma}(x,\boldsymbol{\varepsilon})}(x) + \varepsilon_{\tilde{\sigma}(x,\boldsymbol{\varepsilon})}\right] \\ &= \mathrm{E}_{\boldsymbol{\varepsilon}}\left[Z_{\tilde{\sigma}(x,\boldsymbol{\varepsilon})}(x) + \varepsilon_{\tilde{\sigma}(x,\boldsymbol{\varepsilon})} + \left(1-\tilde{\beta}\right)\delta\int V(x')\pi(x'|x,\tilde{\sigma}(x,\boldsymbol{\varepsilon}))dx'\right] \\ &= \mathrm{E}_{\boldsymbol{\varepsilon}}\max_{i\in\mathcal{I}}\left[Z_i(x) + \varepsilon_i\right] + \left(1-\tilde{\beta}\right)\delta\mathrm{E}_{\boldsymbol{\varepsilon}}\int V(x')\pi(x'|x,\tilde{\sigma}(x,\boldsymbol{\varepsilon}))dx' \\ &= \mathrm{E}_{\boldsymbol{\varepsilon}}\max_{i\in\mathcal{I}}\left[Z_i(x) + \varepsilon_i\right] + \left(1-\tilde{\beta}\right)\delta\sum_{j\in\mathcal{I}}\tilde{P}_j(x)\int V(x')\pi(x'|x,j)dx' \end{aligned} \tag{9}$$

where the second equality just follows from (8); and the third equality follows from (4) and thus

$$\mathrm{E}_{\boldsymbol{\varepsilon}}\left[Z_{\tilde{\sigma}(x,\boldsymbol{\varepsilon})}(x) + \varepsilon_{\tilde{\sigma}(x,\boldsymbol{\varepsilon})}\right] = \mathrm{E}_{\boldsymbol{\varepsilon}}\max_{i\in\mathcal{I}}\left[Z_i(x) + \varepsilon_i\right];$$

and the fourth equality follows from the fact that

$$\begin{aligned} \mathrm{E}_{\boldsymbol{\varepsilon}}\int V(x')\pi(x'|x,\tilde{\sigma}(x,\boldsymbol{\varepsilon}))dx' &= \sum_{j\in\mathcal{I}}\Pr\left(\tilde{\sigma}(x,\boldsymbol{\varepsilon}) = j\right)\int V(x')\pi(x'|x,j)dx' \\ &= \sum_{j\in\mathcal{I}}\tilde{p}_j(x)\int V(x')\pi(x'|x,j)dx'. \end{aligned}$$

Now we make two additional assumptions about the transition of the state variables and the distribution of the shocks.

Assumption 3 ***(Conditional Independence):***

$$\pi(x_{t+1}, \boldsymbol{\varepsilon}_{t+1}|x_t, \varepsilon_t, d_t) = q(\boldsymbol{\varepsilon}_{t+1}|x_{t+1})\pi(x_{t+1}|x_t, d_t)$$
$$q(\boldsymbol{\varepsilon}_{t+1}|x_{t+1}) = q(\boldsymbol{\varepsilon}).$$

Assumption 4 ***(Extreme Value Distribution):*** $\boldsymbol{\varepsilon}_t$ *is i.i.d extreme value distributed.*

Remark 1 *It is well-known that the distribution of the choice-specific shocks to payoffs in discrete choice models are not non-parametrically identified (see Magnac and Thesmar 2002, for example). Thus one has to make an assumption about the distribution of ε. We make the extreme value distribution assumption for simplicity, but it could be replaced by any other distribution G .*

With the above preliminary notations, now we can describe how the agent will make the choices when the state variables are given by x. From Definition 3 for perception perfect strategy profile and Equation (2), we know that the current period decision maker will choose i if and only if

$$i \in \arg\max_{i\in\mathcal{I}} \left\{W_i(x) + \varepsilon_i\right\}.$$

That is, the perception-perfect strategy profile $\sigma^*(x, \boldsymbol{\varepsilon})$ is:

$$\sigma^*(x, \boldsymbol{\varepsilon}) = \arg\max_{i\in\mathcal{I}} \left\{W_i(x) + \varepsilon_i\right\}.$$

Under Assumption 3, the probability of observing action i being chosen at a given state variable x is:

$$P_i(x) = \Pr\left[W_i(x) + \varepsilon_i > \max_{j\in\mathcal{I}\backslash\{i\}} \left\{W_j(x) + \varepsilon_j\right\}\right] = \frac{\exp\left[W_i(x)\right]}{\sum_{j=0}^{I} \exp\left[W_j(x)\right]}. \tag{10}$$

$P_i(x)$ is the current-period self's equilibrium choice probabilities and will be observed in the data.

Now we derive some important relationships that will be used in our identification exercise below. First, note that by combining (2) and (3), we have that:

$$Z_i(x) - u_i(x) = \frac{\tilde{\beta}}{\beta}\left[W_i(x) - u_i(x)\right]. \tag{11}$$

Since both $Z_i(\cdot)$ and $W_i(\cdot)$ depends on $V(\cdot)$, we would like to use (9) to derive a characterization of $V(\cdot)$. Note that under Assumptions 3, we have:

$$\mathrm{E}_{\varepsilon}\max_{i\in\mathcal{I}}\{Z_i(x)+\varepsilon_i\}=\ln\left\{\sum_{i\in\mathcal{I}}\exp\left[Z_i(x)\right]\right\}. \tag{12}$$

Moreover, from (5), we have that

$$\tilde{P}_j(x)=\frac{\exp\left[Z_j(x)\right]}{\sum_{i=0}^{I}\exp\left[Z_i(x)\right]}. \tag{13}$$

Using (12) and (13), we can rewrite (9) as

$$V(x)=\ln\left\{\sum_{i\in\mathcal{I}}\exp\left[Z_i(x)\right]\right\}+\left(1-\tilde{\beta}\right)\delta\sum_{j\in\mathcal{I}}\frac{\exp\left[Z_j(x)\right]}{\sum_{i=0}^{I}\exp\left[Z_i(x)\right]}\int V(x')\pi(x'|x,j)dx'. \tag{14}$$

The three set of equations (3), (11) and (14) will form the basis of our identification argument below. Let us first make a few useful remarks.

Remark 2 *We have three value functions* $\{W_i(x), Z_i(x), V_i(x) : x\in\mathcal{X}\}$ *as defined respectively in (2), (3) and (6). Both* $W_i(\cdot)$ *and* $Z_i(\cdot)$ *are related to* V_i*. It is worth emphasizing that* $W_i(x)$ *will regulate the current self's choice behavior as demonstrated by (10); and* $Z_i(x)$ *will regulate the current self's perception of future selves choices as demonstrated by (13).* $V_i(x)$ *is an auxiliary value functionn that simply uses the long-run discount factor* δ *to evaluate the payoffs from the choices that the current self perceives that will be made by her future selves.*

Remark 3 *If* $\tilde{\beta}=1$*, i.e., if the decision maker is completely naive, we can see from (8) that* $V_i(x)=Z_i(x)$ *for all* x*. This makes sense because when* $\tilde{\beta}=1$*, the current self perceives her future selves to be time consistent. Thus the current self is already perceiving her future selves to be bahaving according to the long run discount factor* δ *only.*

Remark 4 *If* $\tilde{\beta}=\beta$*, i.e., when an agent is sophisticated, we have that* $W_i(x)=Z_i(x)$*. if the decision maker is sophisticated, then the current self's own choice rule will be identical to what she perceives to be her future self's choice rule.*

Remark 5 *When the decision maker is partially naive, there are two distinct value functions* $W_i(x)$ *and* $Z_i(x)$ *that separately regulate the choice of the current self and the perceived choice of* *her future selves. Equation (11) clarifies that it is the fact that we allow for potential* naivety *in the hyperbolic model that is creating the wedge between $W_i(x)$ and $Z_i(x)$: if $\tilde{\beta} = \beta$, i.e., if agents are sophisticated (even when they suffer from present bias), it would be true that $V_i(x) = W_i(x)$. This is an important point because, as we see below in (10), the observed choice probabilities (our data) would provide direct information about $W_i(x)$, without needing any information about the discount factors. When $\tilde{\beta} = \beta$, it also provides direct information about $Z_i(x)$; but when $\tilde{\beta}$ and β are potentially not equal, we can no longer learn about $Z_i(x)$ directly from the observed choice probabilities.*

Relationship with the Dynamic Games Literature. We analyze the observed outcome of the dynamic disrete choice problem of a hyperbolic discounting decision process as the equilibrium outcome of an *intra-personal game* with the players being *the selves at different periods.* Thus our paper represents an interesting intermediate case between the literature on classical estimating single-agent dynamic discrete choice decision problems and the more recent literature on estimating dynamic games. It is worth pointing out that there are two crucial differences between the intra-personal games we analyze for agents with time-inconsistent time preferences and those in the existing dynamic games literature.

The first key difference is that in our case, we *do not observe* the actions of all the players. More specifically, the outcomes – choices and the evolutions of the state variables – we observe in the data are affected only by the current selves, even though the current selves' choices are impacted by their perception of future selves' actions. The current self's perception of how her future selves will play has to be inferred by the researcher using the equilibrium restriction imposed by the theory. As can be seen from the above discussion, $\tilde{P}_j(x)$ as defined in (13), captures the current self's perception of how her future selves will play, which is crucial for us to understand the current self's actual choices. However, as a researcher, we do not observe $\tilde{P}_j$, only observe P_j, the choice probabilities by the current self. In the standard dynamic games literature, it is always assumed that the action of all the players are observed.

Secondly, the dynamic games literature (e.g. Bajari, Benkard and Levin 2007) may allow for players to have different contemporaneous payoff functions, in our setting, however, the payoffs for the players – the current self and the future selves – differ only in their time preferences; moreover, under hyperbolic discounting, we are assuming a rather restricted form of difference in time preferences between the selves in different periods.

2.3 Data and Preliminaries

Before we describe our results on identification, let us assume that we the data provides us with the following information:

DATA:

- **(Conditional Choice Probabilities)** For all $x \in \mathcal{X}$, we observe the choice probabilities $P_i(x)$ for all $i \in \mathcal{I}$;
- **(Transitional Probabilities for Observable State Variables)** For all $(x, x') \in \mathcal{X}^2$, all $i \in \mathcal{I}$, we observe the transition probabilities $\pi(x'|x, i)$; we denote

$$\boldsymbol{\pi} \equiv \left\{ \pi\left(x'|x, i\right) : \left(x, x'\right) \in \mathcal{X}^2, i \in \mathcal{I} \right\};$$

- **(Short Panels)** We have access to at least two periods of the above data, even though the data results from a stationary infinite horizon model.

Because we assume that our data is short panel as in Magnac and Thesmar (2002), we assume that the *structure* of the model, denoted by b, is defined by parameters

$$b = \left\{ \left(\beta, \tilde{\beta}, \delta\right), G, \left\langle \left\{ u_i(x), Z_i\left(x'\right), V_i\left(x'\right) : i \in \mathcal{I}, x \in \mathcal{X}, x' \in \mathcal{X} \right\} \right\rangle \right\}.$$

Note that the elements in b in our setting differs from those in Magnac and Thesmar (2002) in that we have two additional parameters β and $\tilde{\beta}$ that measure present bias and naivety; moreover, the interpretation of $V_i(x')$ in our paper differs from Magnac and Thesmar (2002). In their paper $V_i(x')$ directly informs about the *actual* choice probabilities of the decision maker in the second period, namely, $\Pr(i|x') = \Pr(V_i(x') + \varepsilon_i > V_j(x') + \varepsilon_j \text{ for all } j \neq i)$. In our paper, $Z_i(x')$ captures the current self's *perception* of the choice probability of the next period's self, which is *never actually* observed in the data; while $V_i(x')$ is just an auxiliary value function to account for the exponentially discounted payoff streams from the perceived choices made according to $\tilde{\sigma}(x, \boldsymbol{\varepsilon})$. Another difference is that in Magnac and Thesmar (2002), the vector $\{V_i(x') : x' \in \mathcal{X}\}$ are completely free parameters; in our setting, however, neither $Z_i(x')$ and $V_i(x')$ are completely free parameters as they are subject to the restriction that they have to satisfy (3), (11) and (14).

We denote by $\mathcal{B}$ the set of all permissible structures. The set $\mathcal{B}$ requires that the structure satisfies the assumptions we adopted in the model, as well as the restrictions (3),(11) and (14).

Given any structure $b \in \mathcal{B}$, the model predicts the probability that an agent will choose alternative $i \in \mathcal{I}$ in state $x \in \mathcal{X}$, which we denote by $\hat{P}_i(x; b)$ and is given by

$$\hat{P}_i(x; b) = \Pr\left\{ u_i(x) + \varepsilon_i + \beta\delta \int V(x')\pi(x'|x, i)dx' = \max_{j \in \mathcal{I}} \left[u_j(x) + \varepsilon_j + \beta\delta \int V(x')\pi(x'|x, j)dx' \right] \Bigg| x, b \right\}.$$

As is standard in the identification literature, we call the predicted choice probabilities $\hat{P}_i(x; b)$ as the *reduced form* of structure $b \in \mathcal{B}$. We say that two structures $b, b' \in \mathcal{B}$ are *observationally equivalent* if

$$\hat{P}_i(x; b) = \hat{P}_i(x; b') \; \forall i \in \mathcal{I} \text{ and } x \in \mathcal{X}.$$

A model is said to be *identified* if and only if for any $b, b' \in \mathcal{B}$, $b = b'$ if they are observationally equivalent.

2.4 Identification Results

We first describe identification of $\langle \{u_i(x), Z_i(x'), V_i(x') : i \in \mathcal{I}, x \in \mathcal{X}, x' \in \mathcal{X}\} \rangle$ with for a given set of discount factors $\left\langle \beta, \tilde{\beta}, \delta \right\rangle$. Then we provide conditions pertinent to the identification of $\left\langle \beta, \tilde{\beta}, \delta \right\rangle$.

For any given joint distribution G of $\tilde{\varepsilon} \equiv (\varepsilon_1, ..., \varepsilon_I)$, the choice probability vector $\mathbf{P}(x) = (P_1(x), ..., P_I(x))$ is a mapping Q of $\mathbf{W}(x) = (W_0(x), W_1(x), ..., W_I(x))$. Hotz and Miller (1993) showed that the mapping Q can be inverted and one of the $W_i(x)$ has to be normalized. That is, one can find

$$D_i(x) \equiv W_i(x) - W_0(x) = Q_i(\mathbf{P}(x); G)$$

where Q_i is the i^{th} component of the inverse of Q. Under our Assumption 3 (that ε_i is iid extreme value distributed), the mapping Q_i is especially simple [following from (10)]:

$$D_i\left(x\right) = W_i\left(x\right) - W_0\left(x\right) = \ln \frac{P_i\left(x\right)}{P_0\left(x\right)}. \tag{15}$$

Since we observe $P_i\left(x\right)$ and $P_0\left(x\right)$ from the data, we immediately learn about $D_i\left(x\right)$. We thus proceed as if $D_i\left(x\right)$ is observable.

From (11), we have, for all $i \in \mathcal{I}$,

$$Z_i\left(x\right) = \frac{\tilde{\beta}}{\beta} W_i\left(x\right) + \left(1 - \frac{\tilde{\beta}}{\beta}\right) u_i\left(x\right) \tag{16}$$

Together with (15), we have, for all $i \in \mathcal{I} \backslash \{0\}$ and $x \in \mathcal{X}$,

$$Z_i\left(x\right) - Z_0\left(x\right) = \frac{\tilde{\beta}}{\beta} D_i\left(x\right) + \left(1 - \frac{\tilde{\beta}}{\beta}\right)\left[u_i\left(x\right) - u_0\left(x\right)\right]. \tag{17}$$

This allows us to rewrite $\tilde{P}_j\left(x\right)$ as follows:

$$\begin{aligned} \tilde{P}_0\left(x\right) &= \frac{1}{1 + \sum_{i=1}^{I} \exp\left[\frac{\tilde{\beta}}{\beta} D_i\left(x\right) + \left(1 - \frac{\tilde{\beta}}{\beta}\right)\left[u_i\left(x\right) - u_0\left(x\right)\right]\right]} \\ \tilde{P}_j\left(x\right) &= \frac{\exp\left[\frac{\tilde{\beta}}{\beta} D_j\left(x\right) + \left(1 - \frac{\tilde{\beta}}{\beta}\right)\left[u_j\left(x\right) - u_0\left(x\right)\right]\right]}{1 + \sum_{i=1}^{I} \exp\left[\frac{\tilde{\beta}}{\beta} D_i\left(x\right) + \left(1 - \frac{\tilde{\beta}}{\beta}\right)\left[u_i\left(x\right) - u_0\left(x\right)\right]\right]} \text{ for } j \neq 0. \end{aligned}$$

Assuming that the set of states $\mathcal{X}$ is finite and contains X elements, Equation (14) can be written as:

$$\begin{aligned} V\left(x\right) &= Z_0\left(x\right) + \ln\left\{\sum_{i \in \mathcal{I}} \exp\left[Z_i\left(x\right) - Z_0\left(x\right)\right]\right\} + \left(1 - \tilde{\beta}\right)\delta \sum_{j \in \mathcal{I}} \frac{\exp\left[Z_j\left(x\right) - Z_0\left(x\right)\right]}{\sum_{i=0}^{I} \exp\left[Z_i\left(x\right) - Z_0\left(x\right)\right]} \sum_{x' \in \mathcal{X}} V(x')\pi(x'|x, j) \\ &\quad \text{for all } x \in \mathcal{X}. \end{aligned} \tag{18}$$

Note that equation (18) is simply a system of X linear equation in $\{V\left(x\right) : x \in \mathcal{X}\}$. In fact, if we write $\mathbf{V}$ as the $X \times 1$ column vector $\left[V\left(1\right), ..., V\left(X\right)\right]^T$, $\mathbf{A}$ as the $X \times 1$ column vector $\left[Z_0\left(x\right) + \ln\left\{\sum_{i \in \mathcal{I}} \exp\left[Z_i\left(x\right) - Z_0\left(x\right)\right]\right\}\right]_{x \in \mathcal{X}}$. If we further write

$$\tilde{\mathbf{P}} = \left[\begin{array}{ccc} \tilde{\mathbf{P}}_0 & \cdots & \tilde{\mathbf{P}}_I \end{array}\right]_{X \times [(I+1)X]},$$

where

$$\tilde{\mathbf{P}}_j = \left[\begin{array}{cccc} \tilde{P}_j\left(0\right) & 0 & \cdots & 0 \\ 0 & \tilde{P}_j\left(1\right) & \cdots & 0 \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ 0 & 0 & \cdots & \tilde{P}_j\left(X\right) \end{array}\right]_{X \times X}.$$

Also properly stack up the transition matrices $\pi(x'|x,j)$ into an $[(I+1)X] \times X$ matrix as follows:

$$\mathbf{\Pi} = \begin{bmatrix} \mathbf{\Pi}_0 \\ \vdots \\ \mathbf{\Pi}_I \end{bmatrix}_{[(I+1)X]\times X}$$

where

$$\mathbf{\Pi}_j = \begin{bmatrix} \Pi_j(1) \\ \Pi_j(2) \\ \vdots \\ \Pi_j(X) \end{bmatrix}_{X\times X}$$

where

$$\Pi_j(x) = \begin{bmatrix} \pi(1|x,j) & ... & \pi(X|x,j) \end{bmatrix}$$

is an $1 \times X$ row vector.

Thus we can write (18) as

$$\begin{aligned} \mathbf{V} &= \mathbf{A} + \left(1-\tilde{\beta}\right)\delta\tilde{\mathbf{P}}\mathbf{\Pi}\mathbf{V} \\ \mathbf{V} &= \left[\mathbf{I} - \left(1-\tilde{\beta}\right)\delta\tilde{\mathbf{P}}\mathbf{\Pi}\right]^{-1}\mathbf{A} \end{aligned} \tag{19}$$

Thus for fixed values of $\left(\beta,\tilde{\beta},\delta\right)$, we can plug (19) into (3) and obtain, for all $x \in \mathcal{X}$,

$$\begin{aligned} Z_0(x) &= u_0(x) + \tilde{\beta}\delta\sum_{x'\in\mathcal{X}} V\left(x'\right)\pi\left(x'|x,0\right) \\ &= u_0(x) + \tilde{\beta}\delta\mathbf{\Pi}_0(x)\mathbf{V} \\ &= u_0(x) + \tilde{\beta}\delta\mathbf{\Pi}_0(x)\left[\mathbf{I} - \left(1-\tilde{\beta}\right)\delta\tilde{\mathbf{P}}\mathbf{\Pi}\right]^{-1}\mathbf{A} \end{aligned} \tag{20}$$

where $\mathbf{\Pi}_0(x) = [\pi(1|x,0), ..., \pi(X|x,0)]$ is an $X \times 1$ vector. Similarly, for any $i \in \{1,...,I\}$ and for all $x \in \mathcal{X}$, we can obtain the following analogously:

$$\begin{aligned} Z_i(x) &= u_i(x) + \tilde{\beta}\delta\sum_{x'\in\mathcal{X}} V\left(x'\right)\pi\left(x'|x,i\right) \\ &= u_i(x) + \tilde{\beta}\delta\mathbf{\Pi}_i(x)\mathbf{V} \\ &= u_i(x) + \tilde{\beta}\delta\mathbf{\Pi}_i(x)\left[\mathbf{I} - \left(1-\tilde{\beta}\right)\delta\tilde{\mathbf{P}}\mathbf{\Pi}\right]^{-1}\mathbf{A} \end{aligned} \tag{21}$$

Now consider the system of equations given by (17), (20) and (21). We know that we have to normalize the utility for the reference alternative 0, without loss of generality set $u_0(x) = 0$ for all $x \in \mathcal{X}$. The unknowns contained in the equation system include $(I+1) \times X$ values for $\{Z_i(x) : i \in \mathcal{I}, x \in \mathcal{X}\}$ and $I \times X$ values for $\{u_i(x) : i \in \mathcal{I}/\{0\}, x \in \mathcal{X}\}$, thus the total unknowns is $(2I+1) \times X$. It is also easy to see that the total number of equations in the system is also equal to $(2I+1) \times X$.

Relationship to the Standard Exponential Discounting Special Case: $\beta = \tilde{\beta} = 1$. Before we discuss whether the non-linear equation system (17), (20) and (21) has a unique solution, let us show how it is related to the existing case in the literature for the identification of dynamic discrete choice models with exponential discounting (i.e., the case with $\tilde{\beta} = \beta = 1$). In that case, (17) is reduced to the well-known relationship

$$Z_i(x) - Z_0(x) = \ln P_i(x) - \ln P_0(x); \tag{22}$$

that is, in standard models the difference in the choice probabilities for alternative i and the reference alternative 0 informs us about the difference in the value from choosing i relative to the value from choosing 0. This is of course also true when $\tilde{\beta} = \beta < 1$. The potential naivety we allow in our setup breaks this direct relationship between $P_i(x)/P_0(x)$ and $Z_i(x) - Z_0(x)$.

Moreover, when $\tilde{\beta} = \beta = 1$, (20) is reduced to (using the normalization that $u_0(x) = 0$):

$$Z_0(x) = \delta \sum_{x' \in \mathcal{X}} Z_0(x') \pi(x'|x, 0) + \delta \sum_{x' \in \mathcal{X}} \ln\left[\sum_{i \in \mathcal{I}} \frac{P_i(x')}{P_0(x')}\right] \pi(x'|x, 0).$$

For simplicity, denote the $X \times 1$ vector $\{Z_0(x)\}_{x \in \mathcal{X}}$ as $\mathbf{Z}_0$; write the $X \times X$ matrix $\pi(x'|x, 0)$ as $\mathbf{\Pi}_0$, and write the $X \times 1$ vector $\left\{\ln\left[\sum_{i \in \mathcal{I}} \frac{P_i(x')}{P_0(x')}\right]\right\}_{x \in \mathcal{X}}$ as $\mathbf{m}$. The above equation can be written as

$$\mathbf{Z}_0 = \delta \mathbf{\Pi}_0 (\mathbf{Z}_0 + \mathbf{m}).$$

Thus,

$$\mathbf{Z}_0 = (\mathbf{I} - \delta \mathbf{\Pi}_0)^{-1} \delta \mathbf{m}.$$

Given this unique solution of $\mathbf{Z}_0$, (22) immediately provides $Z_i(x)$ for all $i \in \mathcal{I}/\{0\}$ and all $x \in \mathcal{X}$. To obtain $u_i(x)$ for $i \in \mathcal{I}\backslash\{0\}$, note that (21) implies that

$$\mathbf{u}_i = \mathbf{Z}_i - \delta\mathbf{\Pi}_i\mathbf{Z}_0 - \delta\mathbf{\Pi}_i\mathbf{m}, \tag{23}$$

where $\mathbf{Z}_i$ and $\mathbf{u}_i$ are $X \times 1$ vectors of $\{Z_i(x)\}_{x\in\mathcal{X}}$ and $\{u_i(x)\}_{x\in\mathcal{X}}$ respectively, $\mathbf{\Pi}_i$ is the $X \times X$ matrix $\pi(x'|x,i)$. Recall that in the standard exponential discounting model we have $Z_i(x) = V_i(x)$, thus we can conclude that , $\{\mathbf{u}_i\}_{i\in\mathcal{I}\backslash\{0\}}$ and $\{\mathbf{V}_i\}_{i\in\mathcal{I}}$ are identified once δ, G and $\{u_0(x)\}_{x\in\mathcal{X}}$ are fixed. This replicates the proof of Proposition 2 in Magnac and Thesmar (2002).[7]

The system of equations is no longer linear in the more general case we consider there. But we can also reduce the system of equations (17), (20) and (21) into a single Denote by $\mathbf{c}$ the $X \times 1$ vector

$$\ln\left\{\sum_{i\in\mathcal{I}}\exp\left[\frac{\tilde{\beta}}{\beta}\ln\frac{P_i(x')}{P_0(x')} + \left(1-\frac{\tilde{\beta}}{\beta}\right)u_i(x')\right]\right\}_{x'\in\mathcal{X}}. \tag{24}$$

Note that there is a crucial difference between the vector $\mathbf{m}$ we defined earlier for the case $\tilde{\beta} = \beta$ and $\mathbf{c}$: $\mathbf{m}$ contains only observables and thus can be treated as known; in contrast $\mathbf{c}$ here depends on unknown current payoffs $\{u_i(x) : i \in \mathcal{I}\backslash\{0\}\}$, thus cannot be considered as an observable. With this in mind, we can write (20) in matrix form as (since $\mathbf{A} = \mathbf{Z}_0 + \mathbf{c}$) :

$$\mathbf{Z}_0 = \tilde{\beta}\delta\mathbf{\Pi}_0\left[\mathbf{I}-\left(1-\tilde{\beta}\right)\delta\tilde{\mathbf{P}}\mathbf{\Pi}\right]^{-1}(\mathbf{Z}_0 + \mathbf{c}).$$

Thus,

$$\mathbf{Z}_0 = \tilde{\beta}\delta\mathbf{Q_A}\mathbf{c}$$

where

$$\mathbf{Q_A} = \left(\mathbf{I}-\tilde{\beta}\delta\mathbf{\Pi}_0\left[\mathbf{I}-\left(1-\tilde{\beta}\right)\delta\tilde{\mathbf{P}}\mathbf{\Pi}\right]^{-1}\right)^{-1}\mathbf{\Pi}_0\left[\mathbf{I}-\left(1-\tilde{\beta}\right)\delta\tilde{\mathbf{P}}\mathbf{\Pi}\right]^{-1}.$$

Now (21) can be written in matrix form as

$$\begin{aligned}\mathbf{Z}_i &= \mathbf{u}_i + \tilde{\beta}\delta\mathbf{\Pi}_i\left[\mathbf{I}-\left(1-\tilde{\beta}\right)\delta\tilde{\mathbf{P}}\mathbf{\Pi}\right]^{-1}(\mathbf{Z}_0 + \mathbf{c}) &(25)\\ &= \mathbf{u}_i + \tilde{\beta}\delta\mathbf{\Pi}_i\mathbf{Q_B}\mathbf{c} &(26)\end{aligned}$$

[7] The only difference is that our argument above indicates that $\mathbf{V}_0$ does not have to be fixed. It can be identified from the model.

where

$$\mathbf{Q_B}=\left[\mathbf{I}-\left(1-\tilde{\beta}\right)\delta\tilde{\mathbf{P}}\mathbf{\Pi}\right]^{-1}\left(\mathbf{I}+\left(\mathbf{I}-\tilde{\beta}\delta\mathbf{\Pi}_0\left[\mathbf{I}-\left(1-\tilde{\beta}\right)\delta\tilde{\mathbf{P}}\mathbf{\Pi}\right]^{-1}\right)^{-1}\tilde{\beta}\delta\mathbf{\Pi}_0\left[\mathbf{I}-\left(1-\tilde{\beta}\right)\delta\tilde{\mathbf{P}}\mathbf{\Pi}\right]^{-1}\right).$$

Finally, note that (17) can be written in matrix form as

$$\mathbf{Z}_i-\mathbf{Z}_0=\frac{\tilde{\beta}}{\beta}\mathbf{D}_i+\left(1-\frac{\tilde{\beta}}{\beta}\right)\mathbf{u}_i. \tag{27}$$

where $\mathbf{D}_i$ denotes the $X\times 1$ vector $\{\ln P_i(x)-\ln P_0(x)\}_{x\in\mathcal{X}}$. Thus, we have

$$\begin{aligned}\mathbf{u}_i &= \mathbf{D}_i+\beta\delta\mathbf{Q_A}\mathbf{c}-\beta\delta\mathbf{\Pi}_i\mathbf{Q_B}\mathbf{c}\\ &= \mathbf{D}_i+\beta\delta\left(\mathbf{Q_A}-\mathbf{\Pi}_i\mathbf{Q_B}\right)\mathbf{c}\end{aligned}$$

Let

$$\mathbf{U}=\begin{bmatrix}\mathbf{u}_1\\ \vdots\\ \mathbf{u}_I\end{bmatrix},\mathbf{D}=\begin{bmatrix}\mathbf{D}_1\\ \vdots\\ \mathbf{D}_I\end{bmatrix}$$

both be $(I\times X)\times 1$ vector with $\mathbf{u}_i$ and $\mathbf{D}_i$ stacked together; and let

$$\mathbf{E}=\begin{bmatrix}\mathbf{Q_A}-\mathbf{\Pi}_1\mathbf{Q_B}\\ \vdots\\ \mathbf{Q_A}-\mathbf{\Pi}_I\mathbf{Q_B}\end{bmatrix}$$

be a $(I\times X)\times X$ matrix. Then we have

$$\mathbf{U}=\mathbf{D}+\beta\delta\mathbf{E}\mathbf{c}. \tag{28}$$

Note that in equation (28), the three parameters $\left(\beta,\tilde{\beta},\delta\right)$ appears in three different combinations: $\beta/\tilde{\beta}$, $\left(1-\tilde{\beta}\right)\delta$ and $\tilde{\beta}\delta$ [Note that $\beta\delta=\tilde{\beta}\delta\times\left(\beta/\tilde{\beta}\right)$]. Despite its seeming simplicity, Equation (28) is a highly nonlinear system of equations in $\mathbf{U}$, which is an $(I\times X)\times 1$ column vector. In order to proceed with our identification arguments, we have to make the following additional assumption:additional assumptions:

Assumption 5 ***(Unique Solution)*** *For any values of* $\left\langle\beta,\tilde{\beta},\delta\right\rangle$ *and* $\{u_0(x):x\in\mathcal{X}\}$, *(28) has a unique solution. Under the above assumption, we can uniquely solve* $u_i(x)$ *for all* $i\in\mathcal{I}\backslash\{0\}$ *and* $x\in\mathcal{X}$ *and* $V_i(x)$ *for all* $i\in\mathcal{I}$ *and* $x\in\mathcal{X}$ *for any given* $\left\langle\beta,\tilde{\beta},\delta\right\rangle$.

Now we discuss conditions for identification related to $\left\langle \beta, \tilde{\beta}, \delta \right\rangle$. This discussion is closely related to that in Magnac and Thesmar (2002). We impose the following *exclusion restriction assumption:*

Assumption 6 ***(Exclusive Restriction)*** *Suppose that there exists variables* $(x_1, x_2) \in \mathcal{X}^2$ *with* $x_1 \neq x_2$, *but:*

- *for* all $i \in \mathcal{I}$, $u_i(x_1) = u_i(x_2)$;
- *for* some $i \in \mathcal{I}$, $\pi(x'|x_1, i) \neq \pi(x'|x_2, i)$.

More specifically, to satisfy the exclusion restriction assumption, there must be a variable that does not directly affect the *contemporaneous utility function* u_i for all $i \in \mathcal{I}$ but the variable may matter for choices because it affects the transition of state variables. The extent to which individuals' choice probabilities differ at state x_1 and x_2 reveals information about the discount factors. This is the key intuition from Magnac and Thesmar's (2002) result where they are interested in identifying a single long-term discount factor δ. In their setting, if $\delta = 0$, i.e., if individuals are completely myopic, then the choice probabilities would have been the same under x_1 and x_2; to the extent that choice probabilities differ at x_1 and x_2, it reveals information about the degree of time discounting. Their intuition, however, can be easily extended to the hyperbolic discounting case, as we will exploit in the proposed estimation strategy below. For notational simplicity, we will divide the state variables into two groups (x_r, x_e) where x_r refers to the state variables that directly enter the contemporaneous payoff function $u_i(x_r)$ and x_e refers to the state variables that satisfy the exclusive restriction assumption (i.e., they do not enter the contemporaneous payoff function but affects the state transition probabilities).

Before we show how we can use the exclusion restriction to construct estimators for the three discount factors $\left\langle \beta, \tilde{\beta}, \delta \right\rangle$, it is useful to provide some intuition as to how $\left\langle \beta, \tilde{\beta}, \delta \right\rangle$ come to affect the observed choice behavior by the current self differently.

β **vs.** δ. It may seem counterintuitive that β and δ could be separately identified in a short two-period panel data set. To provide some intuition, let us consider the case that $\tilde{\beta} = \beta$. The question is: "Can we distinguish the behavior of an agent with exponential discounting rate $\hat{\delta} = \beta\delta$ from the behavior of a sophisticated time-inconsistent agent with preference (β, δ)?" Under stationarity assumption, if an agent has time consistent exponential discounting rate $\hat{\delta} = \beta\delta$, her expected continuation utilities is completely determined by the observed choice probabilities. To see this, observe that in equation (18), if one replaces $\tilde{\beta}$ by 1 and δ by $\hat{\delta}$, we will have

$$V(x) = Z_0(x) + \ln\left\{\sum_{i\in\mathcal{I}} \exp\left[Z_i(x) - Z_0(x)\right]\right\},$$

which only depends on $D_i(x)$ when $\beta = \tilde{\beta}$.

However, for a sophisticated time-inconsistent agent with preference (β, δ), there is an incongruence between current self and her perceived future self regarding how they evaluate the future stream of payoffs. Though the current self has to defer to her next-period self in terms of the actual next-period choice that will be chosen, they disagree on how much weight to put on payoffs two-periods from now. It is this incongruence that leads to the last term in Equation (18), which in turn breaks the tight link between observed choice probabilities and the continuation utilities.

β **vs.** $\tilde{\beta}$. To help provide intuition for why β could be distinguished from $\tilde{\beta}$, let us suppose that $\delta = 1$. First note that the ratio $\tilde{\beta}/\beta$ appears in term $Z_i(x)$ [see Eq. (**??**)]. This ratio regulates the incongruance between the current self's own behavior and her perception of the behavior of her future selves. Eq. (17) shows that if $\tilde{\beta}/\beta = 1$, then $Z_i(x) - Z_0(x)$ is uniquely determined by the observed $D_i(x)$; thus the current self's perception about her future self's action is identical to her own action. Since we do not directly observe the current self's perceptions, this insight could not directly be used to test whether $\tilde{\beta}/\beta$ is equal to 1. Only with the exclusion restriction can we use the above intuition to help identify $\tilde{\beta}/\beta$.

For notational simplicity, we will divide the state variables into two groups (x_r, x_e) where x_r refers to the state variables that directly enter the contemporaneous payoff function $u_i(x_r)$ and x_e refers to the state variables that satisfy the exclusive restriction assumption (i.e., they do not enter the contemporaneous payoff function but affects the state transition probabilities).

If is also useful to note that so far, our discussion has focused on short-panel (two period) data sets under stationarity assumption. Having two-period data allows one to non-parametrically estimate the transition probabilities $\pi(x'|x,i)$; stationarity ensures that looking at a two-period slice of a potentially long panel is sufficient.[8]

2.5 Estimation Strategies

We propose two related two-step estimation strategies. The first step for the two estimation strategies are the same: estimate from the data the choice probabilities $P_i(x)$ for all $i \in \mathcal{I}$ and all $x \in \mathcal{X}$, as well as the state transition probabilities $\pi(x'|x,i)$ for all $i \in \mathcal{I}$ and all $(x', x) \in \mathcal{X}^2$.

The second step of both estimation strategies involves two loops. (1) In the inner loop, we solve equation (28) for $u_i(x)$ for a given triple of values for $\left\langle \beta, \tilde{\beta}, \delta \right\rangle$ for all $i \in \mathcal{I} \backslash \{0\}$ and all $x \in \mathcal{X}$ where we normalize $u_0(x) = 0$ for all $x \in \mathcal{X}$. Note the role played by Assumption 5. (2). In the outer loop, we maximize an objective function (to be stated below) over values of $\left\langle \beta, \tilde{\beta}, \delta \right\rangle$. The two estimation strategies differ in the objective function used in the outer loop.

In the first estimation strategy, we take the $\hat{u}_i(x)$ solved from the inner loop and impose (by Assumption 6) the restriction that

$$\hat{u}_i(x_r) = \sum_{\{(x_r, \tilde{x}_e): \tilde{x}_e \in \mathcal{X}_e\}} \hat{u}_i(x_r, \tilde{x}_e) \tag{29}$$

where $\mathcal{X}_e$ is the set of possible values for the payoff irrelevant state variables x_e we discussed in Assumption 6. We then use the above utility function $\hat{u}_i(x_r)$ to predict the pseudo-choice probabilities for the individuals in a given state $x \in \mathcal{X}$. Denote the pseudo-choice probabilities as $\hat{P}_i(x)$. The first estimation strategy will maximize the pseudo-likelihood by iterating over combinations of $\left\langle \beta, \tilde{\beta}, \delta \right\rangle$. Under the maintained model, at the true values of $\left\langle \beta, \tilde{\beta}, \delta \right\rangle$, $\hat{u}_i(x_r, \tilde{x}_e)$ as solved from the inner loop should be independent of $\tilde{x}_e$, the payoff irrelevant state variables, and thus $\hat{u}_i(x_r)$ as calculated in (29) would be exactly equal to $\hat{u}_i(x_r, \tilde{x}_e)$ for all $\tilde{x}_e \in \mathcal{X}_e$. Thus the pair of $\left\langle \beta, \tilde{\beta}, \delta \right\rangle$ that maximizes the pseudo-likelihood of the observed choices is a consistent estimator.

[8] Fang and Silverman (2006) considered a case without stationarity (specifically a finite horizon model) and showed that β and δ could be potentially identified without exclusion restriction if the researcher has access to at least three-period panel data.

In the second estimation strategy, we exploit the identifying assumption 6 in a somewhat different manner. We know that at the true values of $\left\langle \beta, \tilde{\beta}, \delta \right\rangle$, for all $\tilde{x}_e \in \mathcal{X}_e$,the standard deviation of $\hat{u}_i\left(x_r, \tilde{x}_e\right)$ with respect to $\tilde{x}_e$ should be 0, because Assumption 6 requires that $u_i(x_r, \tilde{x}_e)$ does not depend on $\tilde{x}_e$. Therefore the estimator for $\left\langle \beta, \tilde{\beta}, \delta \right\rangle$ in our second estimation strategy is

$$\begin{aligned} \left\langle \beta, \tilde{\beta}, \delta \right\rangle &= \arg \min_{\left\{\beta, \tilde{\beta}, \delta\delta\right\}} f(\beta, \tilde{\beta}, \delta) \\ &= \arg \min_{\left\{\beta, \tilde{\beta}, \delta\right\}} \sum_{x_r \in \mathcal{X}_r} \left| \text{std}\left[\hat{u}_i \left(x_r, \tilde{x}_e; \left\langle \beta, \tilde{\beta}, \delta \right\rangle \right) \right] \right|. \end{aligned}$$

2.6 Monte Carlo Experiments

[Yang, please update this section with the new monte carlo results, please explain the details of the experiments so the readers can understand what is going on] In this section we provide Monte Carlo evidence for the identification of discount factors in a dynamic discrete choice model using the two estimation methods described above. In this simple Monte Carlo exercise, we consider a 2-choice decision problem facing an agent with infinite horizon and stationary state transition. There are two state variables x_1 and x_2. The state variable $x_1 \in \{0, 1, 2, 3, 4, 5\}$, affects both instantaneous utility and state transition; while state variable $x_2 \in \{0, 1, 2, 3\}$, affects only the state transition.[9] When $a = 1$, instantaneous utility is $u_1(x_1) = \alpha_0 + \alpha_1 x_1$; when $a = 0$, instantaneous utility is simply $u_0(x_1) = 0$ for all x_1. The true parameters are $\alpha_0 = -0.1$, $\alpha_1 = 0.5$, $\tilde{\beta}/\beta = 1.2$, and $\tilde{\beta}\delta = 0.8$. We estimate the discount factors and corresponding utility parameters $5,000$ times for various sample sizes, to show the differences in performance of these two estimation methods. Results are shown in Table 1. Table 1 shows that both estimation methods do an excellent job in recovering the true parameter values of in large samples. For the Monte Carlo sample size analogous to our actual estimation sample (12,500), the maximum pseudo-loglikelihood method

[9]The state transition matrices are generated as follows. We first generate two random matrices M_0 and M_1 each with dimension $\#X_1 \times \#X_2 = 6 \times 4 = 24$, with each entry a random number generated from a uniform $[0, 1]$ distribution. We then normalize the entry in each row by its row sum to ensure a proper probability matrix. The resulting matrices are denoted Π_0 and Π_1. The (j, k)-th entry of matrix Π_i where $(j, k) \in \{1, ..., 24\}^2$ is the probability that (x_1', x_2') takes on k-th combination condition on (x_1, x_2) taking on j-th combination in this period and action chosen is $i \in \{0, 1\}$. The matrices Π_1 and Π_0 are assumed to be known by the decision maker; and are directly taken to be the state transition probabilities in our Monte Carlo exercise reported in Table 1.

Estimation Method	Maximizing Pseudo-Loglikelihood				Minimizing Variance of Utilities			
Parameters	$\tilde{\beta}\delta$	$\tilde{\beta}/\beta$	α_0	α_1	$\tilde{\beta}\delta$	$\tilde{\beta}/\beta$	α_0	α_1
True Values	0.8	1.2	-0.1	0.5	0.8	1.2	-0.1	0.5
Sample Size: $240,000$								
Mean	0.828	1.246	-0.100	0.500	0.824	1.240	-0.100	0.500
Std. Dev.	0.076	0.124	0.008	0.003	0.084	0.121	0.009	0.003
Sample Size: $120,000$								
Mean	0.823	1.244	-0.100	0.500	0.817	1.235	-0.100	0.500
Std. Dev.	0.086	0.146	0.012	0.005	0.096	0.136	0.012	0.005
Sample Size: $12,500$								
Mean	0.766	1.243	-0.104	0.503	0.765	1.253	-0.105	0.503
Std. Dev.	0.172	0.234	0.037	0.016	0.186	0.234	0.038	0.016

Table 1: Monte Carlo Results Under the Two Proposed Estimation Methods.

Notes: For each sample size, we generate 5000 random simulation samples. The Mean and Standard Deviations of the estimated parameters are with respect to the 5000 samples.

seems to perform better. As a result, we will use the maximum pseudo-loglikelihood method in our empirical exercise.

3 Application: Empirical Application: Mammography Decisions

3.1 Background on Mammography

Breast cancer is the third most common cause of death, and the second leading cause of cancer death, among American women. From birth to age 39, one woman in 231 will get breast cancer ($<0.5\%$ risk); from age 40-59, the chance is 1 in 25 (4% risk); from age 60-79, the chance is 1 in 15 (nearly 7%). Assuming that a woman lives to age 90, the chance of getting breast cancer over the course of an entire lifetime is 1 in 7, with an overall lifetime risk of 14.3%.[10]

[10] As a useful comparison, breast cancer has a higher incidence rate than lung cancer in the US. In 2004, there were $217,440$ new cases for breast cancer in US (American Cancer Society).

Breast cancer takes years to develop. Early in the disease, most breast cancers cause no symptoms. When breast cancer is detected at a localized stage (it hasn't spread to the lymph nodes), the 5-year survival rate is 98%. If the cancer has spread to nearby lymph nodes (regional disease), the rate drops to 81%. If the cancer has spread (metastasized) to distant organs such as the lungs, bone marrow, liver, or brain, the 5-year survival rate is 26%.

A screening – mammography – is the best tool available to find breast cancer before symptoms appear. Mammography can often detect a breast lump before it can be felt and therefore save lives by finding breast cancer as early as possible. For women over the age of 50, mammography have been shown to lower the chance of dying from breast cancer by 35%.[11] Leading experts, the National Cancer Institute, the American Cancer Society, and the American College of Radiology recommend annual mammography for women over 40. The U.S. Preventive Services Task Force recommends mammography screening for women beginning at age 50 every 12-24 months in order to reduce the risk of death from breast cancer (DHHS 2002).

3.2 Data

The data used in this analysis are from the Health and Retirement Study (HRS). The HRS is a nationally representative biennial panel study of birth cohorts 1931 through 1941 and their spouses as of 1992. The initial sample includes 12,652 persons in about 7,600 households who have been interviewed every two years since 1992. The most recent available data are for year 2006 (wave 8). The survey history and design are described in more details in Juster and Suzman (1995). Since the HRS started asking women questions about their usage of mammography in 1996, our sample is limited to women interviewed in the HRS from 1996 to 2006. We focus on the age group 51 to 64, and exclude those observations with missing values for any of the critical variables.[12]

[11]Source: American Cancer Society. Also note that finding breast cancers early with mammography has also meant that many more women being treated for breast cancer are able to keep their breasts. When caught early, localized cancers can be removed without resorting to breast removal.

[12]The key variables are marital status, non-Hispanic white, self-rated health, household income, and mammography usage (see the summary statistics table).

We also exclude those who have ever been diagnosed of (breast) cancer,[13] since those who are diagnosed of cancer might be of a different group who do not make decisions on mammography or any other preventive health care the same way as do others. Our final sample consists of 12,506 observations (each observation is a two-period short panel pooled from two consecutive waves) for 7,067 individuals.

3.3 Descriptive Statistics

Table 2 provides summary statistics of the key variables for the sample we use in our empirical analysis. The sample of women we select are aged from 51 to 64 (note that we are combining two-period panels using individuals that appear in two consecutive samples), with an average age of 57.8. A large majority of the sample are non-Hispanic white (78%) and married (70.5%). The average household income is about 50K dollars. 23% of the sample has a self-reported bad health and about 76% of the women undertook mammogram in the survey year. About 1.5% of the women who were surveyed in a wave died within two years. Finally, about 75% of the mothers of the women in our sample are either still alive or died at age greater than 70 at the time of the interview.

3.4 Decision Time Line

Figure 1 depicts the time line for mammography decisions for women in our sample. As we mentioned earlier, we only consider women who are alive and have not yet been diagnosed with any cancer (thus not breast cancer) in the first period. Given her period-1 state variables, she makes the decision of whether to undertake mammography. Mammography detects breast cancer with very high probability, though not for certain, if the woman has breast cancer. In the event that the woman has breast cancer, early detection of breast cancer will lead to higher survival probability.

To fully capture the diagnostic nature of mammography, we would need to have information about whether the woman has breast cancer at any period, and estimate the probability of detecting

[13]For those who entered in 1992 (i.e. the first wave), we know whether they have been diagnosed of breast cancer as of 1992. But for those who entered HRS in later waves, we only know whether cancer has been diagnosed of since the survey from 1994 did not ask about breast cancer specifically.

Variable	Mean	Std. Dev.	Min	Max	Obs.
Age	57.81	3.95	51	64	12506
White (Non-Hispanic)	.781	.413	0	1	12506
Married	.705	.456	0	1	12506
Household Income ($1000)	50.04	69.12	0.101	2,140	12506
Log of Household Income	10.32	1.07	4.61	14.57	12506
Bad Health	.230	.421	0	1	12506
Mammogram	.758	.428	0	1	12506
Death	.015	.121	0	1	12506
Mother Still Alive or Died After Age 70	.749	.433	0	1	12506
Bad Health $(t+1)$	.249	.432	0	1	12319
Household Income $(t+1)$ ($1000)	48.98	177.52	0.103	17,600	12319

Table 2: Summary Statistics of Key Variables in the Estimation Sample.

NOTE: The last two variables in the table are observed only for those who survive to the second period.

breast cancer with (P1 in Figure 1) and without (P2) mammogram. However, we do not have access to such data. In HRS, even though we have information on women's mammography choices from 1996 on and we know whether their doctors have told them that they have *any* cancer, we do not have information on which kind of cancer they have been diagnosed of.[14]

Due to these data issues, we decide to go directly from the mammography decision to the live/death outcome and health status if alive (see our empirical specification below), without going through the intermediate step (having breast cancer or not). That is, we simply capture the ultimate effect of undertaking mammography as to lower the probability of dying, and to lower the probability of being in bad health status if alive.

[14] Even for those over age 65 with matching Medicare claim data, the number of observations is not big enough for us to get the needed probabilities of being diagnosed with breast cancer with and without mammography stratified by all the state variables we want to control in our model such as age, race, health status, income, and marital status.

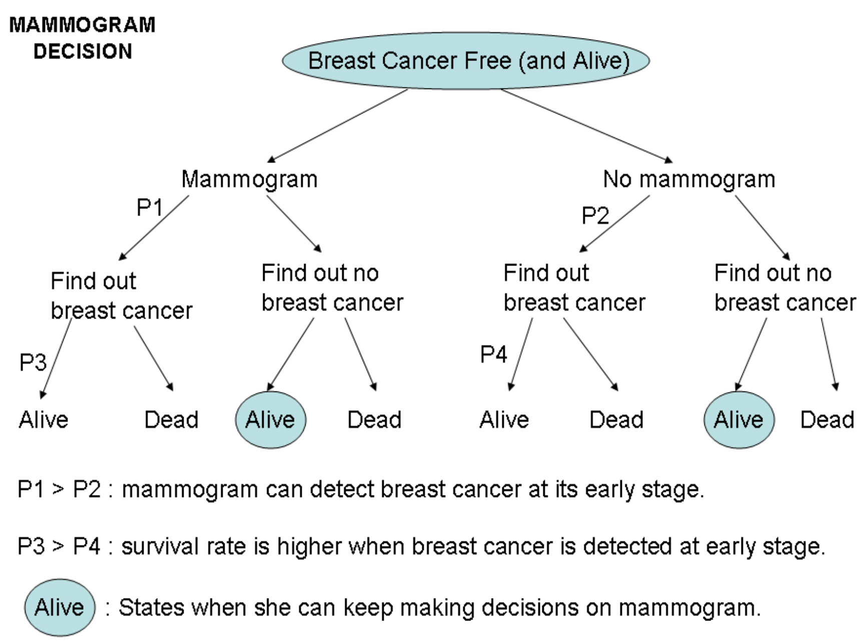

Figure 1: The Timeline for Mammography Decisions

3.5 Empirical Specification

In this dynamic discrete choice model, each agent decides whether to get mammography or not. It is clear from the previous section on identification, we need to impose some normalization of the contemporaneous utilities.

Let us first be specific about the state variables we use in our empirical specification. They include: Age (Age); Marital Status (Married); Bad Health (BadHealth) which indicates whether the individual self reports bad health; Log of Household Income (LogIncome), Death (Death) and whether her mother is still alive or died at age greater than 70 (Mother70)

Each woman decides whether to undertake mammography ($i = 1$ if she does and $i = 0$ if she does not). We normalize the individual's instantaneous utility at the death state to be zero. Note that no decision is necessary if one reaches the death state.

If an individual stays alive, then we specify her utility from taking mammogram relative to not taking mammogram as:

$$u_1(x) - u_0(x) = \alpha_0 + \alpha_1 \text{BadHealth} + \alpha_2 \text{LogIncome} + \varepsilon_t, \tag{30}$$

where "BadHealth" is a binary variable indicating whether the agent is in bad health or not at time t; "LogIncome" denotes the logarithm of household income of the agent at time t, α_0 and α_1 are the utility parameters, and ε_t denotes difference in the choice-specific utility shocks at time t. It is important to remark that even though the other state variables Age, White and Married do not show up in the above specification, it does not mean that these variables do not affect the instantaneous utility of the individual; what it means is that these variables affect the instantaneous utility under action 1 (mammogram) and action 0 (no mammogram) in exactly the same way.

The agents make their decisions about whether to get mammography by comparing the expected summations of current and discounted future utilities from each choice. Individuals are uncertain about their future survival probabilities, and if alive, the transition probabilities of future health and household income. These probabilities depend on their choices about whether or not to get mammography; time-variant state variables including their lagged health status, their lagged household income, and their age, denoted as Age; and time-invariant state variables including their race, denoted by a binary variable White, their marital status Married, and the longevity of their mothers, denoted by the binary variable Mother70 which takes value 1 if the mother is still alive or died at the age older than 70 and 0 otherwise.

The exclusion restriction variable for our empirical analysis is Mother70. We make the plausible identifying assumption that Mother70 affects the transition matrix of the key state variables (BadHealth and Death) that directly enter into the instantaneous utilities, but Mother70 itself does not directly affect the instantaneous utilities. In the preliminary results below, we will show evidence for this assumption.

3.6 Preliminary Results

3.6.1 First Step Estimates

As we noted earlier, our estimation strategy has two steps. In the first step, we need to use the data to estimate choice probabilities, and the state transitions. Here we report these first-step estimation results. The choice probabilities and the death probability are estimated using Logit regressions; but the transition of BadHealth and LogIncome are estimated non-parametrically.

Reduced Form Result for the Determinant of Mammography Table 3 produces the reduced form Logit regression results for the determinants of whether a woman will undertake mammogram in a given year. There are two statistically significant coefficient estimates: high income women are more likely and whites are less likely to undertake mammograms with both coefficient estimates significant at 1%. While the rest of the coefficient estimates are not significant, their signs are plausible: women whose mothers are still alive or died after age 70 are less likely, women with bad health are less likely, and married women are more likely, to undertake mammograms.[15]

[15]Table 3 is a shorter version of the actual regressions we used to measure the choice probabilities where we also include many interactions among the variables included in Table 3: Mother70×Age, Mother70×Age^2, BadHealth×Age, BadHealth×Age^2, White×Mother70, White×hLogIncome, White×Age, White×Age^2, Married×Mother70, Married×LogIncome, Married×Age, Married×Age^2, White×Married, White×Married×LogIncome. The results from this flexible Logit regression

Variable	Coeff. Est.	Std. Err.	t-stat.
Mother70	-.028	.050	-0.57
BadHealth	-.0067	.0531	-0.13
LogIncome	.378***	.024	15.57
Age	.172	.174	0.98
Age^2	-.0014	.00151	-0.95
White	-.209***	.0547	-3.82
Married	.0427	.052	0.82
Constant	-7.65	5.03	-1.52

Table 3: Determinants of Mammography Decisions: Reduced Form Logit Regression Results.

Determinants of the Probability of Dying in Two Years Table 4 reports the Logit regression results for the probability of dying in two years. It shows that on net, undertaking mammogram lowers the probability of death (notice that the average age of the sample is about 58 years). Not surprisingly, women with bad health are more likely to die, but mammogram reduces the probability of dying conditional on bad health. Also note that women whose mothers are either still alive or died after age 70 are less likely to die, suggesting a genetic link of longevity between mothers and daughters.

Evolution of Bad Health in Two Years Figure 2 depicts a subset of the results from the non-parametric estimation of the evolution of BadHealth for a selective combinations of the other state variables and the mammogram choice. For example, Panel (a) shows that probability of having bad health in period 2 is higher for women with bad health in period 1 conditional on undertaking mammogram in period 1 and their mothers are either still alive or died after age 70. Panel (b) showed that women with longer living mothers are less likely to experience bad health. Panel (c) and (d) showed that undertaking mammogram lowers the probability of bad health in period 2. Also note from all panels that the probability of bad health decreases with household

are harder to interpret than those in Table 3.

Variable	Coeff. Est.	Std. Err.	t-stat.
Mammogram	1.94	2.550	0.76
Mammogram×Age	-0.04	0.043	-0.92
Mammogram×BadHealth	-0.04	0.333	-0.12
Mother70	-0.25*	0.160	-1.54
BadHealth	1.72***	0.275	6.25
LogIncome	-0.183***	0.078	-2.35
Age	0.294	0.667	0.44
Age^2	-0.0016	0.006	-0.29
White	-0.126	0.167	-0.75
Married	-0.100	.174	-0.57
Constant	-14.1	19.6	-0.72

Table 4: Determinants of Probability of Dying in Two Years.

income.

3.6.2 Utility Parameters and Discount Factors

[**Yang, please rewrite here to update the results in this section**] Here we report the preliminary estimation results for the parameters in the utility function specification (30) and the identified discount factor combinations $\tilde{\beta}/\beta$ and $\tilde{\beta}\delta$. We found that having bad health lowers the utility of undertaking mammography relative to not undertaking mammography, consistent with our earlier finding in Table 3 that women with bad health are less likely to undertake mammography. We found that the relative utility of undertaking mammography increases with household income, consistent with the finding in Table 3 that women with higher household incomes are more likely to undertake mammography.

More interestingly, we estimate $\tilde{\beta}/\beta$ to be 1.80 and $\tilde{\beta}\delta$ to be around 0.86. We still need to calculate the standard error for these estimates. But these point estimates shows that women exhibit substantial present bias ($\beta < 1$) as well as naivety about their present bias ($\tilde{\beta} > \beta$) when

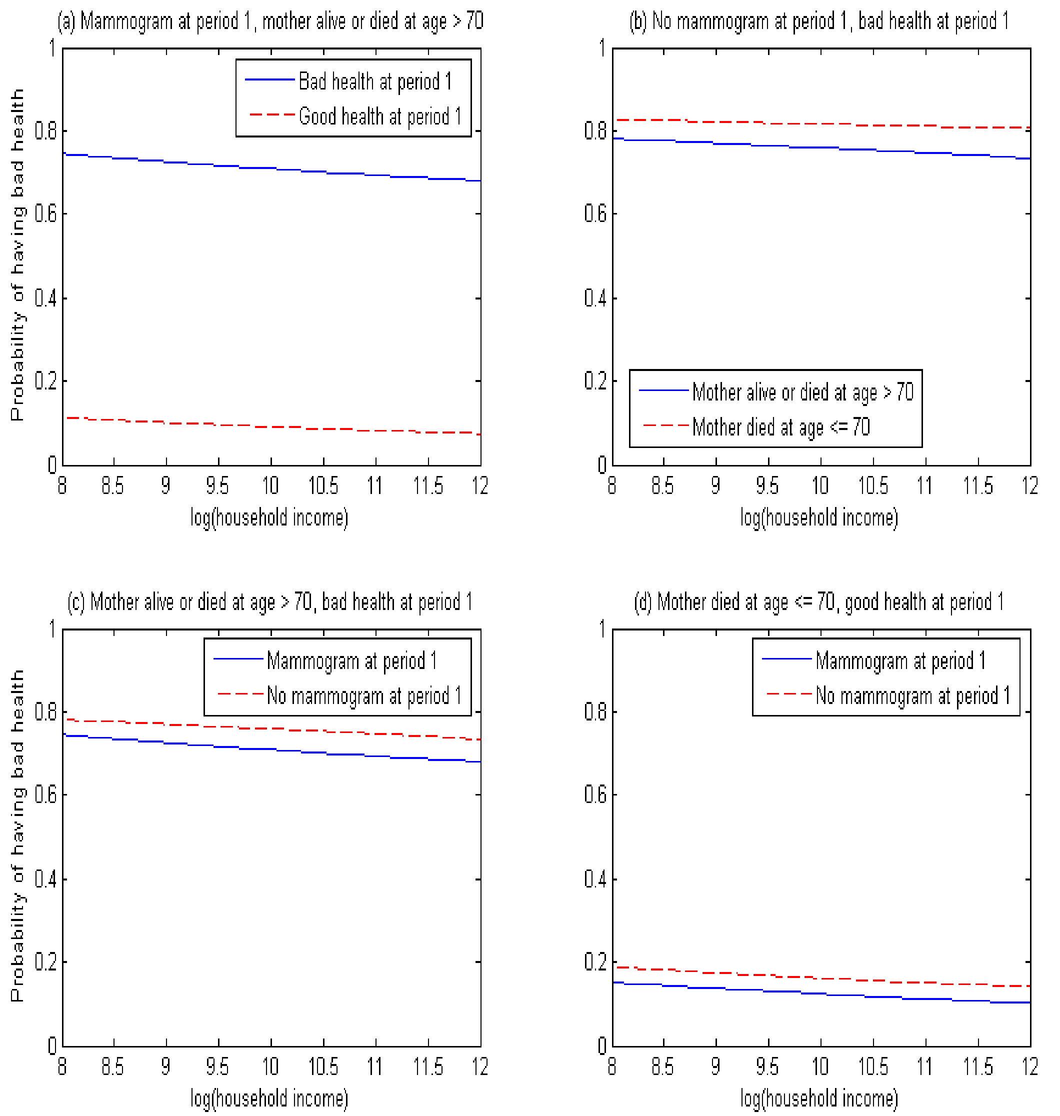

Figure 2: Non-parametric Estimate of the Determinants of Bad Health in Two Years

Variable	Coeff. Est.	Std. Err.
Utility Function Parameters		
BadHealth	-0.29	
LogIncome	0.51	
Constant	-4.66	
Discount Factor Combinations		
$\tilde{\beta}/\beta$	1.80	
$\tilde{\beta}\delta$	0.86	

Table 5: Preliminary Parameter Estimates for Utility Function and Combinations of Discount Factors.

making mammography decisions.

Our estimates for $\beta\delta = \tilde{\beta}\delta \times \beta/\tilde{\beta} = 0.86/1.80 \approx 0.48$. This can be compared with the estimate in Fang and Silverman (2007) where they estimate β to be 0.338 and δ to be 0.88, with $\beta\delta \approx 0.30$ for a group of single mothers with dependent children. It is important to note that our sample period is two years, while Fang and Silverman's sample period is one year; also the sample of women here are older and have very different social economic status (e.g. education, income, marital status) from the sample in Fang and Silverman.

[**Also please add some calculations using sample statistics regarding the threshold income level above which $u_1 - u_0$ is positive, thus present bias will lead to higher, not lower, take up of mammography.**]

3.7 Counterfactual Experiments

Simulation of the Mammography Take Up Rate in the Absence of Naivety, i.e. $\tilde{\beta} = \beta$. **Here keep** β and δ at the estimated value, but set $\tilde{\beta}$ at the estimated value of β.

Simulation of the Mammography Take Up Rates when $\tilde{\beta} = \beta = 1$. Here set $\tilde{\beta} = \beta$ at 1, but let $\hat{\delta}$ at the estimated value of $\beta \times \delta$.

4 Discussions

4.1 Unobserved Heterogeneity

4.2 Unobserved State Variables

4.3 Role of Non-stationarity and Longer Panels

5 Conclusion

[Here need to change as well] This paper extends the semi-parametric identification and estimation method for dynamic discrete choice models using Hotz and Miller's (1993) conditional choice probability (CCP) approach to the setting where individuals may have hyperbolic discounting time preferences and may be naive about their time inconsistency.

Our analysis showed that the three discount factors, the present bias factor β, the standard discount factor δ and the perceived present bias factor $\tilde{\beta}$ for naive agents can not be separately identified; however, with identifying exclusive restrictions, we can identify two combinations of the above three parameters, namely, $\tilde{\beta}/\beta$ and $\tilde{\beta}\delta$. The key identifying restriction is that there exists variables that do not directly enter the instantaneous utility function but affect the transition of other payoff relevant state variables.

Our discussion on identification also makes it clear that, with variables that satisfy the identifying exclusive restrictions, β and δ can not be separately identified in a fully sophisticated model with hyperbolic discounting time preferences (i.e. $\tilde{\beta} = \beta$) with two-period short panel data; but they could be separately when the agents are assumed to be naive about their present bias (i.e. $\tilde{\beta} = 1$).

We proposed two estimation strategies based on the identification argument, and implement the proposed estimation method to the decisions of undertaking mammography to evaluate the importance of present bias and naivety in the under-utilization of mammography. Preliminary results show evidence for both present bias and naivety. The preliminary estimate of $\tilde{\beta}/\beta$ is about 1.80 and $\tilde{\beta}\delta$ is about 0.86.

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