Estimating a Dynamic Oligopolistic Game with Serially Correlated Unobserved Production Costs

SS223B-Empirical IO

Motivation

- There have been substantial recent developments in the empirical literature on estimation of dynamic games.
- However, incorporating unobserved (*to the researcher*) state variables that are **serially correlated and endogenous** remains prohibitively difficult.
- In this paper the authors propose a likelihood based method relying on sequential importance sampling to estimate dynamic discrete games of complete information with **serially correlated unobserved endogenous state variables.**

Motivation

- They apply the method to a dynamic oligopolistic model of entry for the generic pharmaceutical industry.
- This application is interesting because the firm specific production costs are serially correlated unobserved state variables that are endogenous to past entry decisions.
- It is worth to note that the proposed method is applicable to similar games that have a Markovian representation of the latent dynamics and an algorithm to solve the game.

Motivation

- The paper also provides evidence on the dynamic spillover effects of experience in one product market on subsequent performance in the market for another product.
- In order to evaluate the effects of current experience on future market performance as measured by future costs and entry, they formulate and estimate a dynamic game theoretic model of oligopolistic competition.
- In a dynamic setting, current entry can have a potential spillover effect on future entry.

Motivation

- In the case of a generic pharmaceutical firm there can be economies of scope that come from experience working with a particular ingredient, therapeutic class, or form of drug (e.g., oral liquid or liquid injectable).
- It allows for serially correlated firm specific costs that evolve endogenously based on past entry decisions.
- Furthermore, endogeneity of costs to past entry decisions induces heterogeneity among firms even if they are identical ex ante, which they need not be.
- They estimate the model parameters using **Bayesian MCMC methods.**

The model

- Firms maximize profits over an infinite horizon $t = 1, ..., \infty$ where each time the market is open counts as one time increment.
- A market opening is defined to be an entry opportunity that becomes available to generic manufacturers each time a branded product goes off patent.
- The actions available to firm i when market t opens are to enter or not, which is denoted as

$$A_{i,t} = \begin{cases} 1, & \text{If firm } i \text{ enter;} \\ 0, & \text{otherwise.} \end{cases}$$

The model

- There are I firms in total so that the number of entrants in market t is given by

$$N_t = \sum_{i=1}^{I} A_{i,t} \tag{1}$$

- The evolution of current costs, C_{it}, is determined by **past entry decisions and random shocks.**
- They consider the convention of $c_{it} = log(C_{it})$.
- The equation governing the log cost of firm i at market t is

$$c_{it} = \mu_c + \rho_c(c_{i,t-1} - \mu_c) - \kappa_c A_{i,t-1} + \sigma_c e_{it}, \tag{2}$$

The model

- The term e_{it} is a normally distributed shock with mean zero and unit variance, σ_c is a scale parameter, κ_c is the entry spillover or immediate impact on cost at market t if there was entry in market $t-1$.
- μ_c is a location parameter that represents the overall average of the log cost over a long period of time.
- The autoregressive parameter ρ_c represents the degree of persistence between the current cost and its long run stationary level

The model

Assumption

All firms are ex ante identical, with the effects of current decisions on future costs creating heterogeneity between firms.

- The log cost can be decomposed into a sum of two components, a **known component** (or observable to the researcher based on past actions), $c_{k,i,t}$ and a component **unobservable to the researcher,** $c_{u,i,t}$ as follows:

The model

$$c_{i,t} = c_{u,i,t} + c_{k,i,t} \tag{3}$$
$$c_{u,i,t} = \mu_c + \rho_c(c_{u,i,t-1} - \mu_c) + \sigma_c e_{it} \tag{4}$$
$$c_{k,i,t} = \rho_c c_{k,i,t-1} - \kappa_c A_{i,t-1} \tag{5}$$

- The total (lump sum) revenue to be divided among firms who enter a market at time t is $R_t = exp(r_t)$, which is realized from the following independent and identical distribution,

$$r_t = \mu_r + \sigma_r e_{I+1,t}, \tag{6}$$

where $e_{I+1,t}$ is normally distributed with mean zero and unit variance.

The model

- In order to solve the model the authors consider dominant firms (3 or 4)
- Under this simplification, they suggest that a reasonable functional form for dominant firm *i*s per period profit at time t is

$$\Pi_{it} = A_{i,t} \times \left\{ \frac{R_t^\gamma}{N_t} - C_{it} \right\}, \tag{7}$$

where $\gamma \in (0.908, 1)$.

- The firms total discounted profit at time t is

$$\sum_{j=0}^{\infty} \beta^j \Pi_{it+j}, \quad 0 < \beta < 1. \tag{8}$$

Solving the Model

- The Bellman equation for the choice specific value function for firm i's dynamic problem at time t is given by

$$V_i(A_{i,t}, A_{-i,t}, C_{i,t}, C_{-i,t}, R_t) = \Pi_{i,t} + \mathbb{E}_{|\Omega_t}(V_i(A_{i,t}, A_{-i,t}, C_{i,t}, C_{-i,t}, R_t)), \tag{9}$$

  where $\Omega_t = (A_{i,t}, A_{-i,t}, C_{i,t}, C_{-i,t}, R_t)$
- The solution concept is given by "*Pure Strategy Perfect Markov Equilibrium*".
- The numerical scheme is as follows:

Estimation: general scheme

- "Nested" Bayesian (MCMC) estimation: equilibrium computation nested within successive iteration of MCMC iteration
- Successive draws of parameters θ drawn according to Metropolis-Hastings approach
- Difficulty is evaluating likelihood function at given θ: use a particle-filtering importance sampling approach.

Details of particle filter

- Observables: $Y_t = (\vec{r}_t, \vec{A}_t)$. Choice variable A_t, obsd state variable r_t
- Unobservables: $\epsilon_t = (\vec{C}_{ut})$: unobserved component of costs
- Likelihood (simplified):

$$\begin{aligned}
\mathcal{L}(\theta) &= \prod_{t=1}^{T} p(y_t | y^{t-1}; \theta) \\
&= \prod \int p(y_t | y^{t-1}, \epsilon^t; \theta) p(\epsilon^t | y^{t-1}) d\epsilon^t \\
&= \prod \left[p(A_t | r_t, y_{t-1}, \epsilon_t; \theta) \cdot p(r_t | y_{t-1}, \epsilon_t; \theta) p(\epsilon^t | y^{t-1}) d\epsilon^t \right] \\
&= \prod \left[\left(\prod_i \mathbb{1} \left\{ A_{it} = A^*_{it}(\epsilon_t, y_{t-1}, r_t : \theta) \right\} \right) \cdot \text{Eq. (6)} \cdot p(\epsilon^t | y^{t-1}) d\epsilon^t \right]
\end{aligned}$$

- $t = 0$: fix y_0, ϵ_0
- $t = 1$: need draws from $p(\epsilon^1 | y_0, \epsilon_0) = p(\epsilon_1 | \epsilon_0)$, which is easy. Draw $\epsilon^{1|0,s}$ for $s = 1, \ldots, S$. Simulate LL for $t = 1$: $\approx \frac{1}{S} \sum_s p(y_t | y_0, \epsilon^{1|0,s}; \theta)$
- $t = 2$: need draws from $p(\epsilon^2 | y^1) = p(\epsilon^1 | y^1) p(\epsilon_2 | \epsilon^1)$. First term is $p(\epsilon^1 | y^1) \propto p(y_1 | \epsilon^1, y_0; \theta) p(\epsilon^1 | y_0)$. So
  1. resample $\epsilon^{1|1,s}$ from $\epsilon^{1|0,s}$ using weights $w_1^s \propto p(y_1 | \epsilon^1, y_0; \theta)$.
  2. Draw $\epsilon_2^s \sim p(\cdot | \epsilon^{1|1,s})$.
  3. Combine for $\epsilon^{2|1,s} = (\epsilon^{1|1,s}, \epsilon_2^s)$.

  Simulate LL for $t = 2$: $\approx \frac{1}{S} \sum_s p(y_t | y_0, \epsilon_2^{2|1,s}; \theta)$
- $t = 3$: ??