Introduction

We go over Esteban and Shum (2002).

In many durable goods industries (notable auto industry), used products are traded in decentralized secondary markets which are not directly controlled by the producers of new goods.

New approach to automobile industry, with following features:

- **Dynamic supply:** Current production "competes" with secondary market
- In oligopoly: strategic durability, to hurt rivals' future profits thru secondary market.
- **Dynamic demand:** Existence of secondary mkt raises consumers' WTP for new product ("investment" motive).

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Relevant literature on durable goods

Theory:

- Large theoretical literature, very little empirical work
- "Coase conjecture" (Coase (1972)): with an infinitely-durable non-depreciating good, monopolist credibly prices only at marginal cost. (Bulow (1982), Stokey (1981), Gul, Sonnenschein, and Wilson (1986), Ausubel and Deneckere (1989))
- w/Secondary market: presence of used cars tomorrow provides extra incentives to lower production today (Liang (1999))

Empirics:

- Dynamic demand models (but exogenous supply): focus on *timing* of new car purchases, due to transactions costs (Adda and Cooper (2000), Attanasio (2000), Berkovec (1985), Eberly (1994), Rust (1985), Stolyarov (2002)).

- Dynamic demand/supply: Ramey (1989), Porter and Sattler (1999) (DG monopoly only).

This model has:

- Dynamic supply: durable goods oligopoly model
- Dynamic demand: forward-looking consumers
- Time consistent production
- Competitive (decentralized) secondary markets

Overcome tractability issues: *linear-quadratic* game.

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Economic Model: Car market

Multiproduct firms producing cars which differ in quality, durability and depreciation schedule.

Empirical model accommodates cost/demand shocks; for simplicity, describe deterministic model.

- Firms $j = 1, \dots, N$. (e.g. Ford, GM, Honda)
- L is total number of brands/models (e.g. Taurus, Accord, Escort).
- Firm j produces L_j models; set of products denoted $\mathcal{L}_j$.
- Model i lasts T_i periods. There are $K \equiv \sum_{i=1}^{L} T_i$ "model-years" actively traded during any given period.
- Each model year differs in one-dimensional quality $\Rightarrow$ quality ladder

  $$[\alpha_1,\ \alpha_2,\ \dots,\ \alpha_K, \alpha_{K+1} = 0]$$

  where α_{K+1} is quality of outside option.

- Notation: depreciation schedules for different models
  - Define: $\eta(i)$ is ranking of model i when new.
  - Define: $v(\eta(i))$ is ranking of 1-yr old; $v^2(\eta(i)) \equiv v\left(v(\eta(i))\right)$ is ranking of 2-yr old, etc.

  $\Rightarrow$ Depreciation schedule of model i described by sequence

$$\left\{\eta\left(i\right), v\left(\eta\left(i\right)\right), \ldots, v^{T_i-1}\left(\eta\left(i\right)\right)\right\}.$$

  Note: each model has its own depreciation schedule.

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Economic model: Dynamic demand

Derive from individual-level optimizing behavior.

- A continuum of infinitely-lived consumers who differ in their preference for quality θ (one dimension)
- Quasilinear per-period utility: $U_t = \theta\alpha_k + m - p_t^k$, where m is total income. Assume no liquidity constraints.
- Choice set: model-years $k = 1, \ldots, K$, plus outside option (utility normalized =0)
- Consumers incur no **transactions costs**: abstract away from timing issues.

  Implies simple form of dynamic decision rule: in period t, consumer θ chooses model-year k yielding maximal "rental utility":

$$k_t = \text{argmax}_k \left\{0, \alpha_k\theta - p_t^k + \delta E_t p_{t+1}^{v(k)},\ k = 1, \ldots, K\right\}$$

  where δ is discount factor and expected rental price is $p_t^k - \delta E_t p_{t+1}^{v(k)}$.

  (Drop E_t for convenience: in deterministic model, consumers have perfect foresight in equilibrium, so $E_t p_{t+1} = p_{t+1}$.

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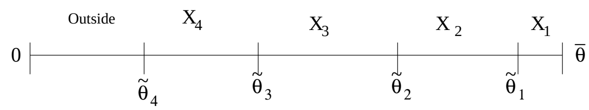
Demand functions

- Given prices $p_t^k,\ p_{t+1}^{v(k)}$ for all $k = 1, \ldots, K$, each period there are K indifferent consumers $\bar{\theta} \geq \tilde{\theta}_t^1 \geq \tilde{\theta}_t^2 \geq \tilde{\theta}_t^3 \geq \ldots \geq \tilde{\theta}_t^K \geq 0$, s.t.



- The indifferent consumers solve

$$\alpha_k \tilde{\theta}_t^k - p_t^k + \delta p_{t+1}^{v(k)} = \quad \alpha_{k+1} \tilde{\theta}_t^k - p_t^{k+1} + \delta p_{t+1}^{v(k+1)}, \quad \text{for } k = 1, \ldots, K-1$$

- Consumer heterogeneity: θ is uniformly distributed (only required for LQ specification of the model).

- Derive inverse demand function

$$p_t^k = (\alpha_k - \alpha_{k+1})\, \bar{\theta} \left(1 - \frac{1}{M} \sum_{r=1}^{k} x_t^r \right) + \delta p_{t+1}^{v(k)} + p_t^{k+1} - \delta p_{t+1}^{v(k+1)}$$

- Equilibrium prices satisfy the following inequalities: $\forall k,\ t$

$$\begin{aligned} & \frac{(p_t^{k-1} - \delta p_{t+1}^{v(k-1)}) - (p_t^k - \delta p_{t+1}^{v(k)})}{\alpha_{k-1} - \alpha_k} \\ \geq & \frac{(p_t^k - \delta p_{t+1}^{v(k)}) - (p_t^{k+1} - \delta p_{t+1}^{v(k+1)})}{\alpha_k - \alpha_{k+1}} \\ \geq & \frac{(p_t^{k+1} - \delta p_{t+1}^{v(k+1)}) - (p_t^{k+2} - \delta p_{t+1}^{v(k+2)})}{\alpha_{k+1} - \alpha_{k+2}} \geq 0. \end{aligned}$$

  Difficult to impose them directly in estimation; we verify that it holds at estimated parameter values.

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Assumptions of model not ad-hoc: 5 crucial assumptions for linear demand functions:

- (*) 1-dim consumer heterogeneity θ: cutoff pt equilibrium

- (**) quasilinear utility + (***) no transactions costs: dynamic demand simplifies to period-by-period "rental" policy
- (****) uniform θ + (*****) secondary markets clear: linear inverse demand functions

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Supply side

- $\boldsymbol{y}_t = [1, x_t^1, \ldots, x_t^K]'$: vector of all cars *transacted* in period t.
- $\boldsymbol{d}_t \equiv \left[x_t^{\eta(1)}, x_t^{\eta(2)}, \ldots, x_t^{\eta(L)}\right]'$: vector of all cars *produced* in period t.
- Define matrices A and B, to get law of motion for y_t:

$$\boldsymbol{y}_t = \boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{B}\boldsymbol{d}_t.$$

- Marginal costs constant; no (dis-)economies of scope: $C_{jt} = \sum_{i \in \mathcal{L}} c_i \cdot x_t^{\eta(i)}$.
- Period t profits for car i is $\Pi_t^i \left(\boldsymbol{y}_t, \boldsymbol{y}_{t+1}, \ldots, \boldsymbol{y}_{t+T_i-1}\right)$: depends on past, current, future prod'n of car i.

  Important: dependence of current profits on future actions leads to a *time-consistency* problem, which is absent from "usual" dynamic problems. Very roughly, time-inconsistency implies that an agent's optimal action in period t differs depending on whether the agent is deciding in period t, or period $t-1$, or period $t-2$, etc.

  Think of durable goods monopoly: in period 1, his optimal optimal period 2 price is the monopoly price (because that would raise his profits in period 1). But when period 2 comes, his optimal period 2 price is actually a lower price (since he wants to sell to people who did not buy in period 1).

- For individual firm: $\forall t$, $\forall j \in \mathcal{N}$, $\forall i \in \mathcal{L}_j$, period-$t$ production $x_t^{\eta(i)}$ maximizes

$$(*) \qquad \max_{x_t^{\eta(i)}, \forall i \in \mathcal{L}_j} \sum_{\tau=0}^{\infty} \sum_{i \in \mathcal{L}_j} \delta^\tau \underbrace{\left[\Pi_{t+\tau}^i \left(\boldsymbol{y}_{t+\tau}, \boldsymbol{y}_{t+\tau+1}, \ldots, \boldsymbol{y}_{t+\tau+T_i-1}\right)\right]}_{\text{period } t+\tau \text{ profits}}, \ \textit{s.t.}$$

$$\boldsymbol{y}_{t+\tau} = \boldsymbol{A}\boldsymbol{y}_{t+\tau-1} + \boldsymbol{B}\boldsymbol{d}_{t+\tau} \ , \quad \text{for } \tau = 1, \ldots, \infty.$$

- Note: obj fxn different in each period t: usual problem is

$$\max_{\left\{x_t^{\eta(i)},\ i \in \mathcal{L}_j\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{i \in \mathcal{L}_j} \delta^t \Pi_t^i \left(\boldsymbol{y}_t, \boldsymbol{y}_{t+1}, \ldots, \boldsymbol{y}_{t+T_i-1}\right).$$

  FOC for $x_t^{\eta(i)}$ contains derivative (say) $\frac{\partial \Pi_{t-1}^i}{\partial x_t^{\eta(i)}}$: in choosing period-$t$ prodn, recognize that it affects period-$(t+1)$ profits $\Rightarrow$ time-inconsistent.

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Time-consistent Equilibrium production

- Restrict attention to Markov strategies: $\boldsymbol{A}\boldsymbol{y}_{t-1}$ is the "payoff-relevant state vector" for period t (stocks of cars produced prior to period t which are still actively traded in period t) $\Longrightarrow$

  Therefore consider production rules $x_t^{\eta(i)} = g_i(\boldsymbol{A}\boldsymbol{y}_{t-1}), \forall i \in \mathcal{L}_j,\ \forall j \in \mathcal{N}$.

- In Markov-perfect equilibrium, $g_1(.), \ldots, g_L(.)$ satisfy Bellman equation

$$V_j(\boldsymbol{A}\boldsymbol{y}_{t-1}) = \max_{x_t^{\eta(i)},\ i \in \mathcal{L}_j} \sum_{i \in \mathcal{L}_j} \Pi_j(\boldsymbol{y}_t, \boldsymbol{y}_{t+1}, \cdots, \boldsymbol{y}_{t+T_i-1}) + \delta V_j(\boldsymbol{A}\boldsymbol{y}_t)$$

  for all firms j, and the Markov decision rules

$$x_t^i = g_i(\boldsymbol{A}\boldsymbol{y}_{t-1}), \text{ for all } i \in \mathcal{L}_j.$$

  Value function $V_j(\boldsymbol{A}\boldsymbol{y}_{t-1})$=(*) (optimal profits from t onwards).

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Linear Quadratic (LQ) Specification

- We focus on *linear* equilibrium decision rule $x_{t+h} = \boldsymbol{G}\boldsymbol{A}\boldsymbol{y}_{t+h-1}$

  No "trigger" strategies (step functions)

- $\Rightarrow$ Quadratic value function $V(\boldsymbol{A}\boldsymbol{y}_{t-1}) = y_t' \boldsymbol{A}' \boldsymbol{S} \boldsymbol{A} y_t$.

- Bellman equation can be rewritten in matrix notation

$$\boldsymbol{y}'_{t-1}\boldsymbol{A}'\boldsymbol{S}\boldsymbol{A}\boldsymbol{y}_{t-1} =$$
$$\max_{x_t^i,\ i\in\mathcal{J}} \left\{ \sum_{i\in\mathcal{J}} \left[\sum_{z=0}^{T_i-1} \boldsymbol{y}'_{t+z}\delta^z \boldsymbol{R}_{v^z(i)}\boldsymbol{y}_t \right] \right\} - \boldsymbol{y}'_t\boldsymbol{C}_j\boldsymbol{y}_t + \boldsymbol{y}'_t\delta\left[\boldsymbol{A}'\boldsymbol{S}_j\boldsymbol{A}\right]\boldsymbol{y}_t$$

- Recursive substitution yields

$$\boldsymbol{y}'_{t-1}\boldsymbol{A}'\boldsymbol{S}\boldsymbol{A}\boldsymbol{y}_{t-1} =$$
$$\max_{x_t^i,\ i\in\mathcal{J}} \boldsymbol{y}'_t \left\{ \left[\sum_{i\in\mathcal{J}} \sum_{z=0}^{T_i-1} (\boldsymbol{A}')^z [(\boldsymbol{I}+\boldsymbol{B}\boldsymbol{G})']^z \delta^z \boldsymbol{R}_{v^z(i)} \right] - \boldsymbol{C}_j + \delta\left[\boldsymbol{A}'\boldsymbol{S}_j\boldsymbol{A}\right] \right\} \boldsymbol{y}_t$$
$$\equiv \max_{x_t^{\eta(i)},\ i\in\mathcal{J}} \boldsymbol{y}'_t\boldsymbol{Q}_j\boldsymbol{y}_t.$$

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Deriving equilibrium production rules

- Value iteration: solve for $\boldsymbol{S}$ and $\boldsymbol{G}$ by iterating over Bellman equation.

- For each $\boldsymbol{S}$, derive corresponding $\boldsymbol{G}$ via FOC of right-hand side:

$$\boldsymbol{B}'_j\left(\boldsymbol{Q}_j+\boldsymbol{Q}'_j\right)\boldsymbol{y}_t = \boldsymbol{B}'_j\left(\boldsymbol{Q}_j+\boldsymbol{Q}'_j\right)\boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{B}'_j\left(\boldsymbol{Q}_j+\boldsymbol{Q}'_j\right)\boldsymbol{B}\boldsymbol{d}_t = 0.$$

  leading to

$$\boldsymbol{d}_{t+h} = -\left(\boldsymbol{W}\boldsymbol{B}\right)^{-1}\left(\boldsymbol{W}\boldsymbol{A}\right)\boldsymbol{y}_{t-1},$$

  where $\boldsymbol{W}_j \equiv \boldsymbol{B}'_j\left(\boldsymbol{Q}_j+\boldsymbol{Q}'_j\right)$ for each firm j and $\boldsymbol{W} \equiv [\boldsymbol{W}_1,\ldots,\boldsymbol{W}_N]'$.

  Verifies that linear strategy is equilibrium.

  Basis for estimating supply side of model.

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Estimation

Model has no sources of error: overidentified relative to data. Introduce shocks to costs:

$$C(x_t^{\eta(i)}) = x_t^{\eta(i)}\left(c_i + \epsilon_{it}\right).$$

Assumptions: $\boldsymbol{\epsilon}_t \equiv [\epsilon_{1t}, \ldots, \epsilon_{Lt}]'$ has zero-mean, *i.i.d.* across t.

Then supply and demand relations:

$$\boldsymbol{d}_t = \boldsymbol{G}\boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{w}_t, \text{ where } E(\boldsymbol{w}_t) = \mathbf{0}.$$

$$\begin{aligned} 0 &= E\left[p_t^{\eta(i)} - \left(\alpha_{\eta(i)} - \alpha_{\eta(i)+1}\right)\left(1 - \frac{1}{M}\sum_{r=1}^{\eta(i)} x_t^r\right) - \delta p_{t+1}^{v(\eta(i))} - p_t^{\eta(i)+1} + \delta p_{t+1}^{v(\eta(i)+1)} \middle| \Omega_t \right] \\ &= E\left[\left(1 - \delta L^{-1}\right)\left(p_t^{\eta(i)} - p_t^{\eta(i)+1}\right) - \left(\alpha_{\eta(i)} - \alpha_{\eta(i)+1}\right)\left(1 - \frac{1}{M}\sum_{r=1}^{\eta(i)} x_t^r\right) \middle| \Omega_t \right] \end{aligned}$$

Supply: can use OLS
Demand: IV are elements of Ω_t

Could also have additive demand shocks (LQ structure quite general!)

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GMM Estimation

- Source of identification: co-movements time series of prices and production.
- Reduced-form is VAR, but (i) will not recover the structural parameters; and (ii) too many parameters.
- Nested GMM procedure: for each value of parameters $\boldsymbol{\psi}$, solve LQ dynamic programming problem for coefficients $\boldsymbol{G}(\boldsymbol{\psi})$ of optimal production rules.
- Let $\boldsymbol{\mu}_T(\boldsymbol{\psi}) \equiv \begin{bmatrix} \boldsymbol{\gamma}_T^s(\boldsymbol{\psi}) \\ \boldsymbol{\gamma}_T^d(\boldsymbol{\psi}) \end{bmatrix}$.
- GMM estimator $\boldsymbol{\psi} \equiv \text{argmin}_{\boldsymbol{\psi}} \boldsymbol{\mu}_T(\boldsymbol{\psi})' \boldsymbol{\Omega}_T^{-1} \boldsymbol{\mu}_T(\boldsymbol{\psi})$.

References

Adda, J., and R. Cooper (2000): "Balladurette and Jupette: A Discrete Analysis of Scrapping Subsidies," *Journal of Political Economy*, 108, 778–806.

Attanasio, O. (2000): "Consumer Durables and Inertial Behavior: Estimation and Aggregation of (s,S) Rules," *Review of Economic Studies*, 67, 667–696.

Ausubel, L., and R. Deneckere (1989): "Reputation in Bargaining and Durable Goods Monopoly," *Econometrica*, 58, 511–531.

Berkovec, J. (1985): "New Car Sales and Used Car Stocks: A Model of the Automobile Market," *RAND Journal of Economics*, 16, 195–214.

Bulow, J. (1982): "Durable-Goods Monopolists," *Journal of Political Economy*, 90, 314–332.

Coase, R. (1972): "Durability and Monopoly," *Journal of Law and Economics*, 15, 143–149.

Eberly, J. (1994): "Adjustment of Consumers' Durables Stocks: Evidence from Automobile Purchases," *Journal of Political Economy*, 102, 403–436.

Esteban, S., and M. Shum (2002): "Durable Goods Oligopoly with Secondary Markets: the Case of Automobiles," mimeo., Penn State University, available at `www.econ.jhu.edu/people/shum/res.html`.

Gul, F., H. Sonnenschein, and R. Wilson (1986): "Foundations of Dynamic Monopoly and the Coase Conjecture," *Journal of Economic Theory*, 39, 155–190.

Liang, M. (1999): "Does a Second-Hand Market Limit a Durable Goods Monopolist's Market Power?," mimeo., University of Western Ontario.

Porter, R., and P. Sattler (1999): "Patterns of Trade in the Market for Used Durables: Theory and Evidence," NBER working paper, #7149.

Ramey, V. (1989): "Durable Goods Monopoly Behavior in the Automobile Industry," mimeo., UC–San Diego.

Rust, J. (1985): "Stationary Equilibrium in a Market for Durable Assets," *Econometrica*, 53, 783–805.

Stokey, N. (1981): "Rational Expectations and Durable Goods Pricing," *Bell Journal of Economics*, 12, 112–128.

Stolyarov, D. (2002): "Turnover of Used Durables in a Stationary Equilibrium: are Older Goods traded More?," *Journal of Political Economy*, 110, 1390–1413.