The Spectral Action and the Standard Model

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Topics in Mathematical Physics

References

- A.H. Chamseddine, A. Connes, M. Marcolli, *Gravity and the Standard Model with Neutrino Mixing*, Adv. Theor. Math. Phys., Vol.11 (2007) 991–1090

The spectral action functional

- Ali Chamseddine, Alain Connes, *The spectral action principle*, Comm. Math. Phys. 186 (1997), no. 3, 731–750.

A good action functional for noncommutative geometries

$$\mathrm{Tr}(f(D/\Lambda))$$

D Dirac, Λ mass scale, $f > 0$ even smooth function (cutoff approx)
Simple dimension spectrum $\Rightarrow$ expansion for $\Lambda \to \infty$

$$\mathrm{Tr}(f(D/\Lambda)) \sim \sum_k f_k \Lambda^k ⨍ |D|^{-k} + f(0)\,\zeta_D(0) + o(1),$$

with $f_k = \int_0^\infty f(v)\, v^{k-1}\, dv$ momenta of f
where $\mathrm{DimSp}(\mathcal{A}, \mathcal{H}, D) =$ poles of $\zeta_{b,D}(s) = \mathrm{Tr}(b|D|^{-s})$

Asymptotic expansion of the spectral action

$$\mathrm{Tr}(e^{-t\Delta}) \sim \sum a_\alpha \, t^\alpha \qquad (t \to 0)$$

and the ζ function

$$\zeta_D(s) = \mathrm{Tr}(\Delta^{-s/2})$$

- Non-zero term a_α with $\alpha < 0 \Rightarrow$ *pole* of ζ_D at -2α with

$$\mathrm{Res}_{s=-2\alpha}\, \zeta_D(s) = \frac{2\, a_\alpha}{\Gamma(-\alpha)}$$

- No $\log t$ terms $\Rightarrow$ regularity at 0 for ζ_D with $\zeta_D(0) = a_0$

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- Get first statement from

$$|D|^{-s} = \Delta^{-s/2} = \frac{1}{\Gamma\left(\frac{s}{2}\right)} \int_0^\infty e^{-t\Delta}\, t^{s/2-1}\, dt$$

with $\int_0^1 t^{\alpha+s/2-1}\, dt = (\alpha + s/2)^{-1}$.

- Second statement from

$$\frac{1}{\Gamma\left(\frac{s}{2}\right)} \sim \frac{s}{2} \quad \text{as} \quad s \to 0$$

contrib to $\zeta_D(0)$ from pole part at $s = 0$ of

$$\int_0^\infty \mathrm{Tr}(e^{-t\Delta})\, t^{s/2-1}\, dt$$

given by $a_0 \int_0^1 t^{s/2-1}\, dt = a_0 \frac{2}{s}$

Spectral action with fermionic terms

$$S = \mathrm{Tr}(f(D_A/\Lambda)) + \frac{1}{2}\langle J\tilde{\xi}, D_A\tilde{\xi}\rangle, \quad \tilde{\xi} \in \mathcal{H}_{cl}^+,$$

D_A = Dirac with unimodular inner fluctuations, J = real structure,
$\mathcal{H}_{cl}^+$ = classical spinors, Grassmann variables

Fermionic terms

$$\frac{1}{2}\langle J\tilde{\xi}, D_A\tilde{\xi}\rangle$$

antisymmetric bilinear form $\mathfrak{A}(\tilde{\xi})$ on

$$\mathcal{H}_{cl}^+ = \{\xi \in \mathcal{H}_{cl} \,|\, \gamma\xi = \xi\}$$

$\Rightarrow$ nonzero on Grassmann variables

Euclidean functional integral $\Rightarrow$ Pfaffian

$$Pf(\mathfrak{A}) = \int e^{-\frac{1}{2}\mathfrak{A}(\tilde{\xi})} D[\tilde{\xi}]$$

avoids Fermion doubling problem of previous models based on symmetric $\langle\xi, D_A\xi\rangle$ for NC space with KO-dim=0

Grassmann variables

Anticommuting variables with basic integration rule

$$\int \xi \, d\xi = 1$$

An antisymmetric bilinear form $\mathfrak{A}(\xi_1, \xi_2)$: if ordinary commuting variables $\mathfrak{A}(\xi, \xi) = 0$ but not on Grassmann variables
Example: 2-dim case $\mathfrak{A}(\xi', \xi) = a(\xi'_1 \xi_2 - \xi'_2 \xi_1)$, if ξ_1 and ξ_2 anticommute, with integration rule as above

$$\int e^{-\frac{1}{2}\mathfrak{A}(\xi,\xi)} D[\xi] = \int e^{-a\xi_1\xi_2} d\xi_1 d\xi_2 = a$$

Pfaffian as functional integral: antisymmetric quadratic form

$$Pf(\mathfrak{A}) = \int e^{-\frac{1}{2}\mathfrak{A}(\xi,\xi)} \, D[\xi]$$

Method to treat Majorana fermions in the Euclidean setting

Fermionic part of SM Lagrangian

Explicit computation of

$$\frac{1}{2}\langle J\tilde{\xi}, D_A\tilde{\xi}\rangle$$

gives part of SM Larangian with

- $\mathcal{L}_{Hf}$ = coupling of Higgs to fermions
- $\mathcal{L}_{gf}$ = coupling of gauge bosons to fermions
- $\mathcal{L}_{f}$ = fermion terms

Bosonic part of the spectral action

$$
\begin{aligned}
S = \quad & \frac{1}{\pi^2}(48\, f_4\, \Lambda^4 - f_2\, \Lambda^2\, \mathfrak{c} + \frac{f_0}{4}\, \mathfrak{d}) \int \sqrt{g}\, d^4x \\
+ \quad & \frac{96\, f_2\, \Lambda^2 - f_0\, \mathfrak{c}}{24\pi^2} \int R\, \sqrt{g}\, d^4x \\
+ \quad & \frac{f_0}{10\,\pi^2} \int (\frac{11}{6}\, R^* R^* - 3\, C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma})\, \sqrt{g}\, d^4x \\
+ \quad & \frac{(-2\,\mathfrak{a}\, f_2\, \Lambda^2 \;+\; \mathfrak{e}\, f_0)}{\pi^2} \int |\varphi|^2\, \sqrt{g}\, d^4x \\
+ \quad & \frac{f_0 \mathfrak{a}}{2\,\pi^2} \int |D_\mu \varphi|^2\, \sqrt{g}\, d^4x \\
- \quad & \frac{f_0 \mathfrak{a}}{12\,\pi^2} \int R\, |\varphi|^2\, \sqrt{g}\, d^4x \\
+ \quad & \frac{f_0 \mathfrak{b}}{2\,\pi^2} \int |\varphi|^4\, \sqrt{g}\, d^4x \\
+ \quad & \frac{f_0}{2\,\pi^2} \int (g_3^2\, G^i_{\mu\nu}\, G^{\mu\nu i} + g_2^2\, F^\alpha_{\mu\nu}\, F^{\mu\nu\alpha} + \frac{5}{3}\, g_1^2\, B_{\mu\nu}\, B^{\mu\nu})\, \sqrt{g}\, d^4x,
\end{aligned}
$$

Parameters:

- f_0, f_2, f_4 free parameters, $f_0 = f(0)$ and, for $k > 0$,

$$f_k = \int_0^\infty f(v) v^{k-1} dv.$$

- $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ functions of Yukawa parameters of νMSM

$$\begin{aligned}
\mathfrak{a} &= \operatorname{Tr}(Y_\nu^\dagger Y_\nu + Y_e^\dagger Y_e + 3(Y_u^\dagger Y_u + Y_d^\dagger Y_d)) \\
\mathfrak{b} &= \operatorname{Tr}((Y_\nu^\dagger Y_\nu)^2 + (Y_e^\dagger Y_e)^2 + 3(Y_u^\dagger Y_u)^2 + 3(Y_d^\dagger Y_d)^2) \\
\mathfrak{c} &= \operatorname{Tr}(MM^\dagger) \\
\mathfrak{d} &= \operatorname{Tr}((MM^\dagger)^2) \\
\mathfrak{e} &= \operatorname{Tr}(MM^\dagger Y_\nu^\dagger Y_\nu).
\end{aligned}$$

Gilkey's theorem using $D_A^2 = \nabla^* \nabla - E$

Differential operator $P = -(g^{\mu\nu} I\, \partial_\mu \partial_\nu + A^\mu \partial_\mu + B)$ with A, B bundle endomorphisms, $m = \dim M$

$$\mathrm{Tr}\, e^{-tP} \sim \sum_{n \geq 0} t^{\frac{n-m}{2}} \int_M a_n(x, P)\, dv(x)$$

$P = \nabla^* \nabla - E$ and $E_{;\mu}{}^{\mu} := \nabla_\mu \nabla^\mu E$

$$\nabla_\mu = \partial_\mu + \omega'_\mu, \qquad \omega'_\mu = \frac{1}{2} g_{\mu\nu} (A^\nu + \Gamma^\nu \cdot \mathrm{id})$$

$$E = B - g^{\mu\nu} (\partial_\mu \omega'_\nu + \omega'_\mu \omega'_\nu - \Gamma^\rho_{\mu\nu} \omega'_\rho)$$

$$\Omega_{\mu\nu} = \partial_\mu \omega'_\nu - \partial_\nu \omega'_\mu + [\omega'_\mu, \omega'_\nu]$$

Seeley-DeWitt coefficients

$$\begin{aligned}
a_0(x, P) &= (4\pi)^{-m/2} \mathrm{Tr}(\mathrm{id}) \\
a_2(x, P) &= (4\pi)^{-m/2} \mathrm{Tr}\left(-\frac{R}{6}\, \mathrm{id} + E\right) \\
a_4(x, P) &= (4\pi)^{-m/2} \frac{1}{360} \mathrm{Tr}(-12 R_{;\mu}{}^{\mu} + 5 R^2 - 2 R_{\mu\nu} R^{\mu\nu} \\
&+ 2 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 60\, R\, E + 180\, E^2 + 60\, E_{;\mu}{}^{\mu} \\
&+ 30\, \Omega_{\mu\nu} \Omega^{\mu\nu})
\end{aligned}$$

Normalization and coefficients

- Rescale Higgs field $H = \frac{\sqrt{\mathfrak{a} f_0}}{\pi} \varphi$ to normalize kinetic term $\int \frac{1}{2} |D_\mu \mathbf{H}|^2 \sqrt{g}\, d^4x$
- Normalize Yang-Mills terms $\frac{1}{4} G^i_{\mu\nu} \overline{G}^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} \overline{F}^{\mu\nu\alpha} + \frac{1}{4} B_{\mu\nu} \overline{B}^{\mu\nu}$

Normalized form:

$$
\begin{aligned}
S = \; & \frac{1}{2\kappa_0^2} \int R \sqrt{g}\, d^4x + \gamma_0 \int \sqrt{g}\, d^4x \\
+ \; & \alpha_0 \int C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma} \sqrt{g}\, d^4x + \tau_0 \int R^* R^* \sqrt{g}\, d^4x \\
+ \; & \frac{1}{2} \int |DH|^2 \sqrt{g}\, d^4x - \mu_0^2 \int |H|^2 \sqrt{g}\, d^4x \\
- \; & \xi_0 \int R\, |H|^2 \sqrt{g}\, d^4x + \lambda_0 \int |H|^4 \sqrt{g}\, d^4x \\
+ \; & \frac{1}{4} \int (G^i_{\mu\nu}\, G^{\mu\nu i} + F^\alpha_{\mu\nu}\, F^{\mu\nu\alpha} + B_{\mu\nu}\, B^{\mu\nu}) \sqrt{g}\, d^4x
\end{aligned}
$$

where $R^* R^* = \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} \epsilon_{\alpha\beta\gamma\delta} R^{\alpha\beta}_{\mu\nu} R^{\gamma\delta}_{\rho\sigma}$ integrates to the Euler characteristic $\chi(M)$ and $C^{\mu\nu\rho\sigma}$ Weyl curvature

Coefficients

$$\frac{1}{2\kappa_0^2} = \frac{96 f_2 \Lambda^2 - f_0 \mathfrak{c}}{24\pi^2} \qquad \gamma_0 = \frac{1}{\pi^2}\left(48 f_4 \Lambda^4 - f_2 \Lambda^2 \mathfrak{c} + \frac{f_0}{4}\mathfrak{d}\right)$$

$$\alpha_0 = -\frac{3 f_0}{10\pi^2} \qquad \tau_0 = \frac{11 f_0}{60\pi^2}$$

$$\mu_0^2 = 2\frac{f_2 \Lambda^2}{f_0} - \frac{\mathfrak{e}}{\mathfrak{a}} \qquad \xi_0 = \frac{1}{12}$$

$$\lambda_0 = \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2}$$

Energy scale: Unification (10^{15} – 10^{17} GeV)

$$\frac{g^2 f_0}{2\pi^2} = \frac{1}{4}$$

Preferred energy scale, unification of coupling constants

1-loop RGE equations for νMSN

$$\partial_t x_i(t) = \beta_{x_i}(x(t))$$

variable $t = \log(\Lambda/M_Z)$

- Coupling constants:

$$\beta_1 = \frac{41}{96\pi^2}\, g_1^3, \quad \beta_2 = -\frac{19}{96\pi^2}\, g_2^3, \quad \beta_3 = -\frac{7}{16\pi^2} g_3^3$$

at 1-loop decoupled from other equations (Notation: $\tilde{g}_1^2 = \frac{5}{3} g_1^2$)

- Yukawa parameters:

$$16\pi^2\ \beta_{Y_u} = Y_u\Big(\frac{3}{2} Y_u^\dagger Y_u - \frac{3}{2} Y_d^\dagger Y_d + \mathfrak{a} - \frac{17}{20}\tilde{g}_1^2 - \frac{9}{4} g_2^2 - 8 g_3^2\Big)$$

$$16\pi^2\ \beta_{Y_d} = Y_d\Big(\frac{3}{2} Y_d^\dagger Y_d - \frac{3}{2} Y_u^\dagger Y_u + \mathfrak{a} - \frac{1}{4}\tilde{g}_1^2 - \frac{9}{4} g_2^2 - 8 g_3^2\Big)$$

$$16\pi^2\ \beta_{Y_\nu} = Y_\nu\Big(\frac{3}{2} Y_\nu^\dagger Y_\nu - \frac{3}{2} Y_e^\dagger Y_e + \mathfrak{a} - \frac{9}{20}\tilde{g}_1^2 - \frac{9}{4} g_2^2\Big)$$

$$16\pi^2\ \beta_{Y_e} = Y_e\Big(\frac{3}{2} Y_e^\dagger Y_e - \frac{3}{2} Y_\nu^\dagger Y_\nu + \mathfrak{a} - \frac{9}{4}\tilde{g}_1^2 - \frac{9}{4} g_2^2\Big)$$

• Majorana mass terms:

$$16\pi^2 \ \beta_M = Y_\nu Y_\nu^\dagger M + M(Y_\nu Y_\nu^\dagger)^T$$

• Higgs self coupling:

$$16\pi^2 \ \beta_\lambda = 6\lambda^2 - 3\lambda(3g_2^2 + g_1^2) + 3g_2^4 + \frac{3}{2}(g_1^2 + g_2^2)^2 + 4\lambda\mathfrak{a} - 8\mathfrak{b}$$

• MSM approximation: top quark Yukawa parameter dominant term: in λ running neglect all terms except coupling constants g_i and Yukawa parameter of top quark y_t

$$\beta_\lambda = \frac{1}{16\pi^2}\left(24\lambda^2 + 12\lambda y^2 - 9\lambda(g_2^2 + \frac{1}{3}g_1^2) - 6y^4 + \frac{9}{8}g_2^4 + \frac{3}{8}g_1^4 + \frac{3}{4}g_2^2 g_1^2\right)$$

where Yukawa parameter for the top quark runs by

$$\beta_y = \frac{1}{16\pi^2}\left(\frac{9}{2}y^3 - 8g_3^2 y - \frac{9}{4}g_2^2 y - \frac{17}{12}g_1^2 y\right)$$

Renormalization group flow

- The coefficients $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ (depend on Yukawa parameters) run with the RGE flow
- Initial conditions at unification energy: compatibility with physics at low energies

RGE in the MSM case

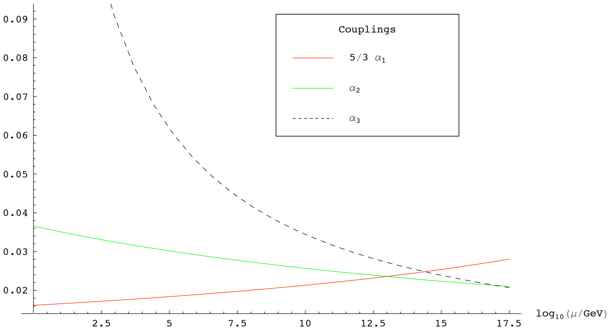
Running of coupling constants at one loop: $\alpha_i = g_i^2/(4\pi)$

$$\beta_{g_i} = (4\pi)^{-2}\, b_i\, g_i^3, \qquad \text{with} \quad b_i = (\frac{41}{6}, -\frac{19}{6}, -7),$$

$$\begin{aligned}
\alpha_1^{-1}(\Lambda) &= \alpha_1^{-1}(M_Z) - \frac{41}{12\pi} \log \frac{\Lambda}{M_Z} \\
\alpha_2^{-1}(\Lambda) &= \alpha_2^{-1}(M_Z) + \frac{19}{12\pi} \log \frac{\Lambda}{M_Z} \\
\alpha_3^{-1}(\Lambda) &= \alpha_3^{-1}(M_Z) + \frac{42}{12\pi} \log \frac{\Lambda}{M_Z}
\end{aligned}$$

$M_Z \sim 91.188$ GeV mass of Z^0 boson

At one loop RGE for coupling constants decouples from Yukawa parameters (not at 2 loops!)



Well known triangle problem: with known low energy values constants don't meet at unification $g_3^2 = g_2^2 = 5g_1^2/3$

Geometry point of view

- At one loop coupling constants decouple from Yukawa parameters
- Solving for coupling constants, RGE flow defines a vector field on moduli space $\mathcal{C}_3 \times \mathcal{C}_1$ of Dirac operators on the finite NC space F
- Subvarieties invariant under flow are relations between the SM parameters that hold at all energies
- At two loops or higher, RGE flow on a rank three vector bundle (fiber = coupling constants) over the moduli space $\mathcal{C}_3 \times \mathcal{C}_1$
- Geometric problem: studying the flow and the geometry of invariant subvarieties on the moduli space

Constraints at unification

The geometry of the model imposes conditions at unification energy: specific to this NCG model

- λ parameter constraint

$$\lambda(\Lambda_{unif}) = \frac{\pi^2}{2f_0} \frac{\mathfrak{b}(\Lambda_{unif})}{\mathfrak{a}(\Lambda_{unif})^2}$$

- Higgs vacuum constraint

$$\frac{\sqrt{\mathfrak{a} f_0}}{\pi} = \frac{2M_W}{g}$$

- See-saw mechanism and $\mathfrak{c}$ constraint

$$\frac{2f_2\Lambda_{unif}^2}{f_0} \leq \mathfrak{c}(\Lambda_{unif}) \leq \frac{6f_2\Lambda_{unif}^2}{f_0}$$

- Mass relation at unification

$$\sum_{generations} (m_\nu^2 + m_e^2 + 3m_u^2 + 3m_d^2)|_{\Lambda=\Lambda_{unif}} = 8M_W^2|_{\Lambda=\Lambda_{unif}}$$

Need to have compatibility with low energy behavior

Mass relation at unification $Y_2(S) = 4g^2$

$$Y_2 = \sum_\sigma (y_\nu^\sigma)^2 + (y_e^\sigma)^2 + 3\,(y_u^\sigma)^2 + 3\,(y_d^\sigma)^2$$

$$\begin{array}{rcl}
(k_{(\uparrow 3)})_{\sigma\kappa} & = & \frac{g}{2M}\, m_u^\sigma\, \delta_\sigma^\kappa \\
(k_{(\downarrow 3)})_{\sigma\kappa} & = & \frac{g}{2M}\, m_d^\mu\, C_{\sigma\mu}\delta_\mu^\rho C_{\rho\kappa}^\dagger \\
(k_{(\uparrow 1)})_{\sigma\kappa} & = & \frac{g}{2M}\, m_\nu^\sigma\, \delta_\sigma^\kappa \\
(k_{(\downarrow 1)})_{\sigma\kappa} & = & \frac{g}{2M}\, m_e^\mu\, {U^{lep}}_{\sigma\mu}\delta_\mu^\rho {U^{lep}}^\dagger_{\rho\kappa}
\end{array}$$

δ_i^j = Kronecker delta, then constraint:

$$\mathrm{Tr}(k_{(\uparrow 1)}^* k_{(\uparrow 1)} + k_{(\downarrow 1)}^* k_{(\downarrow 1)} + 3(k_{(\uparrow 3)}^* k_{(\uparrow 3)} + k_{(\downarrow 3)}^* k_{(\downarrow 3)})) = 2\,g^2$$

$\Rightarrow$ mass matrices satisfy

$$\sum_\sigma (m_\nu^\sigma)^2 + (m_e^\sigma)^2 + 3\,(m_u^\sigma)^2 + 3\,(m_d^\sigma)^2 = 8\,M^2$$

See-saw mechanism: $D = D(Y)$ Dirac

$$\begin{pmatrix} 0 & M_\nu^* & M_R^* & 0 \\ M_\nu & 0 & 0 & 0 \\ M_R & 0 & 0 & \bar{M}_\nu^* \\ 0 & 0 & \bar{M}_\nu & 0 \end{pmatrix}$$

on subspace $(\nu_R, \nu_L, \bar{\nu}_R, \bar{\nu}_L)$: largest eigenvalue of $M_R \sim \Lambda$ unification scale. Take $M_R = x\, k_R$ in flat space, Higgs vacuum v small (w/resp to unif scale) $\partial_u \mathrm{Tr}(f(D_A/\Lambda)) = 0 \quad u = x^2$

$$x^2 = \frac{2\, f_2\, \Lambda^2\, \mathrm{Tr}(k_R^* k_R)}{f_0\, \mathrm{Tr}((k_R^* k_R)^2)}$$

Dirac mass $M_\nu \sim$ Fermi energy v

$$\frac{1}{2}(\pm m_R \pm \sqrt{m_R^2 + 4\, v^2})$$

two eigenvalues $\sim \pm m_R$ and two $\sim \pm \frac{v^2}{m_R}$
Compare with estimates

$$(m_R)_1 \geq 10^7\, GeV\,, \quad (m_R)_2 \geq 10^{12}\, GeV\,, \quad (m_R)_3 \geq 10^{16}\, GeV$$

Low energy limit: compatibilities and predictions
Running of top Yukawa coupling (dominant term):

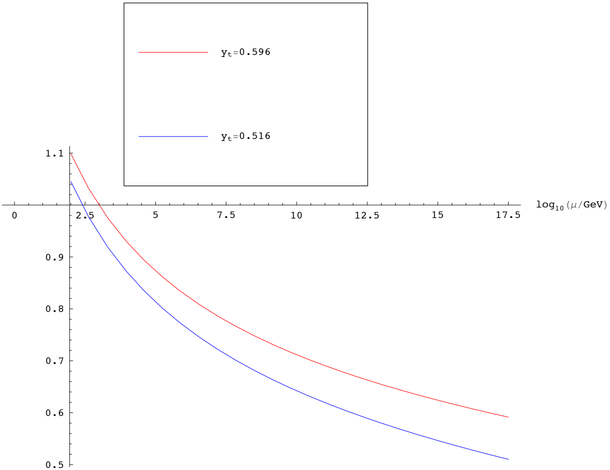
$$\frac{v}{\sqrt{2}}(y^\sigma_\cdot) = (m^\sigma_\cdot),$$

$$\frac{dy_t}{dt} = \frac{1}{16\pi^2}\left[\frac{9}{2}y_t^3 - \left(a\, g_1^2 + b\, g_2^2 + c\, g_3^2\right) y_t\right],$$

$$(a, b, c) = (\frac{17}{12}, \frac{9}{4}, 8)$$

$\Rightarrow$ value of top quark mass agrees with known (1.04 times if neglect other Yukawa couplings)

Top quark running using mass relation at unification



correction to MSM flow by y_ν^σ for τ neutrino (allowed to be comparably large by see-saw) lowers value

Higgs mass prediction using RGE for MSM

Higgs scattering parameter:

$$\frac{f_0}{2\pi^2}\int b\,|\varphi|^4\,\sqrt{g}\,d^4x = \frac{\pi^2}{2\,f_0}\frac{b}{a^2}\int |\mathbf{H}|^4\,\sqrt{g}\,d^4x$$

$\Rightarrow$ relation at unification ($\tilde{\lambda}$ is $|\mathbf{H}|^4$ coupling)

$$\tilde{\lambda}(\Lambda) = g_3^2\,\frac{b}{a^2}$$

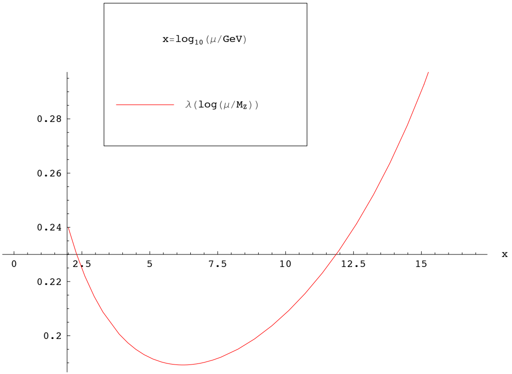
Running of Higgs scattering parameter:

$$\frac{d\lambda}{dt} = \lambda\gamma + \frac{1}{8\pi^2}(12\lambda^2 + B)$$

$$\gamma = \frac{1}{16\pi^2}(12y_t^2 - 9g_2^2 - 3g_1^2) \quad B = \frac{3}{16}(3g_2^4 + 2g_1^2g_2^2 + g_1^4) - 3y_t^4$$

Higgs estimate (in MSM approximation for RGE flow)

$$m_H^2 = 8\lambda \frac{M^2}{g^2}, \quad m_H = \sqrt{2\lambda}\,\frac{2M}{g}$$



$\lambda(M_Z) \sim 0.241$ and Higgs mass ~ 170 GeV (w/correction from see-saw ~ 168 GeV) ... Problem: wrong Higgs mass! too heavy