Natural Language Processing

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Reference

- C.D. Manning, H. Schütze, *Foundations of Statistical Natural Language Processing*, MIT Press, 1999.

• Setting based on Probabilistic Linguistics

• Electronic Corpora
- Linguistic Data Consortium
- European Language Resources Association
- International Computer Archive of Modern English
- Oxford Text Archive
- Child Language Data Exchange System

• Stemming: stripping off affixes and word formation and extract stem of words from a word list

• Markup: syntactic structure is marked

• Penn Treebank: Lisp-like bracketing to mark binary tree structure of sentence

• SGML (Standard Generalized Markup Language): HTML is a type of SGML encoding; Text Encoding Initiative (TEI) encoding scheme suitable for marking parts of various texts, XML simplified form good for web applications

• Grammatical Tagging: automated tagging for categories (parts of speech: nouns, verbs,...)

• Tag Sets: American Brown Corpus (developed to tag the Lancaster–Oslo–Bergen corpus and British National Corpus)

• Penn Treebank tag set: most widely used in computational setting (simplified version of previous)

• rule: least marked category is used as default whenever a word cannot be placed in any other more precise subcategory with additional markings

• Example: "Adjectives" used if cannot further place into "comparatives, superlatives, numerals,..."

• available tag sets are very different (some coarser, some more refined)

- good criterion for tag sets: want to use tag of parts of speech that "best predicts" the behavior of nearby words
- some tags may be too coarse to be good predictors
- Example: adverb *not* has very different distribution than other adverbs, but if tagged in the same way the distinction is lost
- Example: should one use same tag for auxiliaries and for other verbs? (Penn Treebank does)
- correctly identifying proper names is an issue

• at both syntactic and semantic level punctuation matters...
- *the panda bear eats shoots and leaves*
- *the panda bear eats, shoots, and leaves*

• Collocations: a collection of two or more words with limited compositionality

• compositionality: meaning of whole expression deduced from meaning of parts

• Example: an idiom is not compositional
I heard it through the grapevine

• Example: a combination of adjective+noun which cannot be replaced by other synonyms
strong tea (cannot replace "strong" with other words referring to "physical strength" in the same context)

• Example: technical terms are also examples of collocation

- Collocations are identified by frequency based search (for combination of words)
- Collocation window: catch combinations of words that are not contiguous in a sentence
- Example: identify "knock – door" as a collocation (cannot replace "knock" with another term like "hit" or "beat", it would not be correct) requires a window of several words
they knocked repeatedly at the metal front door
- the collocation in such cases can be identified by computing the mean and variance of the signed distance between the two words over the corpus
- collocations = pairs with low deviation (if two unrelated words, variance would be higher)

• still high frequency and low variance can happen by accident: need to test hypothesis

• null hypothesis (no association) and compute probability of event under this hypothesis, reject if too low
(below a given significance level)

• under null hypothesis (independence): $\mathbb{P}(w_1, w_2) = \mathbb{P}(w_1) \cdot \mathbb{P}(w_2)$

• t-test for collocations: $\mathbb{P}_f(w_1, w_2) =$ frequency, number of occurrences normalized by corpus size N

$$t = \frac{\mathbb{P}_f(w_1, w_2) - \mathbb{P}(w_1) \cdot \mathbb{P}(w_2)}{\sqrt{\sigma^2/N}}$$

$\sigma =$ variance

• if t-value is not larger than critical value (for a desired "confidence level") then cannot reject null hypothesis

- use for disambiguation for synonyms
- Example: *strong* and *powerful* can sometime be used as synonyms but not always
- look for other words that form a collocation with one but not with the other (as a way to separate their meaning)
- Examples: *powerful computer*; *strong support*
- w_1 and w_2 the words being compared; w another word

$$t \sim \frac{\mathbb{P}_f(w, w_1) - \mathbb{P}_f(w, w_2)}{\sqrt{\mathbb{P}_f(w, w_1) + \mathbb{P}_f(w, w_2)}}$$

comparison of the means of two independent normal populations

- t-test assumes probabilities normally distributed
- more general test: χ^2-test
- form a 2×2 matrix of frequencies $\mathbb{P}(v_1, v_2)$

$$(O_{ij}) = \begin{pmatrix} \mathbb{P}_f(v_1 = w_1, v_2 = w_2) & \mathbb{P}_f(v_1 = w_1, v_2 \neq w_2) \\ \mathbb{P}_f(v_1 \neq w_1, v_2 = w_2) & \mathbb{P}_f(v_1 \neq w_1, v_2 \neq w_2) \end{pmatrix}$$

then compute

$$\chi^2 = \sum_{i,j} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

with expected values E_{ij} under null hypothesis

- can apply to other sizes than 2×2
- use χ^2 as measure of corpus similarity on an $N \times 2$ matrix $N =$ list of words, $2 =$ two corpora being compared, record frequencies of words in the two corpora

- use χ^2 for identification of translation pairs in aligned corpora
- alignment: know which sentence matches which sentence in the overall text
- count occurrences of words in matching sentences (by 2×2-matrix: frequency of both occurring, of one but not other, of neither)
- high value of χ^2 (with respect to E_{ij} of independence null-hypothesis) reveals a matching translation pair with high confidence

- Likelihood ratio another method for hypothesis testing
- compare independence hypothesis formulated as

$$\mathbb{P}(w_1 \mid w_2) = \mathbb{P}(w_1 \mid \neg w_2) = p$$

with dependence case

$$p_1 = \mathbb{P}(w_1 \mid w_2) \neq \mathbb{P}(w_1 \mid \neg w_2) = p_2$$

- occurrences counting: c_1, c_2, c_{12} for w_1, w_2 and both
- Example: assume a binomial distribution

$$b(k; n, x) = \binom{n}{k} x^k (1-x)^{n-k}$$

- then c_{12} out of c_1 bigrams are $w_1 w_2$:
$b(c_{12}; c_1, p)$ in first case; $b(c_{12}; c_1, p_1)$ in second case
- $c_2 - c_{12}$ out of $N - c_1$ bigrams are $\neq w_1 w_2$
$b(c_2 - c_{12}; N - c_1, p)$ in first case; $b(c_2 - c_{12}; N - c_1, p_2)$ in second

- likelihood of independence hypothesis is product
$\mathcal{L}_1 = b(c_{12}; c_1, p) \cdot b(c_2 - c_{12}; N - c_1, p)$;
likelihood of dependence hypothesis is product
$\mathcal{L}_2 = b(c_{12}; c_1, p_1) \cdot b(c_2 - c_{12}; N - c_1, p_2)$

- likelihood ratio

$$\lambda = \frac{\mathcal{L}_1}{\mathcal{L}_2}$$

or in logarithmic form $\log \lambda$

- this test works better than t-test and χ^2-test for sparse data

- Information theoretic test for collocations:

pointwise mutual information

$$\mathcal{I}(X,Y) = \log_2 \frac{\mathbb{P}(X,Y)}{\mathbb{P}(X)\mathbb{P}(Y)} = \log_2 \frac{\mathbb{P}(X \mid Y)}{\mathbb{P}(X)} = \log_2 \frac{\mathbb{P}(Y \mid X)}{\mathbb{P}(Y)}$$

- how much more information about the occurrence of a word w_2 if know occurrence of word w_1
- case of total dependence

$$\mathcal{I}(X,Y) = \log \frac{\mathbb{P}(X,Y)}{\mathbb{P}(X)\mathbb{P}(Y)} = \log \frac{\mathbb{P}(X)}{\mathbb{P}(X)\mathbb{P}(Y)} = -\log \mathbb{P}(Y)$$

- case of perfect independence

$$\mathcal{I}(X,Y) = \log \frac{\mathbb{P}(X,Y)}{\mathbb{P}(X)\mathbb{P}(Y)} = \log \frac{\mathbb{P}(X)\mathbb{P}(Y)}{\mathbb{P}(X)\mathbb{P}(Y)} = 0$$

values close to zero: approximate independence

• Caveat: this measure of increased information does not explain nature of the relation

• Example: *Chambre des Communes* in French Canadian translates *House of Commons* so in a corpus of text from Canadian parliament would find high $I(X, Y)$ for both pairs *(house, chambre)* and *(house, communes)*

• χ^2-test identifies correctly which pair is better match (because of other unmatched occurrences)

• but $\mathcal{I}(X, Y)$ can give higher value of second

$$\mathcal{I}(X, Y) = \frac{\mathbb{P}(X, Y)}{\mathbb{P}(Y \mid X) + \mathbb{P}(Y \mid \neg X)} \cdot \frac{1}{\mathbb{P}(X)}$$

depending on the entries $\mathbb{P}(Y \mid X)$ and $\mathbb{P}(Y \mid \neg X)$ of test matrix (for same X and for the two choices of Y being compared)

• Note: all these test *not* very good for low frequency cases (rare digrams)

• the above is pointwise mutual information; while usual information theoretic mutual information is averaged (expectation value)

$$\mathbb{I}(X,Y) = \mathbb{E}_{\mathbb{P}(x,y)}\left(\mathcal{I}(x,y)\right) = \mathbb{E}_{\mathbb{P}(x,y)}\left(\log \frac{\mathbb{P}(x,y)}{\mathbb{P}(x)\mathbb{P}(y)}\right)$$

$$= \sum_{x\in X, y\in Y} \mathbb{P}(x,y) \log \frac{\mathbb{P}(x,y)}{\mathbb{P}(x)\mathbb{P}(y)}$$

• the pointwise version measures reduction of uncertainty about occurrence of one word knowing about occurrence of the other

• the averaged version has similar meaning, but only measures it in an averaged sense

More about word disambiguation

- when the same word can have several different meanings, find a way to attribute the appropriate one in the context of the sentence
- also same word (in English) can often be used as a noun or a verb: problem of tagging correctly as part of speech
- supervised versus unsupervised learning: know status (label) of each item used for training or not
- large scale training sets can be created by the use of pseudowords (artificial conflation of two different words to create a disambiguation problem in a text)
- disambiguation algorithms use a limited representation of context: e.g. up to 3 words on each side of the ambiguous word

Supervised disambiguation

- a disambiguated corpus available for training
- Bayesian classification method: Bayesian decision rule

choose s' if $\mathbb{P}(s' \mid c) > \mathbb{P}(s \mid c)$ for all other $s \neq s'$ where

$$\mathbb{P}(s \mid c) = \frac{\mathbb{P}(c \mid s)}{\mathbb{P}(c)} \mathbb{P}(s)$$

prior probability $\mathbb{P}(s)$ (not knowing anything about context), the other factor is evidence about the context

$$s' = \arg\max_s \mathbb{P}(s \mid c) = \arg\max_s \frac{\mathbb{P}(c \mid s)}{\mathbb{P}(c)} \mathbb{P}(s)$$

$$= \arg\max_s \mathbb{P}(c \mid s)\mathbb{P}(s) = \arg\max_s (\log \mathbb{P}(c \mid s) + \log \mathbb{P}(s))$$

assign meaning s to ambiguous word w

Naive Bayes classifier assumption

$$\mathbb{P}(c \,|\, s) = \mathbb{P}(\{v_j \,:\, v_j \in c\} \,|\, s) = \prod_j \mathbb{P}(v_j \,|\, s)$$

v_j = words that occur in the context c of the ambiguous word

$$s' = \arg\max_s \left(\log \mathbb{P}(s) + \sum_j \log \mathbb{P}(v_j \,|\, s) \right)$$

- the $\mathbb{P}(v_j \,|\, s)$ and $\mathbb{P}(s)$ computed from labelled training corpus

$$\mathbb{P}(v_j \,|\, s) = \frac{C(\nu_j, s)}{\sum_r C(\nu_r, s)}, \quad \mathbb{P}(s) = \frac{C(s)}{C(w)}$$

$C(\nu, s)$ = counting of occurrences of pair

- naive assumption does not work when there are strong conditional dependencies between them

- Information theoretic supervised disambiguation

instead of using all words in a context window for disambiguation, tries to identifies a single feature that forces disambiguation

- Example: want to distinguish the fact that *prendre une mesure* translates as *to take a measurement* while *prendre une décision* translates as *to make a decision*

- use (averaged) mutual information

$$\mathbb{I}(X, Y) = \sum_{x \in X, y \in Y} \mathbb{P}(x, y) \log \frac{\mathbb{P}(x, y)}{\mathbb{P}(x)\mathbb{P}(y)}$$

- Flip-flop algorithm: linear-time algorithm computing best partition for given values of an indicator (requires labelled training set: supervised)
- $P = \{t_1, \ldots, t_m\}$ set of possible translations of ambiguous word; $Q = \{x_1, \ldots, x_m\}$ set of possible values of indicator;
- start with a random partition $P = P_1 \cup P_2$
- find partition $Q = Q_1 \cup Q_2$ maximizing $\mathbb{I}(P, Q)$
- find partition $P = P_1 \cup P_2$ maximizing $\mathbb{I}(P, Q)$ given partition of Q, etc.
- with each iteration $\mathbb{I}(P, Q)$ non-decreasing: stop when stationary

Dictionary based disambiguation

- $D_1, \ldots, D_k$ dictionary definitions of the meanings $s_1, \ldots, s_k$ of a word w
- list of words occurring in these definitions
- v_j = words occurring in a context window c around w
- E_{v_j} = vocabulary definition of v_j (list of words for all its meanings)
- score: $\sigma(s_k) = \#(D_k \cap \cup_j E_{v_j})$ count overlap in dictionary definitions
- choose meaning that maximizes: $s = \arg\max_k \sigma(s_k)$

Unsupervised disambiguation

- "sense tagging" cannot be completely unsupervised (assigning meaning to occurrences), but "sense discrimination" can
- cluster contexts of an ambiguous words into a number of (unlabeled) groups
- Context-group discrimination algorithm:

parameters $\mathbb{P}(v_j \mid s_k)$ and $\mathbb{P}(s_k)$

- initialize parameters randomly
- compute likelihood of corpus C given model μ

$$\mathbb{L}(C \mid \mu) = \sum_i \log \sum_k \mathbb{P}(c_i \mid s_k)\mathbb{P}(s_k)$$

c_i = individual contexts within corpus

$$\mathbb{P}(c_i \mid s_k) = \prod_j \mathbb{P}(v_j \mid s_k)^{C(v_j \in c_i)}$$

naive Bayes assumption

- estimate posterior probability

$$h_{ik} = \frac{\mathbb{P}(s_k)\mathbb{P}(c_i \mid s_k)}{\sum_k \mathbb{P}(s_k)\mathbb{P}(c_i \mid s_k)}$$

- re-estimate parameters

$$\mathbb{P}(v_j \mid s_k) = \frac{\sum_i C(v_j \in c_i) h_{ik}}{\sum_j \sum_i C(v_j \in c_i) h_{ik}}$$

$$\mathbb{P}(s_k) = \frac{\sum_i h_{ik}}{\sum_{ik} h_{ik}}$$

- stop when likelihood $\mathbb{L}(C \mid \mu)$ no longer increasing significantly
- choose $s' = \arg\max \left(\log \mathbb{P}(s_k) + \sum_j \mathbb{P}(v_j \mid s_k) \right)$
- can distinguish usage-types more fine grained than listed in dictionaries

Machine Translation

- increasingly refined levels of machine translation
- Word-by-word translations: riddled with lexical ambiguities, messes up word order
- Syntactic parsing and syntactic transfer: restores correct word order; ambiguities remain in ambiguities of parsing trees
- Semantic representation and semantic transfer: correcting for syntactic parse trees ambiguities
- Interlingua: knowledge representation formalism, instead of translation systems for each pair of languages $O(n^2)$ translate between each and interlingua elevel $O(n)$... problem: extremely difficult to design an efficient comprehensive representation formalism

Text Alignment first step for Machine Translation

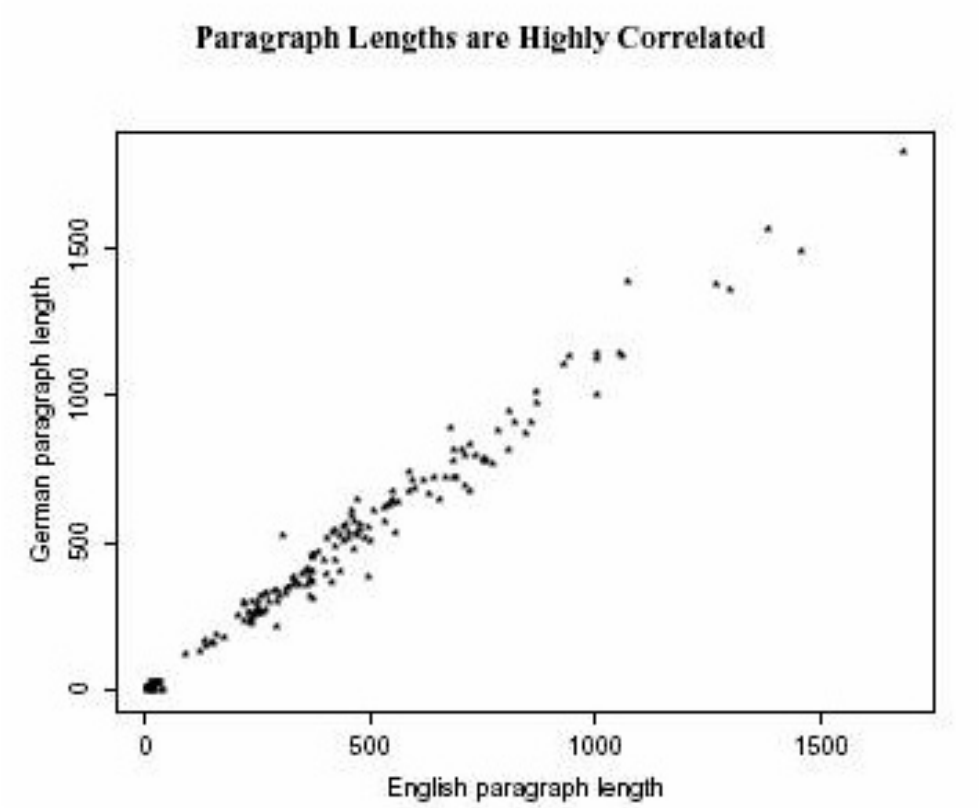
- matching sentences in two parallel texts in different languages
- order of subordinate sentences can vary in different languages: need to match which sentence pairs with which
- good training corpora are legislation documents in bilingual or multilingual countries (Canada, Switzerland, Belgium,...) large supply of documents and accurately translated for reason of legal requirements

Length-based methods: want alignment A with highest probability

$$\arg\max_A \mathbb{P}(A \mid S, T)$$

given parallel texts S, T; assume $\mathbb{P}(A \mid S, T) \sim \prod_k \mathbb{P}(B_k)$ sequence B_k of alignment beads statistically independent

- estimate $\mathbb{P}(B_k)$ by comparing lengths of texts in B_k
- assign a cost function to each choice of alignment B_k and minimize (e.g. based on square of differences of lengths of paragraphs/sentences)
- this method breaks down easily on noisy data like optical character recognition (OCR)

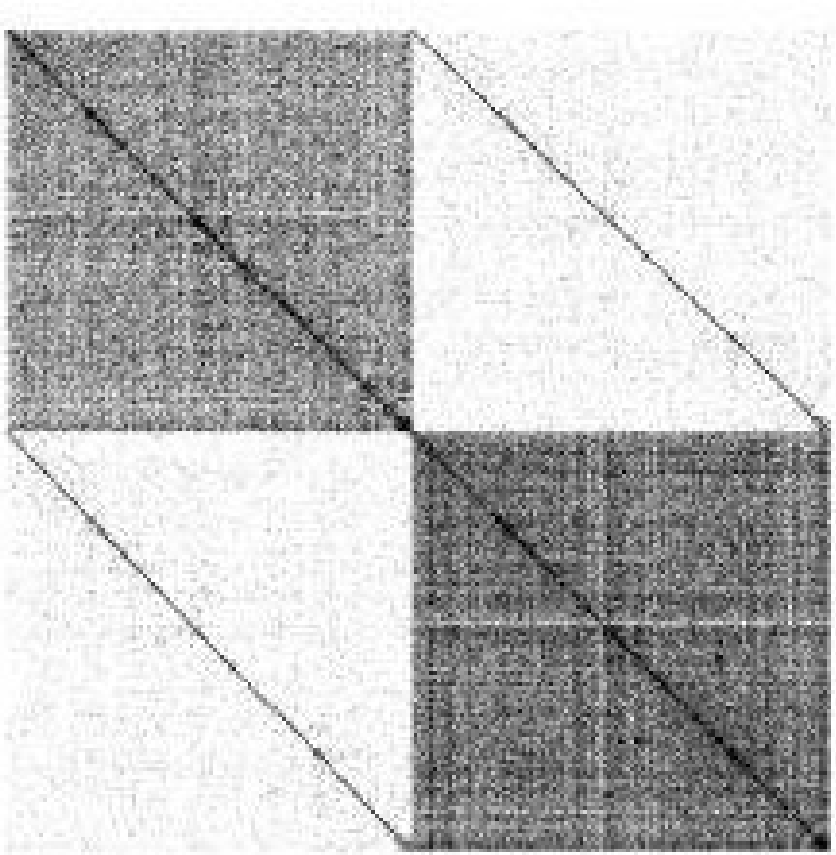


Offset alignment methods by signal processing techniques:

• induce alignment based on cognate words: works best for languages in same family with many cognate words, but can work in other cases too based on proper nouns, numbers, and other such strings of characters

• dot-plot: concatenate both text and make big matrix: place dot at (x, y) if there is a match between positions x and y in the concatenated text... bitext map in off-diagonal quadrants

• can refine with counting positions in text where a word occurs and a vector of distances between subsequent positions, can compare with a candidate match: if distances too different discard... works better when punctuation may be lost (OCR) and languages are not in same family



- W.A.Gale, K.W.Church, *A Program for Aligning Sentences in Bilingual Corpora*, Computational Linguistics (1993) 19(3), 75–102

Word Alignment: next step after sentences matched, done from a bitext using association methods like χ^2-test

Statistical Machine Translation: based on word alignment

$$\mathbb{P}(S_f \mid S_e) = \frac{1}{Z} \sum_{a_1 \dots a_m, =0}^{\ell} \prod_{j=1}^{m} \mathbb{P}(w_{f,j} \mid w_{e,a_j})$$

S_e, S_f English and French sentences, $\ell = \ell(S_e)$, $m = \ell(S_f)$ lengths in number of words, $w_{f,j}$ = j-th word of S_f, a_j = position of S_e that $w_{f,j}$ is aligned to, with word w_{e,a_j}, and $\mathbb{P}(w_{f,j} \mid w_{e,a_j})$ = translation probability (of finding $w_{f,j}$ as a translation of w_{e,a_j})

• sum counts all possible alignments ($a_j = 0$ if word with no matching translation on other side)

Decoder

$$S'_e = \arg\max_{S_e} \mathbb{P}(S_e \mid S_f) = \arg\max_{S_e} \frac{\mathbb{P}(S_e)\mathbb{P}(S_f \mid S_e)}{\mathbb{P}(S_f)}$$

$$= \arg\max_{S_e} \mathbb{P}(S_e)\mathbb{P}(S_f \mid S_e)$$

need a search algorithm (in principle searching over an infinite space: need to reduce to a manageable search)

• Stack search: build sentence S'_e incrementally, keeping track of partial translation hypotheses, extend hypotheses (more words) and prune the stack of least likely extended hypotheses...

Translation Probabilities: estimate the $\mathbb{P}(w_f \mid w_e)$

- initialize $\mathbb{P}(w_f \mid w_e)$ randomly
- count occurrences in all pairs (S_e, S_f) of aligned sentences containing w_e and w_f

$$z_{w_f, w_e} = \sum_{(S_e, S_f)} \mathbb{P}(w_f \mid w_e)$$

- re-estimate probabilities

$$\mathbb{P}(w_f \mid w_e) = \frac{z_{w_f, w_e}}{\sum_v z_{v, w_e}}$$

Ethical issues in machine translation

• try Google Translate between English and German:

- *math teacher* $\longrightarrow$ *Mathlehrer*
- *cooking teacher* $\longrightarrow$ *Kochlehrerin*
- *physics teacher* $\longrightarrow$ *Physiklehrer*
- *French teacher* $\longrightarrow$ *Französisch-Lehrerin*

• because machine translators are primed on counting occurrences in available text corpora, they carry along with them whatever cultural bias exists in the society that produces those texts at the time in which they are produced

• is it possible to construct unbiased machine translators that avoid this kind of problem? It certainly would be desirable...

Information Retrieval

- developing algorithms for retrieving information from repositories of written texts
- inverted index: lists for each word all the documents that contain that word
- with additional position information of all the occurrences
- allows for search for phrases (combinations of words)
- stop list eliminates words not useful for search (too general such as "and")
- stemming of words

Ranking of relevant documents is the main problem

- Probability ranking principle: ranking documents by descrasing probability of relevance is optimal
- view retrieval as a "greedy search" that aims at the most valuable document with highest $\mathbb{P}$
- method assumes that documents are independent

Vector space model:

- it uses spatial proximity for semantic proximity
- very high-dimensional space with one dimension for each word in the document collection
- measure of vector similarity

$$\frac{\langle v, w \rangle}{\|v\| \cdot \|w\|} = \cos(\theta(v, w))$$

• Term weighting: weight gives coordinates d_w to document-vectors $d = (d_w)$ in the vector space model

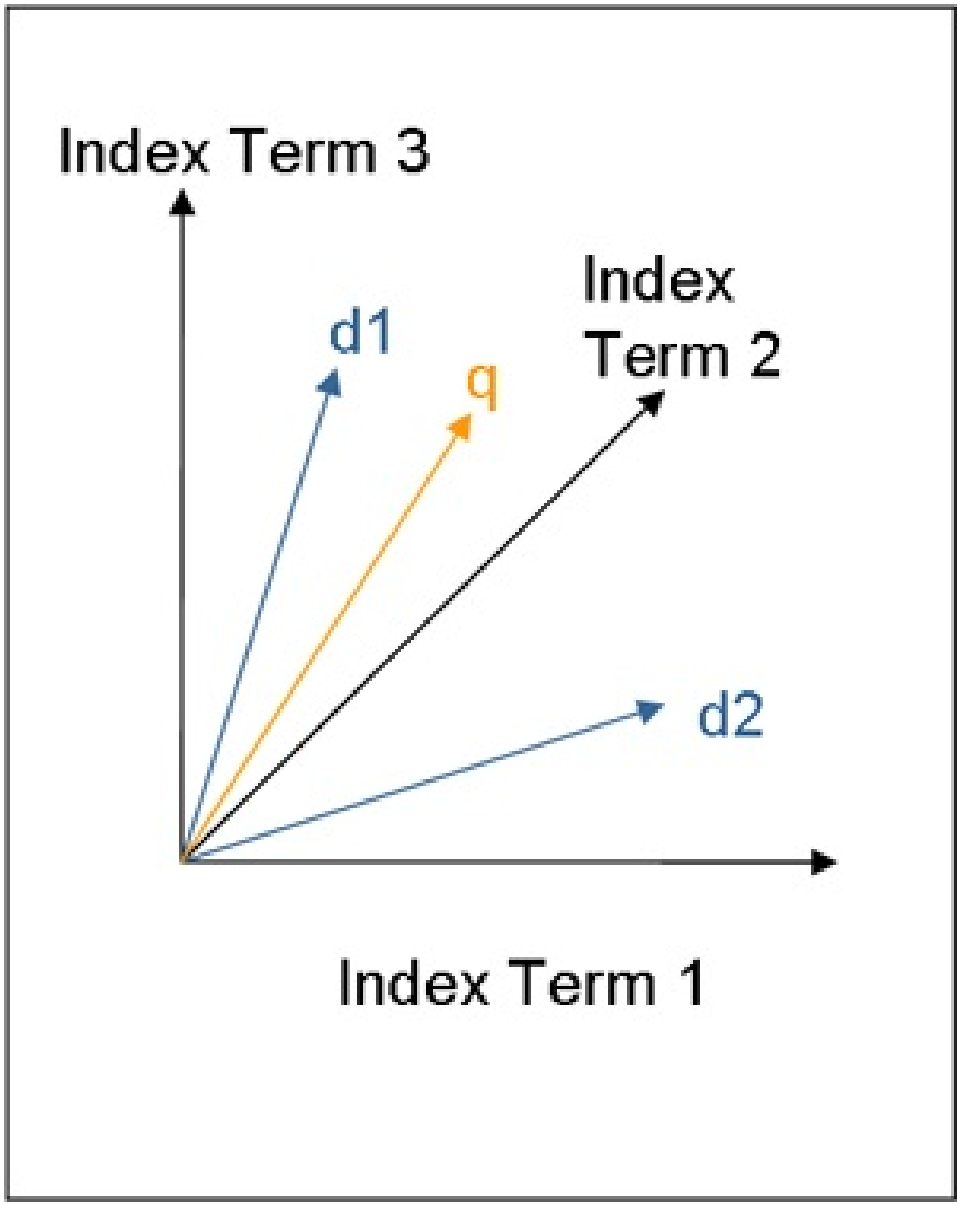
- term frequency: $tf_{i,j}$ = occurrences of word w_i in document d_j
- document frequency: df_i = number of documents in the collection in which w_i occurs
- collection frequency: cf_i = total number of occurrences of w_i in the collection

• weight: if $tf_{i,j} \neq 0$

$$\text{weight}(i,j) = (1 + \log(tf_{i,j})) \log \frac{N}{df_i}$$

and zero otherwise, N = number of documents

• note inverse document frequency: max weight to words occurring in only one document, zero weight to words occurring in all


Index Term 3
Index
Term 2
d1
q
d2
Index Term 1

Term Distribution Models: alternative to weights, use model of occurrences of words (Zipf's law, or a distribution like Poisson) and decide how informative a word is

- Poisson distribution typical for events over units of a fixed size

$$p(k;\lambda) = e^{-\lambda}\ \frac{\lambda^k}{k!}$$

mean and variance both equal to λ

- use with $\lambda = cf_i/N$ average number of occurrences per document
- observed deviation from a Poisson distribution can signal "clustering" of term occurrences in only certain types of documents in the collection

Other probability distributions

- two-Poisson

$$p(k;\pi,\lambda_1,\lambda_2) = \pi e^{-\lambda_1}\ \frac{\lambda_1^k}{k!} + (1-\pi)e^{-\lambda_2}\ \frac{\lambda_2^k}{k!}$$

separates documents containing w into a class with low number of occurrences and one with high

- K-mixture

$$p(k) = (1-\alpha)\delta_{k,0} + \frac{\alpha}{\beta+1}(\frac{\beta}{\beta+1})^k$$

α, β parameters can be fit using observed mean $\lambda = cf/N$ and observed inverse document frequency N/df