Lecture 14: Adjunctions
Ma 191c: Mathematical Models of Generative Linguistics

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this part based on

- from Section 3.7.1 of Matilde Marcolli, Noam Chomsky, Robert C. Berwick, “Mathematical structure of syntactic Merge”, MIT Press.

Adjunctions

recall from previous discussion: unlike arguments, adjuncts can accompany predicates but are not required

arguments:

Jill likes **Jack**.

Sam fried **the meat**.

The old man helped **the young man**.

Sam put the pen **on the chair**.

Larry does not put up **with that**.

Bill is getting **on my case**.

adjuncts:

Jill **really** likes Jack.

Jill likes Jack **most of the time**.

Jill likes Jack **when the sun shines**.

Jill likes Jack **because he's friendly**.

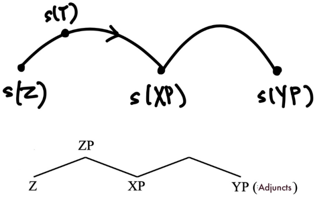
- verbs take a finite number of arguments but an arbitrarily large number of adjuncts
- adjunction can be seen as an instance of syntactic objects $\{XP, YP\}$ which do not have a well defined head function

- complements are selected by their host but adjuncts select their host
- the problem of "invisibility of adjuncts to syntax": adjunctions are "opaque to syntax" (in terms of movement), "adjunct islands"
- the "Pair Merge problem" (an asymmetric form of Merge justifying asymmetric structure of adjunctions)
- mapping to semantics of adjunctions $\notin \mathrm{Dom}(h)$
- characterizing hierarchical structures generated by adjunctions

Pair-Merge proposals

- a Pair-Merge $\langle XP, YP \rangle$ signifies that the first element should be taken to be the "head" while the second element is to be seen as an "adjunct"
- not extendable to other types of syntactic objects $\{XP, YP\} \notin \mathrm{Dom}(h)$ that are not adjunctions
- one proposal of Pair-Merge is "two-peaked structures" but *they cannot be generated through the Merge mechanism*

proposal: the Pair-Merge mechanism is *not* part of the generative structure (usual Merge) but happens on the image $s : \mathcal{SO} \to \mathcal{S}$ in semantic space, and is therefore opaque to syntax because syntax calculator is not located in $\mathcal{S}$

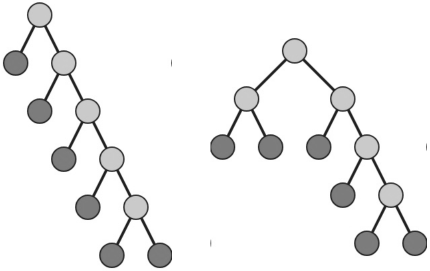


- syntactic object of the form $\{XP, YP\} \notin \mathrm{Dom}(h)$, with both XP and YP in $\mathrm{Dom}(h)$
- consider in $\mathcal{S}$ the points $s(XP)$ and $s(YP)$ and geodesic arc between (since no head, no orientation and assigned root-point)
- suppose also syntactic object $T = \{Z, XP\} \in \mathrm{Dom}(h)$: point $s(T)$ on the geodesic arc between $s(Z)$ and $s(XP)$
- "two-peaked" structure in $\mathcal{S}$, orientation on second geodesic arc induced by first

Pair-Merge by "two-peaked" structure not very satisfactory: different proposal (Huijbregts) identifying asymmetric (α, β) with $\{\{\alpha\}, \{\alpha, \beta\}\}$ a combination of IM/EM (without cancellation of deeper copy)

hierarchical structures generated by adjunctions

- question: characterizing the region that unbounded adjunct sequences span inside the overall structure of recursivity in language
- given arbitrarily large number of adjunctions possible: can "most" of the possible structures be achieved by adjunctions
- focus on case of a verb with a sequence of adjunctions
- assumptions:
  - syntactic object that contains the adjunct substructure is in $\mathrm{Dom}(h)$
  - planarization associated to head function

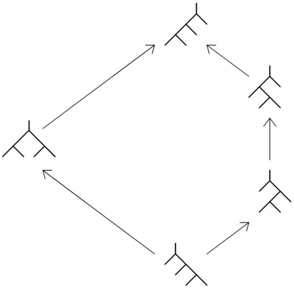
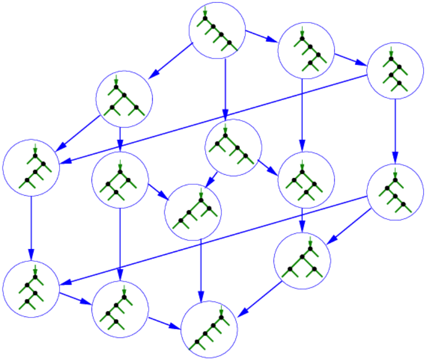


Planar trees with a sequence of adjunctions ⇒ comb trees

key structures: Tamari order and Loday's operations of left and right sum on planar binary rooted trees

Tamari order (or Tamari lattice)

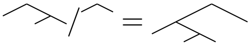
- planar binary rooted trees
- 1-skeleton of the associahedron
- oriented arrows relate planar binary rooted trees obtained by moving one edge from right to left across a vertex
- $T < T'$ if there is an oriented arrow $T \to T'$ in the Tamari lattice
- induces partial order on the set $\mathfrak{T}^{pl}_{n+1}$ of planar binary rooted trees on fixed number $n+1$ of leaves
- comb trees (simplest chains of adjunctions) occupy extreme points of Tamari lattice

Tamari order for four and five leaves

Loday's sums of trees

- "over/under" operations on planar binary rooted trees: S/T and $S\backslash T$



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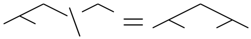


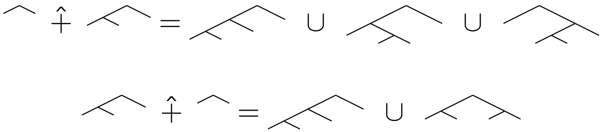
- satisfy $S/T \leq S\backslash T$ in the Tamari order
- Loday's *noncommutative* sum of trees

$$S \hat{+} T = \bigcup_{S/T \leq T' \leq S\backslash T} T'$$

where all $\deg(T') = \deg(S) + \deg(T)$

- noncommutative $S \hat{+} T \neq T \hat{+} S$ but associative
- distributivity with respect to unions $\cup_i T_i \hat{+} \cup_j S_j = \cup_{ij} (T_i \hat{+} S_j)$

Example



other properties

- sum of trees as combination of two operations

$$S \hat{+} T = S \dashv T \cup S \vdash T$$

- defined recursively

$$S \dashv T = S^L \quad (S^R \hat{+} T)$$

$$S \vdash T = (S \hat{+} T^L) \quad T^R$$

where

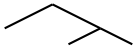
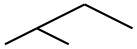
$$S = S^L \quad S^R \quad \text{and} \quad T = T^L \quad T^R$$

with L, R left/right subtrees of planar binary rooted tree

- consider element

$$\mathbb{I} :=$$ 

- then have the simplest adjunctions structures realized as

$$\mathbb{I} \hat{+} \mathbb{I} = \mathbb{I} \dashv \mathbb{I} \cup \mathbb{I} \vdash \mathbb{I} =$$  $\cup$ 

- simple chains of adjunctions (comb-like):
    - head-initial form:

$$\underbrace{\mathbb{I} \dashv \mathbb{I} \dashv \mathbb{I} \dashv \cdots \dashv \mathbb{I} \dashv \mathbb{I}}_{N \text{ times}}$$

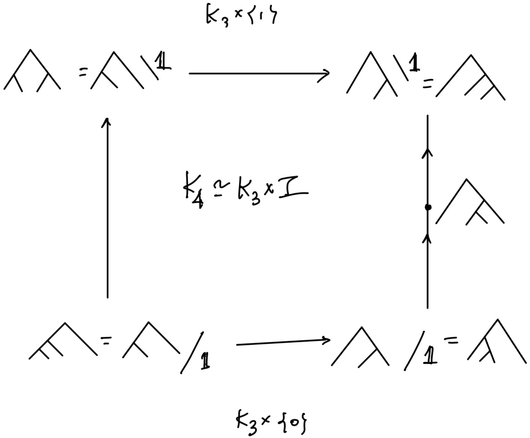
    - head-final form:

$$\underbrace{\mathbb{I} \vdash \mathbb{I} \vdash \mathbb{I} \vdash \cdots \vdash \mathbb{I} \vdash \mathbb{I}}_{N \text{ times}}$$

- consider head-initial case (the other analogous)

adjunctions and the geometry of the associahedron

- K_{n+1} can be identified with a cylinder $K_n \times \mathcal{I}$ (with $\mathcal{I} = [0,1]$) over associahedron K_n
- K_4 as a subdivision of $K_3 \times \mathcal{I}$



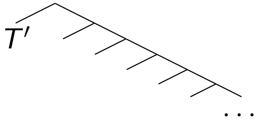
- boundary $K_n \times \{0\}$ has vertices given by the trees $T/\mathbb{I}$ for T a vertex of K_n while $K_n \times \{1\}$ has vertices $T\backslash\mathbb{I}$

- directed segment $\mathcal{I} = [0,1]$ between two such vertices has intermediate vertices given by trees S with $T/\mathbb{I} \leq S \leq T\backslash\mathbb{I}$
- can identify these segments with $T \hat{+} \mathbb{I}$ with the orientation given by Tamari order
- set $\mathfrak{T}_n^{pl}$ of planar binary rooted trees on n leaves is obtained as

$$\mathfrak{T}_{n+1}^{pl} = \mathfrak{T}_n^{pl} \hat{+} \mathbb{I}$$

- i.e. segments $T/\mathbb{I} \leq S \leq T\backslash\mathbb{I}$ cover the entire set of vertices of the associahedron when T varies over the vertices of K_n

- repeated adjunctions



$$= T \backslash \mathbb{I} \backslash \mathbb{I} \backslash \mathbb{I} \cdots \backslash \mathbb{I}$$

- chain of adjunctions of length N map a tree T placed at one of the vertices of the associahedron K_n to a corresponding vertex in the associahedron $K_{n+N} = K_n \times \mathcal{I}^N$
- a subdivision of) an N-cube with vertices $T \hat{+} \mathbb{I} \hat{+} \mathbb{I} \hat{+} \cdots \hat{+} \mathbb{I}$ and oriented edges given by the Tamari order relation
- adjunctions $T \backslash \mathbb{I} \backslash \mathbb{I} \backslash \mathbb{I} \cdots \backslash \mathbb{I}$ located at the topmost vertex $(1, 1, \ldots, 1)$ of the cube $\mathcal{I}^N$

- more general chains of the adjunctions

$$T = T_0 \backslash \underbrace{\mathbb{I} \backslash \mathbb{I} \backslash \cdots \backslash \mathbb{I}}_{N_0 \text{ times}} \backslash T_1 \backslash \underbrace{\mathbb{I} \backslash \mathbb{I} \backslash \cdots \backslash \mathbb{I}}_{N_1 \text{ times}} \backslash \cdots \backslash T_k \backslash \underbrace{\mathbb{I} \backslash \mathbb{I} \backslash \cdots \backslash \mathbb{I}}_{N_k \text{ times}}$$

- with trees of the form $T_i = \widehat{T_i^L\quad}$
- these account for the fact that adjunctions can be sentences with nontrivial structure
- first block

$$S_0 = T_0 \backslash \underbrace{\mathbb{I} \backslash \mathbb{I} \backslash \cdots \backslash \mathbb{I}}_{N_0 \text{ times}}$$

  ias before: gives a vertex S_0 in associahedron $K_{n_0+N_0}$, for T_0 vertex of K_{n_0}
- next step $S_0 \backslash T_1$ gives vertex in associahedron $K_{n_0+K_0+n_1}$, for T_1 vertex of K_{n_1}, etc
- conclusion: chains of adjunctions span high-codimension strata of the associahedra: simplicial subcomplex of codimension $N_0 + \cdots + N_k$ in the associahedron $K_{n_0+\cdots+n_k+N_0+\cdots+N_k}$, where $T_i' \in K_{n_i}$

Some References

problem of adjunctions

- P.M. Pietroski, Function and Concatenation, in "Logical form and Language", Pxford University Press, 2002, 91–117.
- T.Graf, The Syntactic Algebra of Adjuncts, Proceedings of CLS 49, 2013.

Tamari order and Loday sums

- J.L.Loday, Arithmetree, Journal of Algebra 258 (2002) 275–309.