ENTROPY AND ART: THE VIEW BEYOND ARNHEIM

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Contents

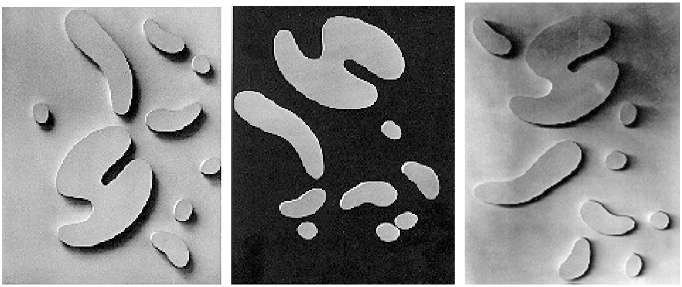


Figure 1. Hans Arp, *Three Constellations of Same Forms*, 1942

In 1971 art theorist Rudolf Arnheim published one of his most famous essays: "Entropy and Art: an Essay on Disorder and Order". In this influential booklet, Arnheim emphasized the ideas of Entropy as Disorder versus Information as Order. Among the art works discussed, an example that figures prominently is Hans Arp's "Three Constellations of Same Forms", where identical forms are disposed *randomly* on a uniform background in varying configurations: the information content lies in the mutual distances and the relative weights of these different shapes. An important question associated to Arnheim's analysis of this piece, which we will be discussing at length in this chapter, is whether one should regard randomness as order or disorder, as measured in terms of Entropy/Information. Although our intuition is naturally inclined to view randomness as disorder, we will see that randomness has structure and complexity and that there are different kinds of randomness. We will see how the notion of randomness, in its multiplicity of forms, has played a crucial role in the development of Contemporary Art.

1. Thermodynamic Entropy

The concept of entropy originates in *Thermodynamics*. It is introduced as a state function S (this means a function that depends only on the present state of the system, regardless of which intermediate states the system went through to get to the present one), which measures the incremental transfer of heat energy Q into the system at a given temperature T. More precisely one sets

$$\Delta S = \frac{\Delta Q}{T}.$$

The Laws of Thermodynamics, which describe the main properties of Energy and Entropy are formulated in the following way.

(1) Conservation of Energy: the total energy of an isolated system is constant
(2) Increase of Entropy: the entropy of an isolated system does not decrease

The first law states that (in an isolated system) energy cannot be created or destroyed, only transformed from one form to another. The second law states that Entropy measures how much energy degrades towards less and less "usable" forms. The laws of thermodynamics are the reason, for instance, why no perpetual motion machine can ever exist.

Indeed, Thermodynamics historically developed out of the need to understand the efficiency of machines turning energy into work: *Entropy limits the efficiency of mechanical machines.* Thermodynamics is, in this way, a science ultimately tied up to the technological era of engines and mechanical devices. In this form, it appears in implicit references throughout the works of the Cubist,

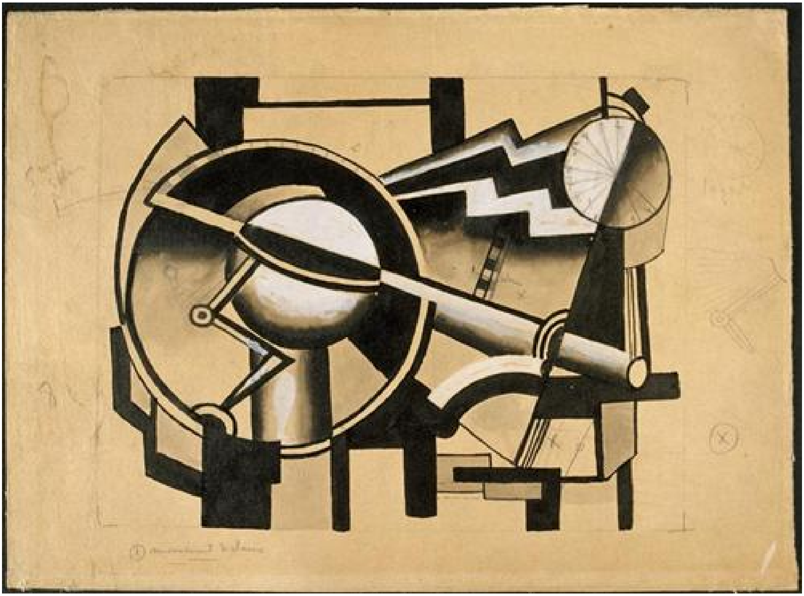


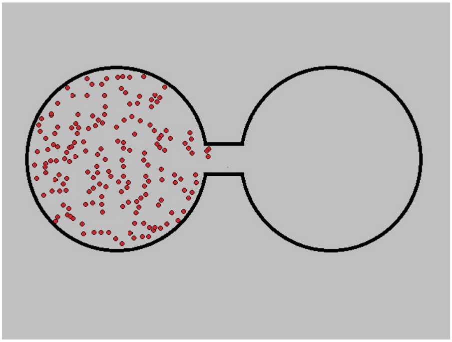
FIGURE 2. Fernand Leger, *Composition Mécanique (Mouvement de Charrue)*

Dadaist, and Futurist avantgarde movements, with their fascination with machines and mechanisms, with engines and energy. The example of Leger's "Composition Mécanique" shows a machine seen as a bubble in itself, an isolated system in the sense of the first law of thermodynamics, existing by itself, without any apparent connection to the outside world, nor any apparent utility. The machine is transforming energy into movement ("mouvement de charrue" as the painting title specifies). The second law of thermodynamics then ensures that this is indeed an impossible machine, at least if we imagine it to run forever, as the painting insinuates given its self sufficient nature. The laws of thermodynamics here are implicitly invoked in the painting in order to suggest its eerie and paradoxical nature.

The thermodynamic notion of Entropy is closely related to the notion of the arrow of time. One can distinguish physical processes into reversible and irreversible processes. In thermodynamic terms this means:

- reversible process $\Delta S = 0$ (entropy stays the same)
- irreversible process $\Delta S > 0$ (entropy increases)

In this sense, one thinks of the arrow of time is simply as the direction of increasing Entropy.



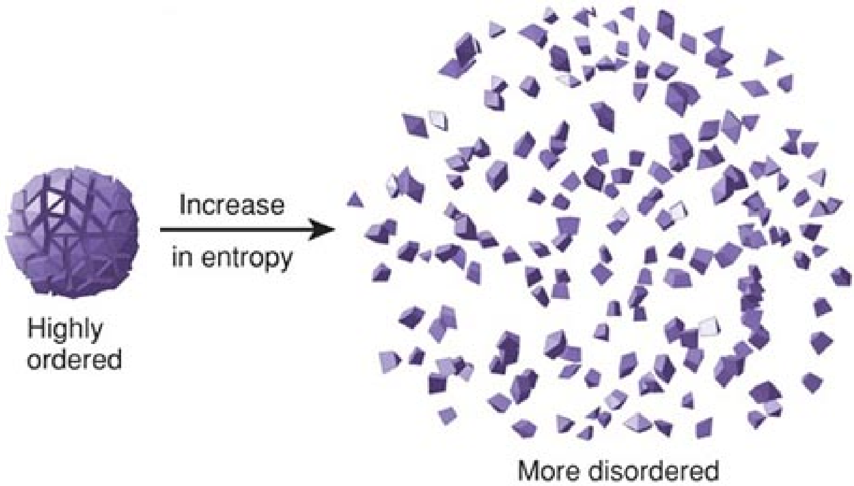
Although at the microscopic level the laws governing the motion of particles in a gas are reversible, hence at the microscopic level it would be impossible to detect if a movie of particles motion would be played in reverse, certainly at the larger macroscopic level one can see that if particles of a gas confined in half of a container, as in the figure above, are released, they will gradually diffuse into the rest of the container: a movie showing the particles naturally grouping together all on one side leaving the other side empty would immediately appear unnatural with

FIGURE 3. Anthony Huber, *Entropy IVa"* Abstract Constructionism Series, 2014

respect to the perceived arrow of time direction. This example reveals an important difference between the individual microscopic behavior of a single particle subject to reversible laws of motion and the collective macroscopic behavior of the gas in the container, which exhibits a clear preferred arrow of time.

The view of Entropy as a measure of irreversible processes and of disorder is captured in some of the paintings of the "Abstract Constructionism Series" of the artist Anthony Huber, as for example in the "Entropy IVa" reproduced here. The overlapping splashes of paint convey the image of a typical irreversible process, while the two large regions of white/black color hint at the usual portrayal of irreversibility in terms of the two chambers with particles randomly moving across the partition, as the the diagram we discussed above.

One typically distinguishes between "high entropy states" that are disordered and "low entropy states", like a crystal structure, that are highly ordered. An irreversible process then transitions from ordered states to more and more disordered states.



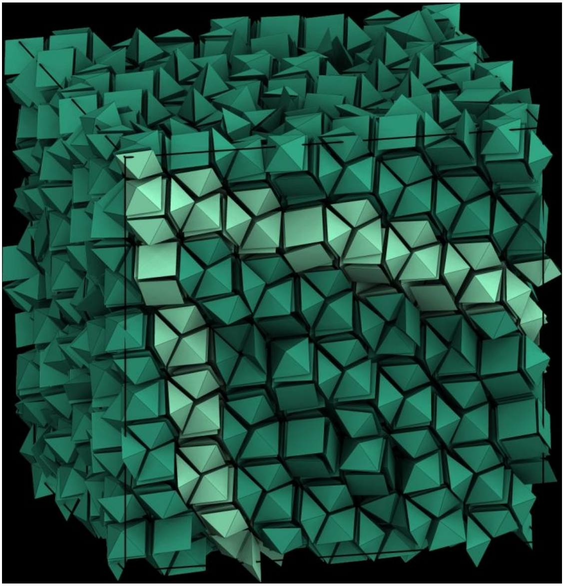
The irreversible transition between Order and Disorder is one of the key concepts in Entropy. There are many works of art that deal with transitions between ordered and disordered structures.

FIGURE 4. Roberto Quintana, *QnTm/FM 2043*, Quantum-Foam Series, 2014

For example, one can see the "melting" of an ordered crystal lattice into a disordered phase portrayed in the piece "QnTm/FM 2043" of the "Quantum-Foam Series" by Roberto Quintana. A similar overlapping of ordered and disordered structures, with splashes of paint used to signify irreversibility, can be seen in another piece, "Entropy IX" of the "Abstract Constructionism Series" of Anthony Huber.

FIGURE 5. Anthony Huber, *Entropy IX*, Abstract Constructionism Series, 2014

However, it is important to point out some subtleties in the identification of Entropy and Disorder. There are situations in physical systems where Entropy does not necessarily lead to a disordered phase. In experiments and simulations carried out by Damasceno, Engel, and Glotzer (see the references section at the end of this chapter), they have observed that experimentally particles find a maximal entropy arrangement which, if there is enough space, looks like a disordered phase, but in a more crowded setting leads to an "ordered" crystal structures, although it is a high entropy state. The figure included here illustrates one such high entropy crystal phases.



1.1. **Probabilistic description of Thermodynamic Entropy.** An important development in the theory of Entropy, in the context of Thermodynamics described above, is a formulation in

Probabilistic terms (Ludwig Boltzmann). As we mentioned already, one typically encounters situations where the laws governing physics ate the microscopic level are time reversible, while at the macroscopic level irreversible processes are clearly present, which determine our sense of an arrow of time, pointing to the direction of increasing Entropy. The relation between microscopic and macroscopic properties of a physical system is understood within the context of Statistical Mechanics.

The view of Entropy based on Statistical Mechanics can be summarized as follows.

- Entropy measures the amount of uncertainty remaining in a system after fixing all its *macroscopic* properties, such as temperature, pressure, volume.
- Entropy is coming from *microscopic states* (degrees of freedom) of the system: it is a measure of the number of different ways in which a system can be arranged.
- Entropy is proportional to the natural logarithm of the number of possible microscopic configurations

$$S = -k_B \sum_i p_i \log p_i$$

  where the sum is over microstates (with probability p_i of finding system in that state).
- If all the microstates are equally probable, then all the $p_i = 1/N$ and the Entropy is simply given by

$$S = k_B \log N$$

  where N is the number of microstates (the number of degrees of freedom at the microscopic level).

We can revisit in this light some of the artwork in the "Quantum-Foam Series" of Roberto Quintana. As we see in the "QnTm/Fm

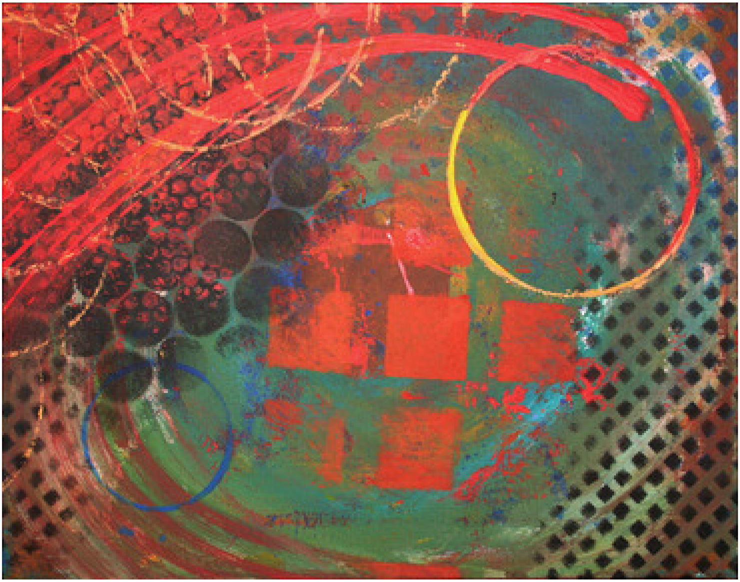


FIGURE 6. Roberto Quintana, *QnTm/Fm 230*, Quantum-Foam Series, 2014

230" and "QnTm/Fm 207" paintings, ensembles of microstates concur to determine the larger scale macroscopic structure, and the bserved properties at macroscopic level (order/disorder) depend on counting all different possible microstates within the macrostate.

1.2. **The Gibbs Measure.** As we mentioned above, if all the microstates would be equally probable, then the Entropy would just be $S \sim \log N$, proportional to the logarithm of the number

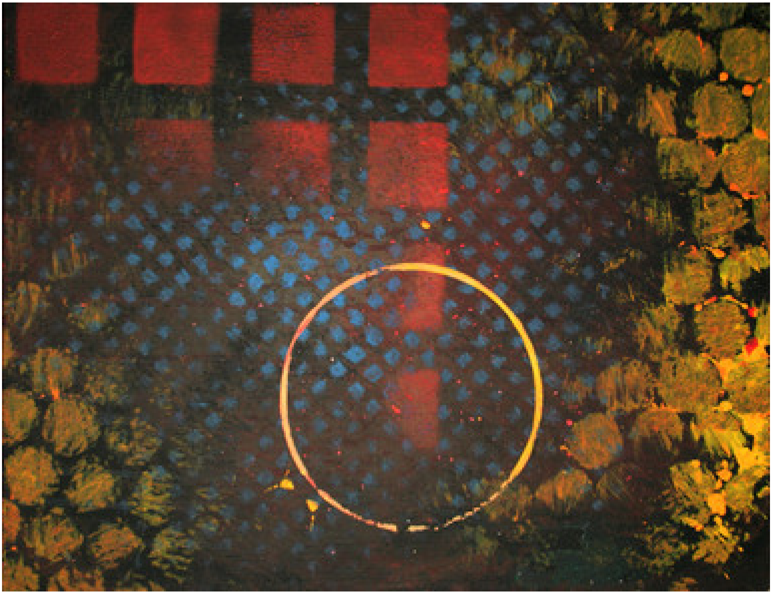


FIGURE 7. Roberto Quintana, *QnTm/Fm 207*, Quantum-Foam Series, 2014

of degrees of freedom. However, in a typical physical system, the probability of occurrence of a certain microstate can depend on the *energy*

$$p_i = \frac{e^{-\beta E_i}}{Z(\beta)}$$

where E_i is the energy level of that microstate, and the partition function

$$Z(\beta) = \sum_i e^{-\beta E_i} = \mathrm{Tr}(e^{-\beta H})$$

is the normalization factor that makes the expression above a probability and that account for the distribution of energy levels

FIGURE 8. Roberto Quintana, *QnTm/FM 2040*, Quantum-Foam Series, 2014

across all the possible microstates. The variable β in this expression is an inverse temperature (up to a conversion factor given by the Boltzmann constant), $\beta = \frac{1}{k_B T}$. This means that, at low temperature, the probability is concentrated on states of lowest energies, while at higher temperatures higher energy states also contribute. The *Gibbs measure* described above has the property that is maximizes the "entropy density" for a fixed "energy density".

Different microscopic states of the system are weighted according to their energies: at low temperature high energy states

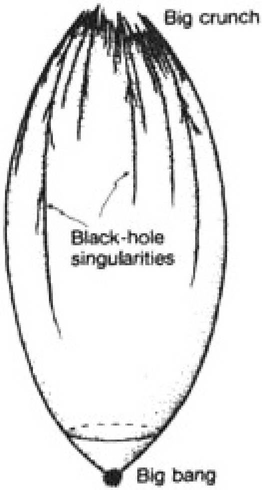
FIGURE 9. Roberto Quintana, *QnTm/FM 2055*, Quantum-Foam Series, 2014

are inaccessible and the Gibbs measure is concentrated on the ground state; at higher temperatures higher energy states also become accessible. Again, we can interpret in this light Quintana's paintings "QnTm/FM 2040" and "QnTm/FM 2055", as a state space subdivided into a series of possible microstates, which are weighted differently (here color coded) to signal a lower or a higher corresponding energy state.

2. Entropy and Cosmology

The Universe is an isolated system. Thus, by Second Law of Thermodynamics, its Entropy is increasing. This implies, going backward in time, and assuming the current standard cosmological model, that the original Big Bang must have been a state of extremely low Entropy. Projecting forward in time, instead, one envisions a scenario of slow Heat Death of the Universe, where all the energy reduces to a homogeneous distribution of thermal energy.

Gravity causes stars to form and it causes sufficiently massive ones to eventually collapse into black holes. Black holes themselves have a form of Entropy (the Bekenstein–Hawking black hole entropy) which is proportional to the area of their event horizon (maximally entropic states). In the case of a closed universe, which starts with a Big Bang and recollapses into a Big Crunch, the initial Big Bang state is at very low entropy, while the final Big Crunch state should be at very high Entropy. A significant contribution to this high Entropy is due to all the black holes that formed in the evolution of the Universe. Roger Penrose's Weyl Curvature hypothesis shows that, in such a closed universe, the Entropy grows like a component of the curvature, the Weyl curvature tensor, as shown in the following diagram taken from Penrose's work. The history of the universe as depicted here starts out at a uniform low-entropy Big Bang with zero Weyl curvature and ends at a high-entropy Big Crunch with a very large Weyl curvature contributed by a congealing of black holes.



However, one should keep in mind that there are several problems with applying the notion of Thermodynamical Entropy to Cosmology. First, this notion depends on a notion of equilibrium, which does not directly apply to a dynamical spacetime. This is related to the problem of "Entropy gap" in an expanding universe. Moreover, when discussing states like Big Bang, or black holes, quantum effects are likely to become relevant in a combination with General Relativity that would require a good model for Quantum Gravity, which is still a main open problem in theoretical physics.

We can see pictures of such evolving universes depicted in the wire sculptures of Ruth Asawa. In the sculpture "Untitled S065" one can see an evolving universe that is undergoing a series of expansions and collapses. Such cosmologies have been studied extensively since the 1970s, based on particular solutions of the

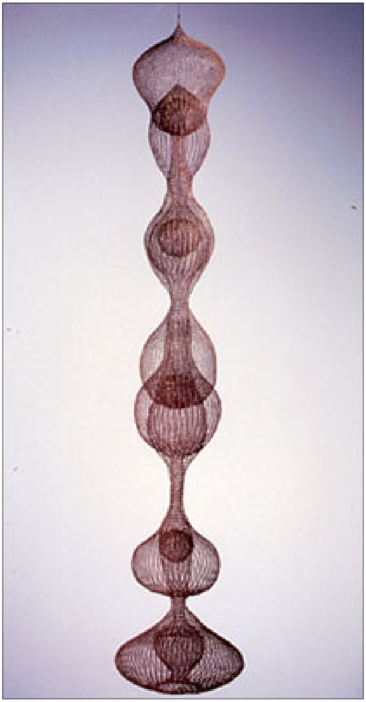


Figure 10. Ruth Asawa, *Untitled S065*, early 1960s, crocheted wire sculpture

Einstein field equations known as Kasner metrics, and a dynamical system that combines a series of these Kasner metrics. They exhibit a chaotic behavior that alternates phases of expansion and collapse. Other forms of cyclic cosmologies have been recently proposed by Penrose, in his model of Conformally Cyclic Cosmologies, also referred to as "cycles of time".

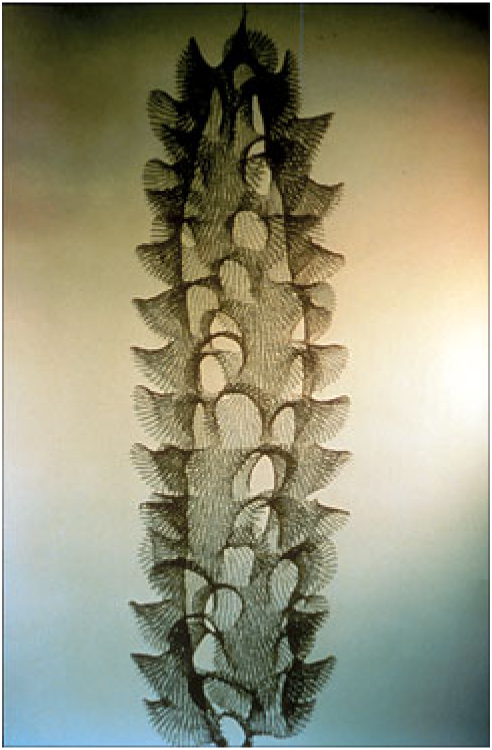


FIGURE 11. Ruth Asawa, *Untitled S039*, 1959, crocheted wire sculpture

In the sculpture "Untitled S039" instead, we see an evolving universe more similar to the Penrose Weyl curvature hypothesis model, where a large quantity of Weyl curvature is injected in a background smooth space, through many singularities that open up in the fabric of spacetime, like black holes in the Penrose model.

3. Entropy and Information

In 1948 mathematician Claude E. Shannon published the seminal article "A Mathematical Theory of Communication", republished as a book with Warren Weaver in 1949. This work introduced the information theoretic meaning of Entropy and ushered the modern era of Information Theory.

While thermodynamic Entropy limits the efficiency of machines (engines), information theoretic Entropy limits the efficiency of communication. In every form of communication, a signal is encoded, transmitted, and decoded: the main problem is how to optimize the efficiency of communication. For instance, efficiency is improved if instead of using uniformly long codings for all letters, one uses a shorter coding for frequently occurring letters, and a longer one for rarely occurring ones. This shows that optimizing is related to frequencies, hence to probabilities.

The Shannon Entropy is defined as

$$S = -\sum_i p_i \log(p_i).$$

Due to the presence of the logarithm function, information is additive over independent events:

$$\log(p_1 p_2) = \log(p_1) + \log(p_2).$$

This definition, compared with our previous discussion of Boltzmann Entropy shows that Information is Negative Entropy.

FIGURE 12. Mika Tajima, *Negative Entropy*, 2012

The "Negative Entropy" series of artist Mika Tajima was produced by recording sounds of old Jacquard loom factories in

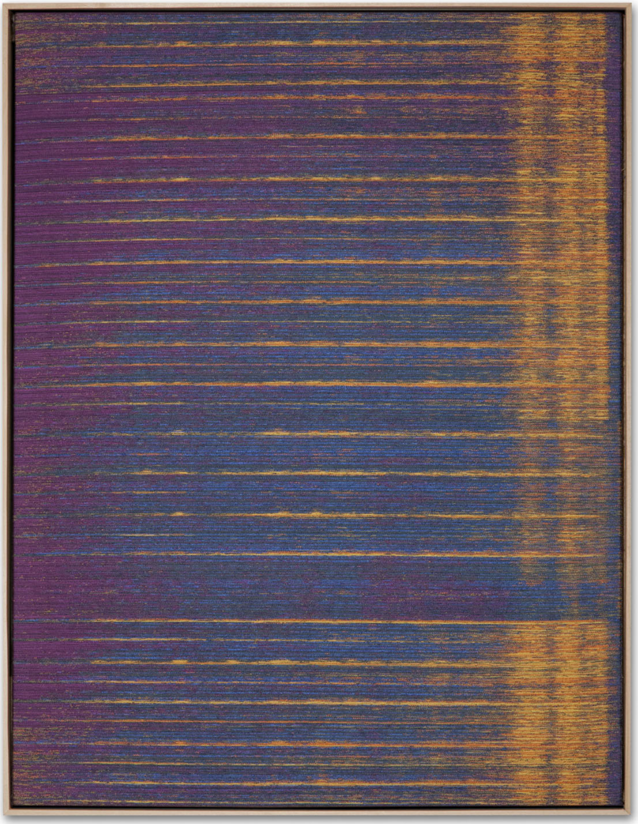


FIGURE 13. Mika Tajima, *Negative Entropy*, 2012

Pennsylvania and turning the sound recording into digital image files, which are then in turn woven into the resulting artworks. The use of Jacquard looms in Tajima's art is a profound reflection on the historical dawn of the Information Age. The programmable Jacquard loom was the historical precursor, and the direct inspiration, of the punchcards used to program the first generation of computers. One of the crucial links in the history of technology, the Jacquard loom can be seen as the last of the age of machines and the first of the age of computers. It marks the transition between Thermodynamic Entropy and Information Theoretic Entropy.

A group of abstract artists in 1960s New York, known as the Park Place Gallery Group, derived their inspiration from various branches of contemporary science, ranging from Cosmology and General Relativity, which inspired the explicit "Reimagining Space" theme of the group, to Information and Entropy, all revisited, in their own words, "with a grain of salt and a lot of irony".

Quoting from Bridgman's thermodynamics reference, which was a source of inspiration to this group of artists, "Like energy, entropy is in the first instance a measure of something that happens when one state is transformed into another". At the same time, the artists of the Park Place Gallery Group were also influenced by Buckminster Fuller's ideas on geometry and synergetics. An analysis of the role of Entropy in 1960s American Abstract Art is given in Robert Smithson's essay *Entropy And The New Monuments*, available in his collected works.

We consider here two paintings from the Park Place Gallery Group: one by Tamara Melcher and one by Dean Fleming. Both consider ordered information patterns, with Fleming's painting akin to a crystal structure. We can read Melcher's painting in

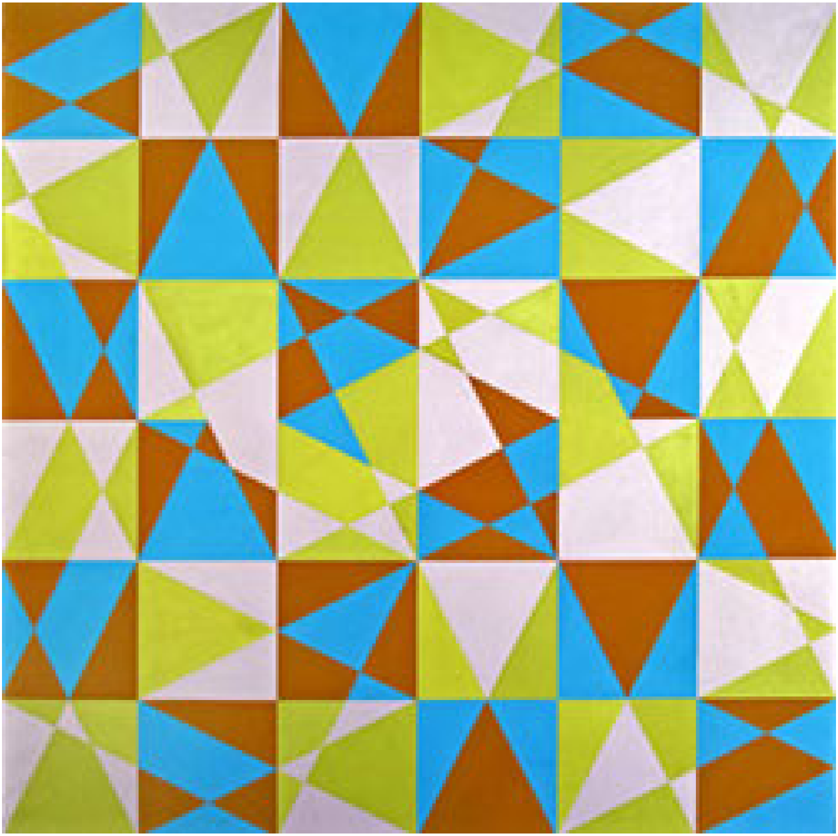


FIGURE 14. Tamara Melcher, *Untitled*, 1965

the sense of the theory of communication. There are several patterns of regularity in the painting are determined by the different superimposed systems of parallel directions, at an angle to each other, and the different pattern of emergence of one stratum over the other in their intersections creates a non-trivial message with

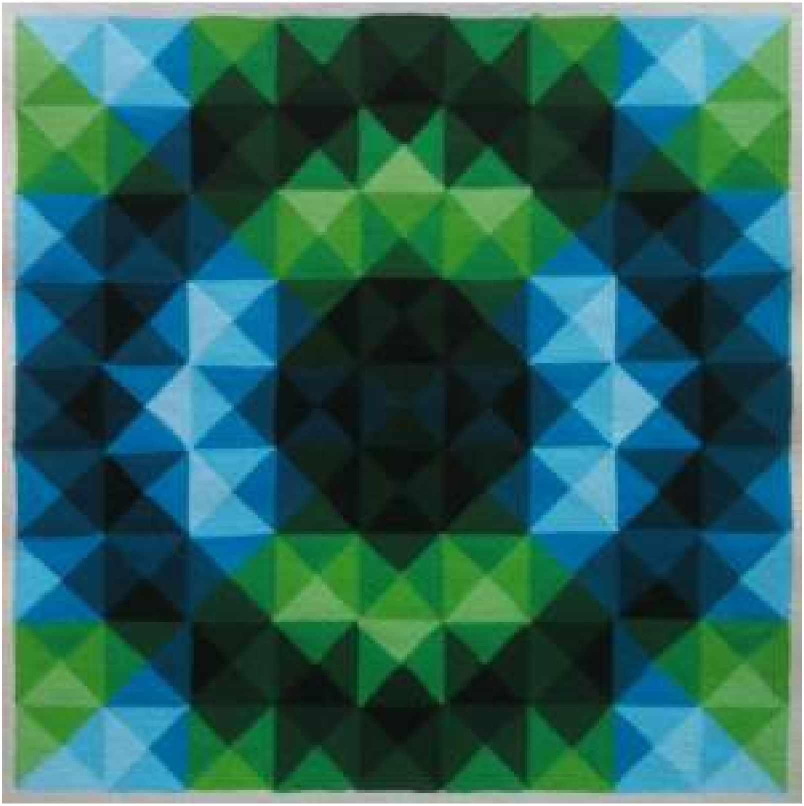


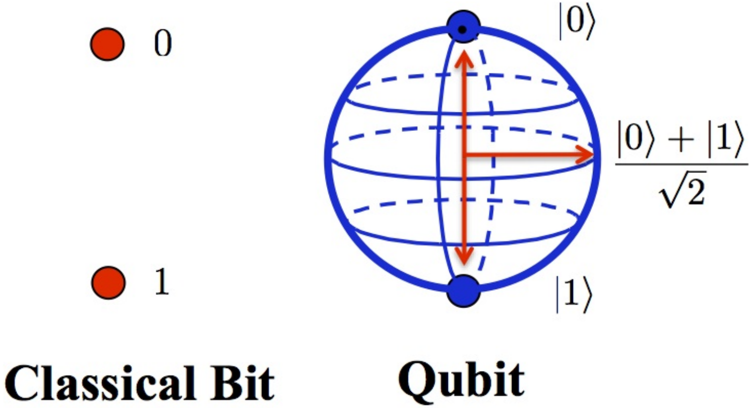
Figure 15. Dean Fleming, *Tunis*, 1964

a higher information content, in the sense of Shannon, than the underlying uniform pattern.

4. Quantum Information

Suppose a fair coin is tossed repeatedly and the outcomes recorded. One can denote by the symbols 0 and 1 the two possible outcomes. After tossing the coin N times, the possibilities are 2^N and the Shannon entropy $S = \log_2(2^N) = N$ counts the number of *bits* that are needed to describe the output.

In passing from classical to quantum information, one replaces the classical notion of a *bit*, which is a single 0 or 1 output, with the quantum mechanical notion of a *qbit*.



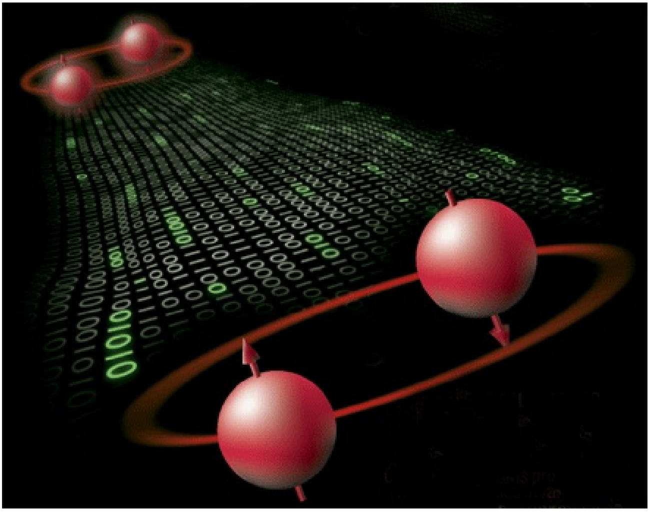
The fundamental difference between the classical bit and the quantum mechanical qbit is that, while the former is a discrete variable taking only two possible values 0 or 1, the latter is a quantum mechanical state, which lives in a superposition of zero and one, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, so that it ouputs 0 with probability $|\alpha|^2$ and 1 with probability $|\beta|^2$, with the normalization condition $|\alpha|^2 + |\beta|^2 = 1$. The set of qbits is a sphere (the Bloch sphere). Thus, while the classical bit is a discrete 0/1 switch, a qbit lives on a sphere of possibilities. The *measurement* operation collapses a qbit to a classical bit: the outcome of measurement on a qbit is a classical bit *with probabilities* assigned to the 0/1 outcomes.

4.1. **Entanglement.** The Shannon information of the classical theory is generalized to quantum information by the von Neumann entropy

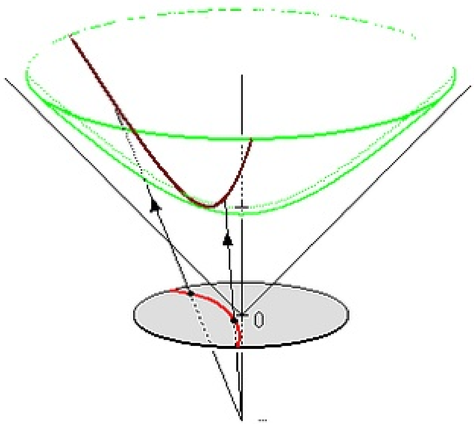
$$S = -\mathrm{Tr}(\rho \log \rho),$$

where the classical probability distribution (p_i) is replaced by a quantum mechanical density matrix ρ, with the trace of matrices replacing the sum.

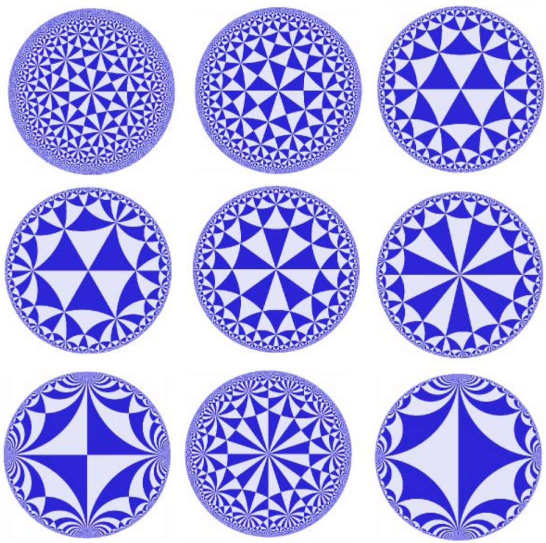
The quantum state for a system of two particles does not always separate out into a separate contribution for each particle: it can be *entangled*. This is a fundamental quantum mechanical property, which is at the basis of the theory of Quantum Communication. The amount of Entanglement in a physical state can be measured using the von Neumann Entropy.



4.2. **Holography: Entropy and Spacetime.** String Theory conjectures an equivalence between certain kind of physical theories, Conformal Field Theory, on *boundary* and Gravity on *bulk* space in a negatively curved AdS spacetime. This correspondence, often referred to as "Holography" because, like holography that reconstructs a three-dimensional object from its image stored on a two-dimensional surface, it reconstructs gravity on the bulk space from a field theory that lives on the lower dimensional boundary. The negatively curved geometry of anti de Sitter (AdS) spacetime is crucial to this correspondence.



AdS spacetime is a Lorentzian geometry. Its analog in Riemannian geometry is the hyperbolic space. The two-dimensional hyperbolic space (hyperbolic plane) was historically one of the first models of non-Euclidean geometries. It can be visualized as a disc (Poincaré disc) with a metric that puts the boundary at infinite distance from the interior points, and where paths of shortest length are not straight lines but circles that encounter the boundary perpendicularly. It can also be represented as a hyperboloid, as in the figure above, with the projection to the disc recovering the Poincaré model, or by a upper half plane where the metric puts the boundary line at infinite distance. There are similar higher dimensional versions of hyperbolic spaces. The hyperbolic metric can be visualized by considering regular tilings of the hyperbolic plane, like the ones shown in the picture below. In each of these tilings the individual tiles are all of the same size, when the size is computed in the hyperbolic metric, although we see them as different sizes with respect to the usual Euclidean metric of the plane. In the hyperbolic metric it is then clear that the boundary is at infinite distance: in order to reach the boundary of the disc one has to traverse an infinite number of tiles of equal size, hence to travel an infinite distance.



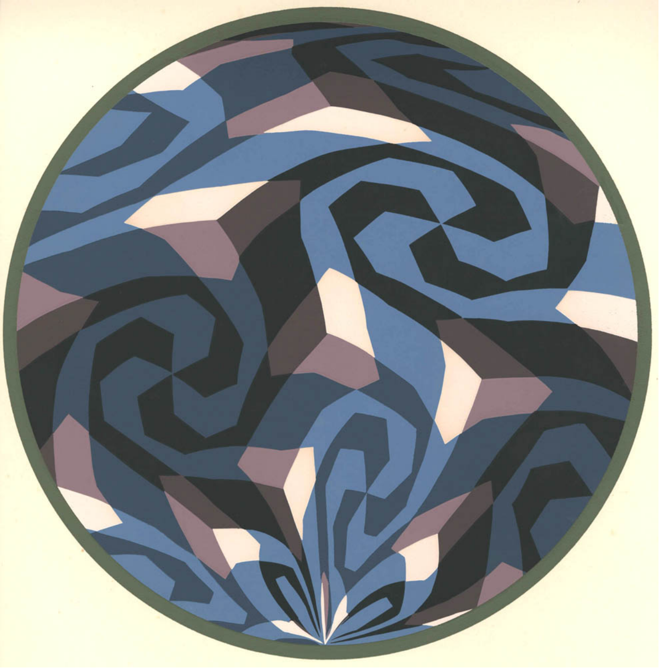


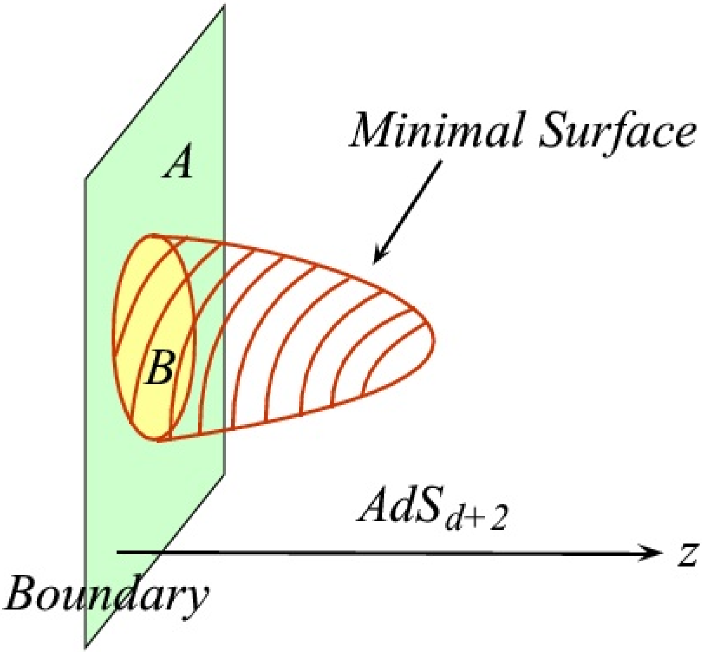
FIGURE 16. Hans Hinterreiter, *Untitled*, 1967

The geometry of the hyperbolic plane was explored in depth the the artist Hans Hinterreiter, who produced a series of drawings and paintings based on the use of different tessellations of the hyperbolic plane and patterns obtained from walks along parts of various hyperbolic tessellations.

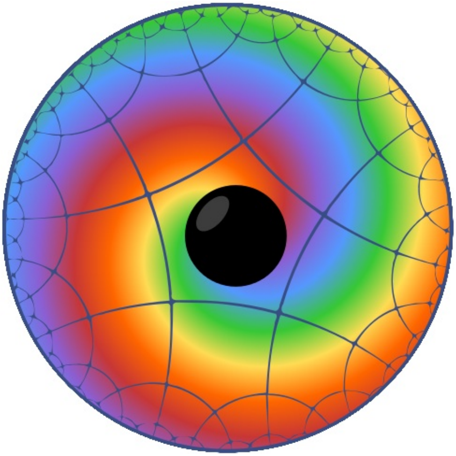
FIGURE 17. Hans Hinterreiter, *Kreis Komposition*, 1978

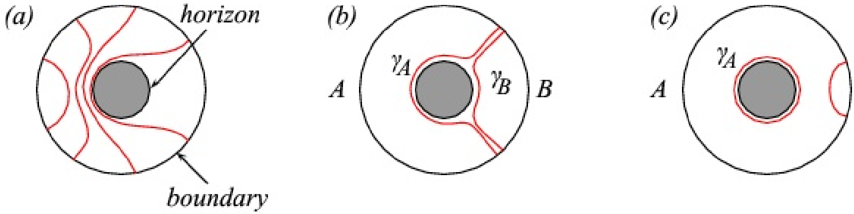
FIGURE 18. Hans Hinterreiter, *Opus 67 B*, 1975–1979

Entanglement Entropy plays a fundamental role in the current models of Holography in String Theory. The recently proposed Ryu–Takayanagi conjecture predicts that the Entanglement Entropy of quantum states corresponding to a region in the conformal boundary at infinity is proportional to the *area* of the minimal surface in the bulk space that has the same region as boundary. This far reaching idea would make it possible to reconstruct gravity (Einstein equations) on the bulk space from the properties of Entropy on the boundary space.



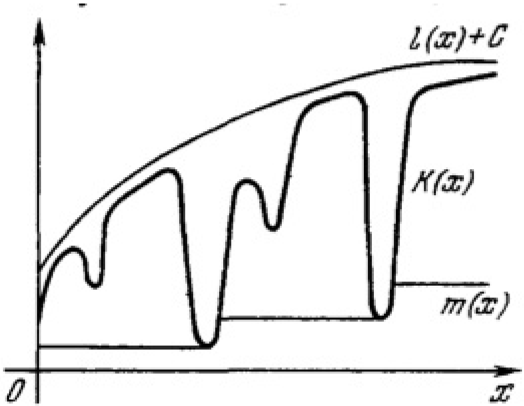
The picture or this bulk/boundary correspondence and the role of Entanglement Entropy becomes more complex when there is a black hole in the bulk space, in which case different surfaces in the bulk with the same boundary region would either wrap around the black hole horizon or avoid it, leading to different contributions, as shown in the figures, reproduced from the Nishioka–Ryu–Takayanagi survey.




(a)
horizon
boundary
(b)
γ_A
A
γ_B
B
(c)
γ_A
A

5. Entropy versus Complexity

Entropy and Complexity are two related but *different* mathematical concepts. The *Kolmogorov Complexity* of an object is the *shortest length of an algorithmic description* of that object, that is the length of the shortest program that produces that output. Thus, for example, a number like 10^{100} has very low complexity compared to a random 100-digits numbers, because it has a very short algorithmic description: "raise the number 10 to the power 100", which is much shorter than having to write out 100 digits. The behavior of complexity in relation to the length (e.g. the number of digits) is illustrated in the following figure (from Li-Vitány). The dips correspond to objects whose complexity is much smaller than their description length, as in the case of the number 10^{100}.



Clearly, any compression algorithm gives an approximate version of Kolmogorov Complexity. However, despite the existence of good approximation (by excess) given by compression algorithms, there are no ways to predict exactly where dips in complexity will occur, that is to say, there are no good approximations from below. This problem is related to the unsolvability of the *halting problem* in the theory of computation: it is impossible to predict whether a program on a given input will output in finite time or will run forever. The reason why the unsolvability of the halting problem affects the computability of Kolmogorov Complexity can be seen in the following way. Whenever we have an algorithm that produces the desired output, whose complexity we wish to compute, that gives us an upper bound on the complexity: an upper bound only, because there may still be a shorter program that produces the same output. However, since we cannot predict whether a program will output or run forever, we cannot a priori tell whether any of the possible shorter programs will give the output we are considering (hence lowering its Kolmogorov complexity) or whether the one we have is really the shorter one. For this intrinsic reason, Kolmogorov Complexity is a *non-computable* function. Saying that something is non-computable seems to be saying it is useless: what good is a function that cannot be computed. However, the existence of good approximations from above generally makes the use of complexity feasible. Moreover, there is a relation between Kolmogorov Complexity and Shannon Entropy, which shows that entropy is an "averaged version" of complexity. This also in general makes it possible to obtain estimates on the behavior of complexity in an average, probabilistic sense.

There is, however, an aspect of Kolmogorov Complexity which is counterintuitive with respect to the heuristic intuitive notion of complexity we all have in our experience and perception of the world. Kolmogorov complexity is maximal for *completely random* patterns. This is not what one would expect: the intuitive idea

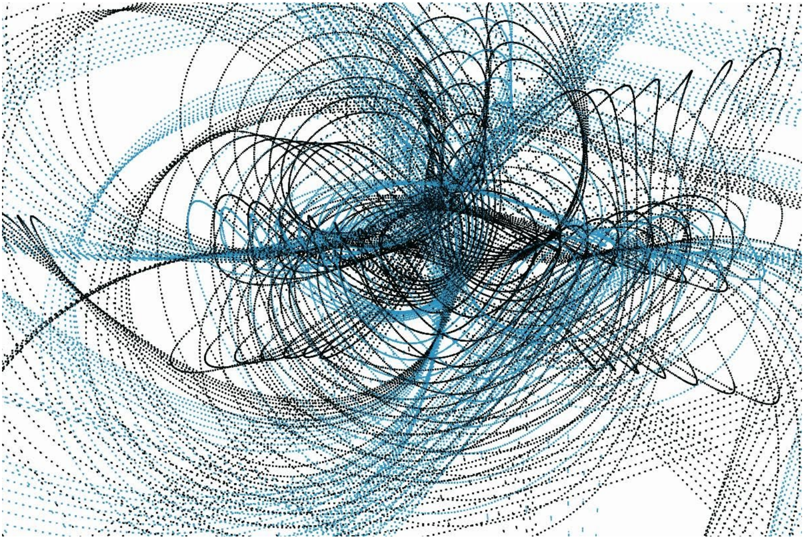


FIGURE 19. Dennis M. Callahan Jr. *Desired Constellations*, 2010

of complexity is related to the existence of "structures" that are sufficiently intricate, but not completely random.

Following this idea of what complexity should be measuring, Murray Gell-Mann proposed a notion of Effective Complexity, which measures the Kolmogorov complexity of all "regular subpatterns" in a given pattern. Effective Complexity is high in an intermediate region between total order and complete randomness.

For instance, let us consider again the two paintings of the Park Place Gallery Group by Tamara Melcher and Dean Fleming of Figures 14 and Figure 15 above. We can say that the painting by

Tamara Melcher has a higher Effective Complexity, in the sense of Murray Gell-Mann, than the painting by Dean Fleming, because Fleming's painting has a simpler structure with fewer very regular patterns, while Melcher's painting has a richer structures of sub-patterns, each of which has a greater Kolmogorov complexity.

One finds explicit use of notions of Complexity Theory in the Generative Art movement. We refer the reader to the "Recovering Infinity" site `http://recoveringinfinity.blogspot.com/` for a forum of Generative Art, and to the writings of Philip Galanter listed below. Generative Art is seen as living in between the "highly ordered" patterns of traditional art, and the "highly disordered" patterns that emerged in Modern Art through the adoption of randomization in artistic expression, ranging from the visual arts to music. The Generative Art movements proposes to focus artistic expression on the intermediate region of high Gell-Mann Effective Complexity, as can be seen in realizations such as the computer generated works of Dennis Callahan.

6. The Problem with Arnheim

We have explored the landscape of the mathematical notions of Entropy and Complexity and parallel forms of artistic expression where such notions manifest themselves. Coming back to the old essay by Arnheim on "Entropy and Art", it is worth pointing out some its main shortcomings and inaccuracies. Although he certainly contributed a very influential essay, Arnheim failed to understand some of the nuances in the mathematical notion of Entropy, failed to correctly and consistently interpret the notions of "order" and "structure", and gave some serious misrepresentations of the artistic movements of the time. Some of these criticisms are already formulated in Peter Lloyd Jones' essay listed below. We discuss here briefly some of the main issues with "Entropy and Art".

First of all, there are problems with the scientific notion of Entropy. Arnheim *overly* emphasizes the identification of Entropy with "disorder". Moreover, he confuses the notions of "equilibrium" and "order". These are definitely not the same thing: a physical system can be in equilibrium in a highly disordered state. Also Arnheim appears to miss entirely the fact that Shannon's "information content" is a *statistical* notion. He confuses the notion of *Entropy/Information* with the notion of *Complexity*. As we discussed in the previous section, although Shannon entropy can be viewed as an averaged version of Complexity in a suitable sense, Entropy and Complexity are two very different mathematical concepts. Arnheim also refers incorrectly to the notion of *redundancy* in Information Theory.

There are also serious problems with the notion of "order". Arnheim's discussion of "order" seems based on several different and at times incompatible meanings. He proposes a point

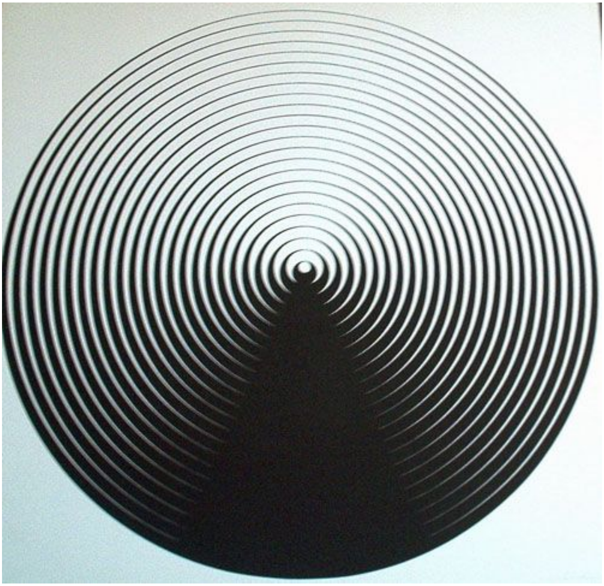


FIGURE 20. Getulio Alviani, *Untitled*, 1964

of view based on *Gestalt Psychology*'s notions of "tension reduction" and "field process" to describe "order" and "structures". This is problematic, since, unlike scientific definitions, these are not *predictive* notions. Gestalt Psychology certainly does have some interesting relations to the mathematics of Pattern Theory, but not so much to the notion of Thermodynamic Entropy that Arnheim repeatedly refers to.

Arnheim also seems unaware of where in the artistic movements of his time Gestalt Psychology had really played a central role, that is, in the "Arte Cinetica e Programmata" movement, developed in Italy in the 1950s, 1960s and 1970s, by artists like Bruno Munari, Gianni Colombo, Getulio Alviani, and Yacoov Agam. An example of their work is given by the painting of Getulio Alviani reproduced above. Gestalt Psychology was a crucial component of these artists chosen language of expression and the whole movement was influenced by the psychologist Gaetano Kanizsa, with whom the artists of this group maintained close connections.

There are many other dubious Art History statements and interpretations of specific art works in Arnheim's essay. He certainly misinterprets Andy Warhol's work: in Warhol's repetitions of images what matters are the details that are *not* identical, and that's where the semantic meaning lies, and the critical message about the mass consumerist society. There are similar widespread misinterpretations of the Minimalist Art movement.

Overall, Arnheim should be praised for initiating a reflection on the theme of Entropy and Art. An in depth reflection on this topic is certainly useful and even necessary to navigate the landscape of the postwar Modern Art movements. However, such a reflection requires a more subtle and precise handling on the correct scientific notions: the different forms of Entropy and the different notions of Complexity. We hope here to stimulate this reflection further.

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