THE RICCI FLOW ON NONCOMMUTATIVE TWO-TORI

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Abstract. The Ricci flow has been one few the most important geometric tools of modern mathematics. In this talk I will describe a version of Ricci flow for noncommutative 2-tori, based on a spectral formulation in terms of the eigenvalues and eigenfunction of the Laplacian and recent results on the Gauss–Bonnet theorem for noncommutative tori. We have found that there is a natural analog of the classical Ricci flow for the noncommutative torus (2 dimensional). In fact we found something even more - a possible analog of the scalar curvature, which has been further investigated by other mathematicians.

The Ricci flow has been one of the most important tools in low dimensional geometry since it's introduction in the early 80's by Richard S. Hamilton. It's most famous application was seen in Grisha Perelman's proof of the century old Poincar conjecture by Grisha Perelman. The most important characteristic of the Ricci flow is probably the fact that it carries information not only about the geometry of the manifold under question but also about the underlying topology.

Since, noncommutative geometry aims to generalize the concept of manifolds and describe a much larger family of geometries, it is natural to try and extend all the important geometric tools to these larger class. The Ricci flow is definitely a very important geometric idea which can lead to a much deeper understanding of noncommutative spaces. An analog of the Ricci flow in noncommutative geometry might allow us to draw parallels with the recent developments in Riemannian geometry including the Poincar conjecture. It might also allow us to extract new knowledge about the topology underlying noncommutative spaces. One early sign of such applications is seen immediately from our description of the noncommutative Ricci flow in the form of a scalar curvature.

In this talk I will begin with a brief description of the Ricci flow. In particular I will discuss it's description in terms of the eigenvalues and eigenfunctions of the Laplacian, which is due to Luca Fabrizio Di Cerbo (2007). Since our goal is to use it for the case of the two dimensional torus, I will briefly discuss some classical results in this particular case.

Then I will give a description of the noncommutative two torus. There are may ways to formulate the noncommutative torus; I will use a algebraic formulation in which the analog to the classical torus is very clear. We will see that most functional concepts have a natural counterpart in the noncommutative case. After this I will describe what we mean by a metric on this noncommutative torus.

Then I will describe some recent results about the noncommutative torus. In particular the Gauss-Bonet theorem by Alain Connes and Paula Tretkopf. This is particularly important because in our work we use several of their results and similar techniques.

Finally I will describe the results we found about the Ricci flow and scalar curvature of the noncommutative torus. We have found that the Ricci flow can be described exactly in the same form as in the classical case in terms of eigenvalues and eigenfunctions of the Laplacian, which depends only on one parameter that characterizes the metric. We also find that there is an analog of the scalar curvature that only depends on this parameter i.e. the metric. Modulo time available, I will discuss the details of the calculation.

This work was done in collaboration with and under the supervision of Professor Matilde Marcolli. Financial support was given by Caltech SFP program, Ryser fund (Caltech mathematics department) and NSF grants held by Professor Marcolli.