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Quantum Nondemolition Measurements

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Scientists have understood since the 1920's that the physical laws which govern atoms, molecules, and elementary particles are very different from the laws of everyday experience. The special laws of the atomic and molecular "microworld" are called quantum mechanics; those of everyday experience are classical mechanics. The laws of quantum mechanics were forced on physicists and chemists in the 1920's as the only possible way to understand the spectral properties of the light emitted by atoms and molecules.

Summary. Some future gravitational-wave antennas will be cylinders of mass ~100 kilograms, whose end-to-end vibrations must be measured so accurately (10^{-19} centimeter) that they behave quantum mechanically. Moreover, the vibration amplitude must be measured over and over again without perturbing it (quantum nondemolition measurement). This contrasts with quantum chemistry, quantum optics, or atomic, nuclear, and elementary particle physics, where one usually makes measurements on an ensemble of identical objects and does not care whether any single object is perturbed or destroyed by the measurement. This article describes the new electronic techniques required for quantum nondemolition measurements and the theory underlying them. Quantum nondemolition measurements may find application elsewhere in science and technology.

Quantum mechanics tells us that, whenever a person measures some property of an electron (or of any other object in the microworld), his measurement inevitably will disturb the electron in a somewhat unpredictable way. The more accurate the measurement, the bigger and more unpredictable the disturbance. The disturbance is not due to the person's incompetence; rather, it is an intrinsic and inevitable feature of the laws of quantum mechanics.

As an example, if a person measures the position of an electron in space with complete accuracy, his measurement inevitably will kick the electron with a totally unpredictable force. A second measurement of the electron's position, immediately after the first one, will give the same position as the first one did; but a measurement of the electron's momentum will give a completely unexpected result.

If, nevertheless, the momentum is measured very carefully and some definite result is obtained, that momentum measurement inevitably will disturb the electron's position by an unpredictable amount. If a second momentum measurement is made, the result is known in advance: it will be the same as just obtained. But if the next measurement is of position, nobody can know the result in advance.

It matters not at all how the person makes his measurements—with the best technology of the 1920's, or the best of the 1980's, or the best of the 23rd century: an accurate position measurement must completely disturb the momentum; an accurate momentum measurement must completely disturb the position.

As bizarre as this situation may seem, it is even more bizarre when studied in greater depth—as was done theoretically by Niels Bohr, Werner Heisenberg, John von Neumann, Wolfgang Pauli, and others in the 1920's and 1930's. [See the textbook of Bohm (*1*) for details; see (*2*) for a detailed illustrative example.] It turns out that the unpredictable disturbance is a direct result of the extraction of information about the particle's position or momentum. It matters not how the information is extracted, nor where it is stored—in a person's brain, on magnetic tape, or in some minute change of the state of some other particle. So long as the information exists somewhere in the universe outside the original particle (more precisely, "outside the particle's wave function"), future measurements of the particle will reveal that the disturbance has occurred. The only way to undo the disturbance is to "run the measuring apparatus perfectly backward" and thereby reinsert all the information back into the particle. Only if no trace of the information remains anywhere, not even in the experimenter's brain, can the particle return to its original undisturbed state.

The quantum theory of measurement (*1*), which tells us these things and more, is very widely but not universally accepted by physicists. Einstein never fully accepted it (*3*); Lamb, a Nobel Prize winner for his experimental work in quantum physics, does not fully accept it (*4*). The authors of this article do accept it, and will presume it to be correct throughout this article.

The quantum theory of measurement tells us that, if a measurement is somewhat imprecise, then the magnitude of its disturbance is somewhat but not entirely predictable. For example, a very careful measurement of the east-west position of an electron, with an imprecision Δx, can

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be guaranteed to disturb its east-west momentum by not much more than $\Delta p = \hbar/(2\Delta x)$, where $\hbar$ ($\equiv 1.054 \times 10^{-27}$ cm g cm/sec) is Planck's constant. However, no matter how careful the measurement may be, the momentum uncertainty afterward will be at least $\hbar/(2\Delta x)$. Similarly, a momentum measurement of precision Δp will leave the position uncertain by at least $\Delta x = \hbar/(2\Delta p)$—but if the measurement is very careful, the position disturbance need not be much larger than this. The limit $\Delta x \Delta p \geq \hbar/2$, which holds for either type of measurement, is called the Heisenberg uncertainty principle.

The ultimate limits imposed by the uncertainty principle have been explored in great detail during the past decade by C. W. Helstrom, R. L. Stratanovich, J. P. Gordon, and others. They have developed a beautiful, mathematical theory of optimum quantum mechanical measurements (quantum detection and estimation theory) (5). Unfortunately, this theory assumes one can make a precise measurement of one observable or another, or of some combination of observables; but it does not spell out how such precise measurements can be realized technically—even in principle.

This gap in the theory is being confronted today in the effort to detect cosmic gravitational waves (6). Gravity-wave detectors consist of aluminum (or sapphire or silicon or niobium) bars, weighing between 10 kilograms and 10 tons, which are driven into motion by passing waves of gravity. The motions are very tiny: for the gravity waves that theorists predict are bathing the earth, a displacement $\delta x \cong 10^{-19}$ centimeter might be typical (6). And this displacement may oscillate, due to oscillations of the gravity wave, with a period $P \sim 10^{-3}$ second. To see the details of the gravity wave, one must thus make repeated measurements of the bar's position with precision $\Delta x \lesssim 10^{-19}$ cm, and with time intervals between measurements of $\hat{\tau} \lesssim 10^{-3}$ second.

For all measurements ever made in the past on a heavy bar, the effects of quantum mechanics were totally negligible; the classical mechanics of everyday experience gave a perfectly adequate description of the bar's behavior. But one never before tried to make measurements of such enormous precision as 10^{-19} cm. If the bar is suspended freely like a pendulum, as it is in some detectors (6), then over time intervals $\hat{\tau} \sim 10^{-3}$ second it will behave as though it were not suspended at all. It will be as free to move horizontally as the electron described above—and like the electron it will be subject to the laws of quantum mechanics: an "initial" measurement of the bar's east-west position with precision $\Delta x_i \cong 10^{-19}$ cm will inevitably disturb the bar's east-west momentum by $\Delta p \geq (\hbar/2\Delta x_i)$, and correspondingly will disturb its velocity by $\Delta v = \Delta p/m \geq (\hbar/2m\Delta x_i)$, where m is the bar's mass. During the time interval $\hat{\tau} \sim 10^{-3}$ second between measurements, the mass will move away from its initial position by an amount, $\Delta x_m = \Delta v\ \hat{\tau} \geq (\hbar\hat{\tau}/2m\Delta x_i)$, which is unpredictable because Δv is unpredictable. Putting in numbers ($\hat{\tau} = 10^{-3}$ second, $m = 10$ tons, $\Delta x_i = 10^{-19}$ cm), we find $\Delta x_m \geq 5 \times 10^{-19}$ cm—which is somewhat larger than the desired precision of our sequence of measurements. If the next measurement reveals a position changed by as much as 5×10^{-19} cm, we have no way of knowing whether the change was due to a passing gravity wave or to the unpredictable, quantum mechanical disturbance made by our first measurement. In effect, our first measurement plus subsequent free motion of the bar has "demolished" all possibility of making a second measurement of the same precision, $\Delta x \sim 10^{-19}$ cm, as the first, and of thereby monitoring the bar and detecting the expected gravity waves.

In principle one can circumvent this problem by making the bar much heavier than 10 tons (recall that Δx_m is inversely proportional to the mass). However, this is impractical. In principle another solution is to shorten the time between measurements (recall that Δx_m is directly proportional to $\hat{\tau}$). However, this will weaken the gravitational-wave signal ($\delta x_{GW} \propto \hat{\tau}^2$ for $\hat{\tau} \lesssim 10^{-3}$ second) even more than it reduces the unpredictable movement of the bar ($\Delta x_m \propto \hat{\tau}$).

The best solution is cleverness: find some way to make the gravity-wave effect stronger; this is being done in laser-interferometer gravity-wave detectors (6), but only at the price of having to make 10^{-16} cm measurements of the relative displacement of two bars as far apart as several kilometers. Alternatively, find some way to circumvent the effects of the Heisenberg uncertainty principle—that is, some way to prevent the inevitable disturbance due to the first measurement, plus subsequent free motion, from demolishing the possibility of a second accurate measurement: a quantum nondemolition (QND) method.

One QND method which could work in principle is this: instead of measuring the position of the 10-ton bar, measure its momentum with a small enough initial error, $\Delta p_i \sim 10^{-9}$ g cm/sec, to detect the expected gravity waves. Thereby inevitably disturb the bar's position by an unknown amount $\Delta x \gtrsim \hbar/2\Delta p_i \sim 5 \times 10^{-19}$ cm. Wait a time $\hat{\tau} \sim 10^{-3}$ second and then make another momentum measurement. As the bar moves freely between the measurements, its momentum remains fixed. The uncertainty Δx in the bar's position does not by free evolution produce a new uncertainty Δp_m in the momentum. Consequently the second measurement can have as good accuracy, 10^{-9} g cm/sec, as the initial measurement; and a momentum change of (a few) $\times\ 10^{-9}$ g cm/sec due to a passing gravity wave can be seen.

Momentum measurements can be quantum nondemolition, but position measurements cannot be, for this simple reason: in its free motion between measurements the bar keeps its momentum constant, but it changes its position by an amount $\delta x = (p/m)\hat{\tau}$ that depends on the momentum, and that therefore is uncertain because of measurement-induced uncertainties of the momentum. We say that momentum is a QND variable, but position is not.

Unfortunately, however, it is far easier to measure position than momentum. Nobody has yet invented a technically realizable way of making momentum measurements with the required precision.

The problem of inventing a technically realizable QND measurement scheme was first posed in 1974 (8). This reference also formalized the concept of QND measurements. Subsequent developments in the theory of QND are due largely to Unruh (9, 10); Braginsky, Vorontsov, and Khalili (11, 12); Caves, Thorne, Drever, Zimmermann, and Sandberg (13-15); and Hollenhorst (16). All of this work has been theoretical: it has shown that in principle QND measurement schemes can completely circumvent the disturbing back-action effects of one's measurements, and it has led to several tentative designs for practical QND measurements in gravity-wave detectors—designs which do not involve measuring momentum.

Actual laboratory work on QND measurement schemes is only now beginning to get under way, and the levels of sensitivity required are so great that we cannot hope for any laboratory results until several years from now. Nevertheless, it is reasonable to expect QND measurements to be a routine part of gravity-wave technology by the late 1980's.

The purpose of this article is to make as wide an audience as possible aware of these developments, so that people can begin to ask whether the QND idea might be useful in other areas of science and technology. To achieve this purpose effectively, we feel it necessary to write the rest of this article at a somewhat technical level. We hope thereby to convey to physicists, engineers, chemists, and others familiar with elementary quantum mechanics and elementary electronics, a deep enough understanding of the QND idea that they can begin to think creatively about it themselves.

Resonant-Bar Gravitational-Wave Antennas

Although the QND idea is explained most easily, as we have done, in terms of bars which move freely (free masses), QND measurements are most needed for a different type of gravity-wave antenna: one made of a bar which oscillates mechanically in its fundamental mode (bar mass, $m \simeq 10$ to 10,000 kg; oscillation frequency, $\omega/2\pi \simeq 500$ to 10,000 hertz) (6). The expected gravity waves should produce changes of oscillation amplitude $\delta x \simeq 10^{-18}$ to 10^{-19} cm, which are less than or of order the width of the quantum mechanical wave packet of the oscillator $\delta x_{\mathrm{QM}} = (\hbar/2m\omega)^{1/2}$, if the oscillator is in its ground state or in a coherent (minimal-wave-packet) state. Here, by contrast with nuclear, atomic, and elementary-particle physics, there is only one quantum mechanical system being measured (the oscillator), rather than an ensemble of systems; and we must make a continuous sequence of measurements on this one system.

Such an oscillator will actually behave quantum mechanically even in the presence of thermal Brownian motion and at bar temperatures $kT >> \hbar\omega$, so long as its quality factor Q is sufficiently high—that is, so long as the fundamental mode of the bar is coupled sufficiently weakly to the other, thermalized modes (7). In particular, when one is making energy measurements which put the oscillator in an energy eigenstate, Brownian motion during one cycle will change the number of quanta n in the oscillator by less than unity if (7)

$$(n + \tfrac{1}{2})kT/Q < \hbar\omega/4\pi \qquad (1a)$$

and if one is making amplitude measurements, Brownian motion during the measurement time $\bar{\tau}$ will change the amplitude by less than the coherent-state wave-packet width $(\hbar/2m\omega)^{1/2}$ if (7)

$$2kT\bar{\tau}/Q < \hbar \qquad (1b)$$

(see Eqs. 40 and 41 below).

In order to monitor the effects of a weak gravity-wave force on such an oscillator, one must use a measurement technique whose back action on the oscillator, together with subsequent free evolution, does not substantially disturb the probability distributions of the observables being measured—a QND technique. In the following sections we shall describe the theory of such QND measurement techniques as applied to oscillators, to free masses, and more generally to any quantum mechanical system. Throughout our description we shall try to give short, elementary proofs of most of the results quoted. More elegant and rigorous proofs will be found in the primary literature. To understand our discussion, the reader must be familiar with elementary quantum mechanics and elementary electronic circuit theory, but little other specialized knowledge should be needed.

General Theory of Quantum Nondemolition Measurements

Consider a system, such as an oscillator, that has some observable $\hat{A}$ which an experimenter wishes to monitor. For the moment, assume that the system's only coupling to the external world is through the experimenter's measuring apparatus. We define a QND measurement of $\hat{A}$ as a sequence of precise measurements of $\hat{A}$ such that the result of each measurement is completely predictable from the result of the first measurement—plus, perhaps, other information about the initial state of the system. This definition, and the ramifications which follow, are a refinement by Caves [in (15)] of Braginsky and Vorontsov's (8) original concept of quantum nondemolition. A similar refinement has been developed independently by Unruh (10).

Quantum nondemolition measurements are ideal tools for use in the detection of weak external forces (such as gravity waves) that act on the system. One need only perform a QND monitoring of the evolution of $\hat{A}$ and watch for deviations from the predicted evolution.

Most observables cannot, even in principle, be monitored in a QND way. In any precise measurement of an observable $\hat{A}$, the back action of the measuring apparatus uncontrollably and unpredictably kicks all observables $\hat{C}$ which fail to commute with $\hat{A}$; and then, in the subsequent free evolution of the system, the contamination in $\hat{C}$ may be fed into $\hat{A}$, making the results of future measurements of $\hat{A}$ unpredictable. Only very special observables can be immune to such feedback contamination; they are called QND observables [or sometimes generalized QND observables (15)]. Mathematically, $\hat{A}$ is a QND observable if and only if, when the system is evolving freely in the Heisenberg picture, $\hat{A}$ commutes with itself at the different moments of time t_j, t_k when one makes one's measurements (10, 15)

$$[\hat{A}(t_j), \hat{A}(t_k)] = 0 \qquad (2)$$

If this condition is satisfied at all times t_j and t_k, then $\hat{A}$ is called a continuous QND observable; if it is satisfied only at special times, then $\hat{A}$ is a stroboscopic QND observable. If $\hat{A}$ is conserved during free evolution ($d\hat{A}/dt = 0$), then it is guaranteed to satisfy Eq. 2 for all t_j, t_k and therefore to be a continuous QND observable.

In the case of a free particle, the energy and momentum are conserved and are continuous QND observables, but the position is not: $\hat{x}(t + \tau) = \hat{x}(t) + \hat{p}\tau/m$, so

$$[\hat{x}(t), \hat{x}(t + \tau)] = i\hbar\tau/m \qquad (3)$$

Precise measurements of $\hat{x}$ perturb $\hat{p}$ uncontrollably, and the contamination in $\hat{p}$ subsequently feeds back into $\hat{x}$ as the particle moves freely.

For a harmonic oscillator the position and momentum satisfy the commutation relations

$$[\hat{x}(t), \hat{x}(t + \tau)] = (i\hbar/m\omega)\sin\omega\tau \qquad (4a)$$

$$[\hat{p}(t), \hat{p}(t + \tau)] = i\hbar m\omega\sin\omega\tau \qquad (4b)$$

These imply that $\hat{x}$ and $\hat{p}$ are not continuous QND observables. However, if one makes one's measurements stroboscopically at times separated by an integral number of half-periods ($\tau = k\pi/\omega$; $\sin\omega\tau = 0$), then the commutators in Eqs. 4a and 4b vanish. This means that $\hat{x}$ and $\hat{p}$ are stroboscopic QND observables (12, 13). Stroboscopic QND measurements (12, 13) of $\hat{x}$ or $\hat{p}$ drive the oscillator into a state where $\hat{x}$ is known precisely—for example, at moments $t = k\pi/\omega$ and $\hat{p}$ is known precisely at $t = (k + \tfrac{1}{2})\pi/\omega$; but at other times $\hat{x}$ and $\hat{p}$ are highly uncertain. For an oscillator the conserved quantities, which are guaranteed to be QND observables at any and all times, include the energy (8) and the real and imaginary parts of the complex amplitude (13)

$$\hat{X}_1 = \hat{x}(t)\cos\omega t - [\hat{p}(t)/m\omega]\sin\omega t \qquad (5a)$$

$$\hat{X}_2 = \hat{x}(t)\sin\omega t + [\hat{p}(t)/m\omega]\cos\omega t \qquad (5b)$$

High-precision measurements of $\hat{X}_1$ or $\hat{X}_2$ (whether fully QND or not) are called back-action-evading measurements (14, 15) because they enable the measured component of the amplitude (for example, $\hat{X}_1$) to avoid back-action contamination by the measuring device, at the price of strongly contaminating the other component ($\hat{X}_2$). (The uncertainty relation

$$\Delta X_1 \Delta X_2 \geq \hbar/2m\omega \qquad (5c)$$

is enforced by the commutation relations $[\hat{X}_1, \hat{X}_2] = i\hbar/m\omega$, which follow from $[\hat{x}, \hat{p}] = i\hbar$.)

Let $\hat{A}$ be a QND observable which is to be monitored by a sequence of perfect QND measurements at times t_0, t_1, t_2, ... Since $\hat{A}(t_0)$ and $\hat{A}(t_j)$ commute (QND assumption; Heisenberg picture), one can perform a perfect "state-preparation measurement" at time t_0, which puts the system into a simultaneous eigenstate $|\psi_0\rangle$ of the observables $\hat{A}(t_0)$, $\hat{A}(t_1)$, $\hat{A}(t_2)$, . . . with some (not previously predictable) eigenvalues $A(t_0)$, $A(t_1)$, $A(t_2)$, ... From the results of this first measurement one can compute the eigenvalues $A(t_0)$, $A(t_1)$, $A(t_2)$, ... Later, as the system evolves freely, its state $|\psi_0\rangle$ remains fixed in time, while its observable $\hat{A}$ evolves through the values $\hat{A}(t_1)$, $\hat{A}(t_2)$, ... Subsequent perfect measurements of $\hat{A}$ at times t_1, t_2, . . . must give the known eigenvalues $A(t_1)$, $A(t_2)$ and must leave the state of the system $|\psi_0\rangle$ unchanged. If $\hat{A}$ is a continuous QND observable, then the QND measurements can be made continuously, and each measurement can last as long or as short a time as one wishes. If $\hat{A}$ is a stroboscopic QND observable, then each measurement must be made very quickly (stroboscopically) to avoid contamination. Examples will be given below, and further detail will be found in section IV of (*15*).

The apparatus used in any measurement consists of a sequence of stages, through which information flows toward the experimenter's eyes and brain. Measurement theory asserts that, although the early stages of the apparatus may behave quantum mechanically, the late stages must be classical. There is no universally accepted definition of classical. We shall regard a stage as classical if the quantum mechanical uncertainties of it and of subsequent stages have no significant influence on the overall accuracy of the measurement. If the system being studied interacts directly with a classical stage, the measurement is called direct. When, between the system and the first classical stage, there is a quantum stage (quantum mechanical readout system, QRS), the measurement is called indirect (*17*). For example, the measurement of the position of a particle by its blackening of a photographic plate is direct. The measurement of position by the particle's scattering of light or of an electron is indirect. The vast majority of measurements are indirect. In electronic measuring systems the first classical stage is often the first amplifier.

In deducing quantum limitations on the sensitivity of a specific measuring scheme, one must analyze quantum mechanically everything that precedes the first classical stage. The overall accuracy of the measurement is governed not only by quantum fluctuations in the quantum stages, but also by the details of the couplings between those stages and between them and the measured system. These all influence the signal which enters the first classical stage, and that signal ultimately determines the quantum errors of measurement.

In practice, if not in principle, the reduction of the wave function occurs when the signal enters the first classical stage. If that signal carries information not only about the observable $\hat{A}$ which interests us, but also about observables $\hat{C}$ that fail to commute with $\hat{A}$, an exact measurement is impossible. This is because any flow of information about $\hat{C}$ into the first classical stage will, according to the uncertainty principle, be accompanied by unpredictable back-action forces into the quantum stages—back-action forces which must ultimately contaminate all observables that fail to commute with $\hat{C}$, including $\hat{A}$.

Because of this back action, the measurement error must always exceed an ultimate quantum limit. We shall derive that limit under the special assumption that in the Heisenberg picture $\hat{A}$ and $\hat{C}$ are time-independent—either because they are constants of the motion such as $\hat{X}_1$ and $\hat{X}_2$, or because they are time-evolving observables [such as $\hat{x}(t)$ and $\hat{p}(t)$] evaluated at some fixed moment of time [such as $\hat{A} = \hat{x}(0)$, $\hat{C} = \hat{p}(0)$]. We assume that the "readout observable" of the last quantum stage, $\hat{Q}_R$, which couples into the first classical stage, is expressible as

$$\hat{Q}_R = f(\alpha\hat{A} + \beta\hat{C}) \qquad (6a)$$

where

$$[\hat{A},\hat{C}] = 2i\gamma\hbar \quad \text{so} \quad \Delta A \Delta C \geq \gamma\hbar \qquad (6b)$$

with γ a real number. The time evolution of the readout observable $\hat{Q}_R$ is embodied in the function f and/or in the real parameters α and β. Typically, α and β will be sinusoidal functions of time, which are used to code and separate the $\hat{A}$ and $\hat{C}$ signals. We assume that the first classical stage (usually an amplifier) is equally sensitive to signals at the $\hat{A}$ and $\hat{C}$ frequencies. Then no matter how accurately the first classical stage monitors $\hat{Q}_R$, it must give errors in $\hat{A}$ and $\hat{C}$ related by $\Delta A = (\bar{\beta}/\bar{\alpha})\Delta C$, where $\bar{\alpha}$ and $\bar{\beta}$ are the root-mean-square (r.m.s.) values of α and β. These relative errors, combined with the uncertainty relation (Eq. 6b), imply the ultimate quantum limit

$$\Delta A \geq [(\bar{\beta}/\bar{\alpha})\gamma\hbar]^{1/2} \qquad (6c)$$

Return now to the general situation where $\hat{A}$ and $\hat{C}$ might be time-dependent. In order that the instantaneous signal at time t not contain any contaminant information about observables $\hat{C}(t)$ which fail to commute with $\hat{A}(t)$, it is necessary and sufficient that $\hat{A}(t)$ commute with that part of the Hamiltonian $\hat{H}_I(t)$ which describes the interaction of the system with the measuring apparatus (*1*)

$$[\hat{A}(t),\hat{H}_I(t)] = 0 \qquad (7)$$

In order that information about $\hat{A}$ enter the measuring apparatus, $\hat{H}_I$ must depend upon $\hat{A}$. Usually one achieves these conditions by direct coupling of $\hat{A}$ to some observable $\hat{M}$ of the measuring apparatus

$$\hat{H}_I = K\hat{A}\hat{M} \qquad (8)$$

In summary, the condition in Eq. 7 guarantees no direct, instantaneous back action of the measuring apparatus on the quantity $\hat{A}$ being measured; and the condition in Eq. 2 guarantees that variables $\hat{C}$ which have been contaminated by back action will not subsequently, by free evolution (with $\hat{H}_I$ turned off), feed their contamination into $\hat{A}$. Often, however, $\hat{H}_I$ is turned on for a long time—even for all time. Then there is danger that $\hat{H}_I$ may catalyze an evolutionary feeding of $\hat{C}$ into $\hat{A}$. One can be sure this does not happen if an analysis of the system plus measuring apparatus, including all couplings, reveals that $[\hat{A}(t_1), \hat{A}(t_2)] = 0$ for all times t_1 and t_2 at which signals enter the classical stage. However, such a full analysis may be prohibitively difficult.

Fortunately, in one common situation a full analysis is not necessary: Caves [in (*15*)] has shown that, if $\hat{A}$ is a continuous QND observable and $\hat{H}_I$ contains no system observables except $\hat{A}$, then the Heisenberg picture evolution of $\hat{A}$ with couplings turned on is identical to its free evolution, and consequently $\hat{A}$ is fully isolated from back action—both direct and indirect. Caves (*15*) has also proved "full isolation of $\hat{A}$ with $\hat{H}_I$ turned on" under more general circumstances.

Just as $\hat{H}_I$ might catalyze an indirect feeding of contaminated variables $\hat{C}$ into $\hat{A}$, so also such feeding might be catalyzed by the coupling of the system to a classical external force $F(t)$ (for example, to a gravitational wave). This coupling is embodied in a piece of the Hamiltonian

$$\hat{H}_F = \mu F \hat{x} \quad (9)$$

where μ is a coupling constant and $\hat{x}$ is a dynamical variable of the system ($\hat{x}$ is position in the case of a gravitational-wave antenna). If $\hat{A}$ satisfies the self-commutation condition (Eq. 2) even in the presence of $\hat{H}_F$, then $\hat{A}$ can remain free from contamination. If, in addition, free evolution with $\hat{H}_F$ turned on causes an eigenvalue $A(t)$ of $\hat{A}(t)$ to evolve in such a way that, from a precise knowledge of $A(t)$, one can deduce $F(t)$, then $\hat{A}$ is called a QNDF observable [(*15*); see also Unruh (*10*), where this is denoted QNDD]. QNDF observables are ideal tools for monitoring weak, classical forces.

If, on the other hand, the term $\hat{H}_F$ in the Hamiltonian catalyzes an evolutionary feeding of contaminated observables $\hat{C}$ into $\hat{A}$ (that is, if $[\hat{A}(t_j), \hat{A}(t_k)] \neq 0$ in the presence of $\hat{H}_F$), then although $\hat{A}$ may be highly sensitive to the presence of an external force, one cannot hope to monitor the details of the force by measurements of $\hat{A}$.

In the case of an oscillator with position $\hat{x}$ coupled to the force (for example, a gravitational-wave detector), $\hat{X}_1$ and $\hat{X}_2$ (Eqs. 5) are QNDF observables and thus can be used for perfect monitoring of the forces (*13, 15*). By contrast, the oscillator's energy, although a QND observable, is not QNDF. As a result, precise measurements of the energy can reveal the presence of an arbitrarily weak force; but they cannot determine the strength of the force with a precision better than a factor of 3 (*13, 10, 15*)—unless the force is so strong that it increases the energy by an amount large compared to the initial energy. Examples and proofs will be sketched later.

When one is using a quantum system to monitor a classical force F, one can increase one's sensitivity by increasing the response of the measured quantity $\hat{A}$ to F. In (*8*) and (*18*) it is shown that, if $\hat{A}$ is the energy of an oscillator, F produces a change of $\hat{A}$ which is larger, the larger the oscillator's initial energy. Formally, such a measuring scheme satisfies Unruh's (*10*) general condition for the dependence of $\hat{A}$'s response on the initial state of the detector (detector-dependent response, or DDR)

$$[\hat{A}, \hat{x}] \neq \text{a } C\text{-number} \quad (10)$$

In the case of (*8*) and (*18*) $\hat{A}$ is the detector's (oscillator's) Hamiltonian, $\hat{x}$ is its position, and $[\hat{A}, \hat{x}] = -(i\hbar/m)\hat{p}$. The larger the initial energy of oscillation, the larger will be $\langle \hat{p}^2 \rangle$, and the larger will be $d\hat{A}/dt$. Further details will be given later.

This completes our sketch of the general theory of QND measurements. This theory will now be applied to various types of measurements of harmonic oscillators, with emphasis on issues relevant to gravitational-wave detection.

Position Measurements

A resonant-bar gravity-wave antenna is an oscillator with mass m, frequency ω, position $\hat{x}$, and momentum $\hat{p}$, which couples to a gravitational wave (classical external force F) with a coupling energy $\hat{H}_F = -\hat{x}F(t)$. In most experiments the antenna's position $\hat{x}$ is coupled by a transducer ($\hat{H}_I = K\hat{x}\hat{q}$; $K \equiv$ coupling constant) to an electromagnetic circuit (quantum readout system), which we shall describe as an oscillator with capacitance C, inductance L, generalized coordinate (equal to charge on the capacitor) $\hat{q}$, and generalized momentum (equal to flux in the inductor) $\hat{\pi}$. More complicated QRS's can be used; but this is the typical case. The voltage on the capacitor, which is proportional to $\hat{q}$, is monitored by an amplifier—the first classical stage of the measuring system. Thus $\hat{q}$ is the readout observable $\hat{Q}_R$ (see Eqs. 6).

The coupled antenna, force, and QRS are governed by the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2 + \frac{\hat{\pi}^2}{2L} + \frac{\hat{q}^2}{2C} + \hat{H}_F + \hat{H}_I$$

$$\hat{H}_F = -\hat{x}F(t) \qquad \hat{H}_I = K\hat{x}\hat{q} \quad (11)$$

for which the Heisenberg evolution equations are

$$\begin{aligned} d\hat{x}/dt &= \hat{p}/m \\ d\hat{p}/dt &= -m\omega^2\hat{x} + F(t) - K\hat{q} \\ d\hat{q}/dt &= \hat{\pi}/L \\ d\hat{\pi}/dt &= -\hat{q}/C - K\hat{x} \end{aligned} \quad (12)$$

Because these equations ignore the first classical stage (amplifier) and its detailed back action on the QRS, they cannot tell us the actual sensitivity of the measuring system. On the other hand, they can tell us the ultimate quantum mechanical limit on the sensitivity.

Suppose, as a first case, that the signal $\hat{Q}_R = \hat{q}$ is fed continuously into the amplifier for a time much longer than a quarter-cycle of the antenna, and that one's goal is to measure $\hat{x}_0$, the initial value of the oscillator's position. During the measurement $\hat{x}(t)$, which feeds $\hat{\pi}$ and thence $\hat{Q}_R \equiv \hat{q}$, oscillates between $\hat{x}_0$ and $\hat{p}_0$. [$\hat{x}(t) = \hat{x}_0 \cos \omega t + (\hat{p}_0/m\omega) \sin \omega t$, aside from minor modifications due to the couplings. Note that $\hat{x}_0 \equiv \hat{X}_1$, $\hat{p}_0/m\omega \equiv \hat{X}_2$; Eqs. 5.] Consequently, the signal $\hat{Q}_R$ entering the amplifier contains not only $\hat{x}_0$ but also, unavoidably, $\hat{p}_0$. Since their relative strengths in the signal are $p_0/x_0 = m\omega$, the measurement determines them with relative precisions $\Delta p_0 = m\omega\Delta x_0$. Taking account of the uncertainty relation $\Delta x_0 \Delta p_0 \geq \hbar/2$, we find (*19, 18, 12, 15*)

$$\Delta x_0 = \Delta p_0/m\omega \geq (\hbar/2m\omega)^{1/2} \quad (13)$$

(This is a specific example of the general quantum limit of Eqs. 6.) Such a measurement is called an amplitude-and-phase measurement because it gives information about both the amplitude $[x_0^2 + (p_0/m\omega)^2]^{1/2}$ and the phase $\psi_0 = \tan^{-1}(p_0/m\omega x_0)$ of the antenna's motions. An ideal amplitude-and-phase measurement with the limiting sensitivity in Eq. 13 drives the antenna into a coherent (minimal-wave-packet) state. If such a measurement (state preparation) has put the antenna into a coherent state with $\langle \hat{x}(t) \rangle = x_0 \cos \omega t + (p_0/m\omega) \sin \omega t$, then a classical force $F = F_0 \cos(\omega t + \varphi)$ acting for a time $\hat{\tau} >> 2\pi/\omega$ will leave the state coherent but change its amplitudes by $\delta x_0 = (F_0\hat{\tau}/2m\omega) \sin \varphi$, $\delta p_0/m\omega = (F_0\hat{\tau}/2m\omega) \cos \varphi$ (*15*). A subsequent ideal amplitude-and-phase measurement can reveal this change if the force F_0 exceeds the quantum limit (*18*)

$$F_0 \simeq (2/\hat{\tau})(m\omega\hbar)^{1/2} \quad (14)$$

No amplitude-and-phase measurement can do better than this.

An alternative derivation of the quantum limits in Eqs. 13 and 14, due to Giffard (*19*), takes detailed account of quantum fluctuations in the amplifier and their back action on the QRS.

The quantum limits in Eqs. 13 and 14 are traceable to the fact that $\hat{x}$ is not a continuous QND observable; a continuous measurement of $\hat{x}$ produces direct back action on $\hat{p}$, which then contaminates $\hat{x}$ through free evolution. On the other hand, $\hat{x}$ is a stroboscopic QND observable (see Eq. 4a). Consequently, by stroboscopic measurements (*12, 13*) at times $t = 0, \pi/\omega, 2\pi/\omega, \ldots$ one can monitor $\hat{x}$ with perfect precision, in principle (except for the ridiculous limit from relativistic quantum theory, $\Delta x \geq \hbar/mc \simeq 10^{-41}$ cm for $m \simeq 10$ kg). Stroboscopic measurements can be achieved with the system of Eq. 11 by pulsing on and off the transducer's coupling constant K. By a sequence of perfect stroboscopic measurements one can monitor an arbitrarily weak force F_0.

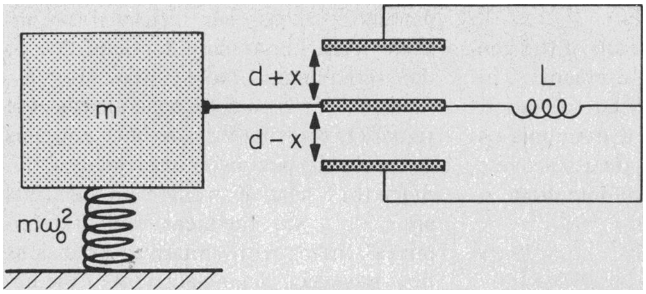

Fig. 1. Scheme for coupling a mechanical oscillator's (position)$^2 \equiv x^2$ to an electromagnetic QRS.

Perfect stroboscopic measurements require that $\hat{x}$ be coupled to the QRS for arbitrarily short time intervals τ at $t = 0$, π/ω, . . . (and also that the QRS transfer its information to the first classical stage in a time less than π/ω). If τ is finite then the momentum spread $\Delta p \geq \hbar/2\Delta x$, produced by a measurement of precision Δx, causes a mean position spread $(\Delta x)_s \simeq (\Delta p/m)\tau \geq \hbar\tau/2m\Delta x$ during the next measurement. The resulting r.m.s. error is (*18, 12, 13*)

$$\Delta x \gtrsim (\hbar\tau/m)^{1/2} \qquad (15)$$

The shorter the measurement time τ, the more accurate the measurement can be.

Unfortunately, short measurements require very strong coupling of the antenna to the QRS in order to surmount the quantum mechanical zero-point energy that accompanies the signal through the QRS and into the amplifier. This is quantified in (*20, 15, 14*) for the case of a mechanical oscillator with transducer and QRS that feed the amplifier a sinusoidal voltage signal

$$V_s = \sqrt{2}\, Kx|g_{21}|\cos \Omega t \qquad (16a)$$

Here Ω (assumed $>> 1/\tau$) is the signal frequency, K is the transducer's coupling constant, and g_{21} is the transfer function of the QRS. This signal carries an r.m.s. power $P_s = (Kx|g_{21}|)^2/4\mathrm{Re}(g_{22})$, where $\mathrm{Re}(g_{22})$ is the real part of the output impedance of the QRS. Accompanying this signal in the experimental bandwidth $1/2\tau$ is a fluctuating quantum mechanical zero-point power $P_{\rm n} = \frac{1}{2}\hbar\Omega/2\tau$—half of it at the known phase of the signal, the other half at the other phase. This zero-point noise leads to $\Delta x \gtrsim (\hbar/2m\omega)^{1/2}\,(1/\beta\omega\tau)^{1/2}$, where

$$\beta \equiv \frac{(K|g_{21}|)^2}{m\omega^2\mathrm{Re}(g_{22}\Omega)} \qquad (16b)$$

is a dimensionless coupling constant (*21, 20*). If one averages over N successive stroboscopic measurements (total bandwidth $1/2N\tau$), then the accuracy improves as $N^{-1/2}$

$$\Delta x \gtrsim (\hbar/2m\omega)^{1/2}\,(1/\beta N\omega\tau)^{1/2} \qquad (17)$$

Optimization of the measurement time τ in Eqs. 15 and 17 leads to an ultimate quantum limit for stroboscopic measurements with finite coupling (*14, 15, 20*):

$$\tau_{\rm optimal} = (\beta N\omega^2)^{-1/2} \qquad (18a)$$

$$\Delta x \gtrsim (\hbar/m\omega)^{1/2}(\beta N)^{-1/4} \qquad (18b)$$

A coupling as large as $\beta = 1$ is difficult to achieve. Therefore, to beat the amplitude-and-phase quantum limit $(\hbar/2m\omega)^{1/2}$, one will probably have to average over a large number N of measurements.

By a sequence of stroboscopic measurements at the quantum limit of Eq. 18b, one can monitor a classical force $F = F_0 \cos(\omega t + \varphi)$. If the phase φ is near $\pi/2$ or $3\pi/2$, then the optimal times for the stroboscopic measurements are $t = 0$, π/ω, $2\pi/\omega$, . . .; the force produces during N half-cycles $\delta x = (\pi/2)(NF_0/m\omega^2)$, and the force is measurable if

$$F_0 \gtrsim (2/\pi)(\hbar m\omega^3)^{1/2}\beta^{-1/4}N^{-5/4} \qquad (19)$$

If the phase φ is near 0 or π, then the precision of Eq. 19 is achieved by stroboscopic measurements at $t = \pi/2\omega$, $3\pi/2\omega$, ... Since the phase of a gravitational wave is not predictable in advance, two antennas are needed; one to be monitored at $t = 0$, π/ω, . . . , the other at $t = \pi/2\omega$, $3\pi/2\omega$, . . . (*12, 13*).

The stroboscopic limits of Eqs. 15, 17, 18, and 19 strictly speaking refer to a harmonic oscillator with only one degree of freedom. Unfortunately, a resonant-bar gravitational-wave antenna has many normal modes which can all be simultaneously perturbed by the back action of each measurement. However, if the strongly perturbed modes have commensurate eigenfrequencies, then stroboscopic QND measurements on the fundamental mode are also QND for the other modes (*12*), and results near the limits of Eqs. 15 to 18 may be achievable.

Stroboscopic measurements can be carried out on electromagnetic oscillators [such as inductance-capacitance (*LC*) circuits] as well as on mechanical oscillators. For example, one could send a collimated pulse of electrons through the capacitor so quickly that it spends a time $\tau << 2\pi/\omega$ between the capacitor plates. The electrons will be deflected by the electric field of the capacitor, which is proportional to the oscillator's generalized coordinate q ($\equiv$ charge on plates); and by measuring the deflection one can infer q. A stroboscopic sequence of such measurements can reveal q, in principle, to an accuracy $(\hbar\tau/L)^{1/2} = (\hbar C\omega^2\tau)^{1/2}$ (see Eq. 15) and can reveal the corresponding voltage in the capacitor to $\Delta V = (\hbar\omega/C)^{1/2}(\omega\tau)^{1/2}$—which is a factor $(\omega\tau)^{1/2}$ better than the standard amplitude-and-phase quantum limit.

Energy Measurements

Suppose that one has developed a method for making accurate measurements of a harmonic oscillator (antenna), and that an initial "state-preparation" measurement has put the oscillator into an eigenstate with energy E_0. A force $F_0 \cos(\omega t + \varphi)$ acting for time $\bar{\tau}$ will change the oscillator's state. Because the phase of the initial state is completely indeterminate, no interference terms show up in the new state's energy expectation value (*22*)

$$\langle E\rangle = E_0 + W \qquad W \equiv F_0{}^2\bar{\tau}^2/8m \qquad (20a)$$

However, interference is a dominant effect in the variance of the new state's energy (*22*)

$$\sigma(E) = (2E_0W)^{1/2} \qquad (20b)$$

The next measurement is likely to reveal a changed energy, and thereby tell us that a force has acted, if $\sigma(E) \gtrsim \hbar\omega$. (Here we assume the force to be weak, $W < E_0$.) Rewritten in terms of F_0, this detection criterion is

$$F_0 \gtrsim \frac{2}{\bar{\tau}}\left(\frac{m\omega\hbar}{n_0 + \frac{1}{2}}\right)^{1/2} \qquad (21)$$

where $n_0 = E_0/\hbar\omega - 1/2$ is the number of quanta in the initial state. This force-detection method can be arbitrarily sensitive if n_0 is made arbitrarily large (*8, 18*). However, because there is no unique relationship between the measured energy and F_0 [$\sigma(E) >> \langle E\rangle - E_0$], this method cannot tell us the precise magnitude of F_0. In other words, the energy is not a QNDF observable (*13, 10, 15*); see the discussion following Eq. 9.

A perfect energy measurement (perfect up to one quantum) is possible only if (i) the interaction Hamiltonian $\hat{H}_{\rm I}$ for the oscillator and QRS involves the oscillator energy $\hat{H}_0$, and (ii) $\hat{H}_{\rm I}$ commutes with $\hat{H}_0$; see Eq. 7 and associated discussion.

If instead $\hat{H}_I = K\hat{x}\hat{q}$, as occurs in most measuring systems, then the directly measured quantity is $\hat{x}(t)$—or the amplitudes $\hat{x}_0$ and $\hat{p}_0$—and the measurement is of the amplitude-and-phase type with quantum limits $\Delta x_0 = \Delta p_0/m\omega \geq (\hbar/2m\omega)^{1/2}$ (Eq. 13). From the measured x_0 and p_0 one can compute the oscillator's energy

$$E_0 = p_0^2/2m + m\omega^2 x_0^2/2 = (n_0 + 1/2)\hbar\omega$$

up to an accuracy, for $n_0 >> 1$

$$\Delta E = p_0 \Delta p_0/m + m\omega^2 x_0 \Delta x_0 \gtrsim n_0^{1/2}\hbar\omega \quad (22)$$

The effect of the force will be discernible if this error is less than $\sigma(E)$ (Eqs. 20), which implies the same force-detection criterion (Eq. 14) as we derived from our original amplitude-and-phase discussion.

One way to achieve an $\hat{H}_I$ which involves $\hat{H}_0$ rather than $\hat{x}$—and to thereby beat the amplitude-and-phase limit (Eq. 14)—is to make the oscillator's mass m and spring constant $m\omega^2$ depend weakly on a variable $\hat{q}$ of the QRS: $m = m_0/(1 + K\hat{q})$, $m\omega^2 = m_0\omega_0^2(1 + K\hat{q})$ (*10*). Then the total Hamiltonian becomes

$$\hat{H} = \hat{H}_0 + \hat{H}_I + \hat{H}_{QRS}$$

$$\hat{H}_0 = \hat{p}^2/2m_0 + \frac{1}{2} m\omega_0^2\hat{x}^2 \quad (23)$$

$$\hat{H}_I = K\hat{q}\hat{H}_0$$

where $\hat{H}_{QRS}$ is the Hamiltonian of the free quantum readout system. Unruh (*10*) has given a pedagogical example of this for an electromagnetic oscillator: the "mass" m is an inductance; the "spring constant" $m\omega^2$ is 1/(a capacitance); and the inductor coil and capacitor plates are attached to a mechanical pivot with angular position $\hat{q}$, which varies the inductance and capacitance in the required manner. One can also vary the inductance and capacitance by letting the mechanical QRS move appropriate materials in and out of the inductor and capacitor (*11*, *13*).

For a mechanical oscillator with electromagnetic QRS there are also several ways to make the mass and spring constant depend on a QRS variable. In Fig. 1 the oscillator's mass is attached to a movable capacitor plate of the QRS; the energy in the capacitor's electric field is

$$E_c = [1 - (\hat{x}/d)^2]\hat{q}^2/2C \quad (24)$$

where C is the capacitance when $x = 0$; and consequently the charge $\hat{q}$ on the central plate of the QRS renormalizes the spring constant to $m\omega^2 = m_0\omega_0^2 - \hat{q}^2/2Cd^2$. The spring constant can also be renormalized by the QRS "momentum" $\hat{\pi}$ ($\equiv$ flux in inductor) by attaching the oscillator's mass to a current-carrying coil that resides between two oppositely wound coils of the QRS inductor. To renormalize the oscillator's mass m one might attach to it a conducting plate that resides in the inductor's magnetic field. The velocity of the plate through the magnetic field would induce an electric dipole moment on the plate, which in turn would couple by its velocity to the magnetic field, giving an interaction energy proportional to $\hat{p}^2\hat{\pi}^2$ and thence a mass renormalization.

Unfortunately, these various ideas have not yet produced a viable design for clean coupling of a mechanical oscillator's energy $\hat{H}_0$ to a QRS. On the other hand, designs without clean coupling can still yield measurements of $\hat{H}_0$ more accurate than the amplitude-and-phase quantum limit $(n_0 + 1/2)^{1/2}\hbar\omega$. An example is a QRS that couples only to $\hat{x}^2$, but that averages $\hat{x}^2$ over a number of cycles before sending it into the first classical stage (amplifier) (*9*, *11*). The measurement scheme of Fig. 1 will do this if the period of the circuit's (QRS) oscillations is much longer than the period of the mechanical oscillations. Then the circuit's capacitance (Eq. 24) and resonant frequency will be sensitive to the time average of $\hat{x}^2$ and thence to $\hat{H}_0$, with only small admixtures of sensitivity to the time-varying part of $\hat{x}^2$ and thence to the oscillator's phase $\hat{\psi}$. This is equivalent to the statement in Eq. 6 that $\hat{Q}_R = f(\hat{H}_0 + \alpha\langle\hat{H}_0\rangle\hat{\psi})$ with $\bar{\alpha} << 1$, which in turn permits accuracies much better than $\Delta E = (n_0 + 1/2)^{1/2}\hbar\omega$. A detailed analysis of this type of scheme is given in (*11*), but for an electromagnetic oscillator with a mechanical QRS and with $\hat{H}_I = K\hat{x}^2\hat{q}$ rather than $K\hat{x}^2\hat{q}^2$ as in Fig. 1 and Eq. 24. That analysis reveals a limiting sensitivity

$$\Delta E \gtrsim \left(n_0 + \frac{1}{2}\right)^{1/2} (\Omega/\omega)^{1/2}\hbar\omega \quad (25a)$$

where $E = (n_0 + 1/2)\hbar\omega$ is the oscillator's energy, ω is its frequency, Ω is the frequency of the QRS, and $\Omega << \omega$. The corresponding limit on the detection of a classical force $F_0 \cos(\omega t + \varphi)$, which drives changes in the oscillator's energy, is

$$F_0 \gtrsim \frac{2}{\hat{\tau}} \left(\frac{m\omega\hbar}{\omega/\Omega}\right)^{1/2} \quad (25b)$$

if $\omega/\Omega < n_0 + 1/2$. If $\omega/\Omega > n_0 + 1/2$, then the limit on ΔE in Eq. 25a is replaced by $\hbar\omega$, the ultimate precision with which one can ever measure energy changes; correspondingly, the force limit in Eq. 25b is replaced by Eq. 21.

In measurements of the time average of $\hat{x}^2$ and thence $\hat{H}_0$, it is not essential that the interaction Hamiltonian $\hat{H}_I$ involve $\hat{x}^2$. Instead $\hat{H}_I$ can be proportional to $\hat{x}$, and then the internal workings of the QRS can produce the average of $\hat{x}^2$ at the entrance to the first classical stage.

Back-Action-Evading Measurements of $\hat{X}_1$

The QNDF observable $\hat{X}_1 = \hat{x} \cos \omega t - (\hat{p}/m\omega) \sin \omega t$ (real part of complex amplitude; Eq. 5), like the position $\hat{x}$, has a continuous spectrum of eigenvalues; and in principle it can be measured arbitrarily quickly and accurately (*13*, *15*). Suppose that an initial state-preparation measurement at $t = 0$ has put the oscillator into an eigenstate $|\xi_0\rangle$ of $\hat{X}_1(0)$ with eigenvalue ξ_0. A classical force $F(t)$ [total Hamiltonian $\hat{H} = \hat{H}_0 - \hat{x}F(t)$] will change $\hat{X}_1$ as seen in the Heisenberg picture

$$\hat{X}_1(t) = \hat{X}_1(0) - \int_0^t [F(t')/m\omega] \sin \omega t' dt' \quad (26)$$

In the Heisenberg picture the oscillator's state remains fixed in time at $|\xi_0\rangle$, but this is an eigenstate of $\hat{X}_1(t)$ with eigenvalue

$$\xi(t) = \xi_0 - \int_0^t [F(t')/m\omega] \sin \omega t' dt' \quad (27a)$$

Subsequent perfect measurements of $\hat{X}_1$ must yield this eigenvalue and will reveal the full details of its evolution. It evolves in exactly the same manner as X_1 would evolve for a classical oscillator (*13*, *15*).

One pays the price, in these measurements, of not knowing anything about the imaginary part of the complex amplitude $\hat{X}_2$ (Eq. 5c). However, if one has a second oscillator coupled to the same force $F(t)$, one can measure the imaginary part $\hat{Y}_2$ of its complex amplitude, giving up all information about the real part $\hat{Y}_1$. One's measurements must give the eigenvalue

$$\eta(t) = \eta_0 + \int_0^t [F(t')/m\omega] \cos \omega t' dt' \quad (27b)$$

which evolves in exactly the same manner as the X_2 or Y_2 of a classical oscillator. From the output of either oscillator, or better from the two outputs, one can deduce all details of the evolution of $F(t)$, no matter how weak $F(t)$ may be (*13*, *15*).

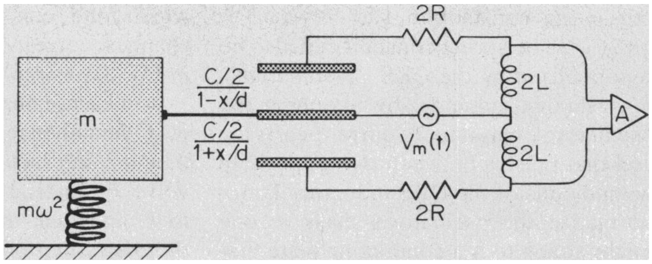

Fig. 2. Scheme for stroboscopic or continuous back-action-evading measurements of a mechanical oscillator. This scheme was devised independently in 1978 by V. B. Braginsky and by R. W. P. Drever, but has not previously been published.

Thus $\hat{X}_1$ and $\hat{Y}_2$ are QNDF observables.

A perfect measurement of $\hat{X}_1$ (or $\hat{Y}_2$) requires (i) that the interaction Hamiltonian $\hat{H}_I$ depend on $\hat{X}_1$ and (ii) that $\hat{H}_I$ commute with $\hat{X}_1$ (Eq. 7 and associated discussion). The simplest example is

$$\hat{H}_I = K\hat{X}_1\hat{q} = K\hat{x}\hat{q}\cos\omega t - (K/m\omega)\hat{p}\,\hat{q}\sin\omega t \quad (28)$$

A coupling of this type can be achieved, for a mechanical oscillator, by using a capacitive position transducer with sinusoidally modulated coupling constant ($\hat{H}_I = K\hat{x}\hat{q}\cos\omega t$), followed by an inductive momentum transducer with modulated coupling constant [$H_I = -(K/m\omega)\hat{p}\hat{q}\sin\omega t$]. The two transducers together produce a voltage output

$$\begin{aligned}\hat{V} &= \partial\hat{H}_I/\partial\hat{q} \\ &= K\hat{x}\cos\omega t - (K/m\omega)\hat{p}\sin\omega t \\ &= K\hat{X}_1 \end{aligned}\quad (29)$$

which drives an electromagnetic circuit, the QRS, in which the charge $\hat{q}$ flows (*15*). While capacitive position transducers and inductive velocity transducers are easy to construct, inductive momentum transducers are not. The momentum and velocity of the oscillator are related by

$$\begin{aligned}\dot{x} &= \partial(H_0 + H_I)/\partial p \\ &= p/m - (K/m\omega)q\sin\omega t\end{aligned} \quad (30a)$$

which means that the classical Lagrangian $L = p\dot{x} - (H_0 + H_I)$ for oscillator plus transducers is

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\omega^2x^2 - Kxq\cos\omega t + (K/\omega)\dot{x}q\sin\omega t + \frac{1}{2}m(K\sin\omega t/m\omega)^2q^2 \quad (30b)$$

The first two terms represent the oscillator, the third is the capacitive position transducer, the fourth is an inductive velocity transducer (wire, physically attached to oscillator, moves through external magnetic field), and the last is a negative capacitor in the QRS circuit. Thus, an inductive momentum transducer is equivalent to an inductive velocity transducer (easy to construct) plus a negative capacitor (hard) (*15*). Although negative capacitors are not standard electronic components, they can be constructed in principle, and in principle they can be noise-free (*15*).

For an electromagnetic oscillator with mechanical QRS, one can achieve the desired $\hat{H}_I = K\hat{X}_1\hat{q}$ using a capacitive transducer for the oscillator's position $\hat{x}$ (≡ charge in oscillator's capacitor) and an inductive transducer for its momentum $\hat{p}$ (≡ flux in oscillator's inductor). The momentum transducer turns out to involve a standard mechanical current transducer (current ≡ $\dot{x}$) plus a negative spring in the QRS (*15*). In principle negative springs can be noise-free (*15*).

The sinusoidal modulations required in the transducers must be regulated by an external, classical clock, which has the same frequency ω as one's oscillator. One cannot use the oscillator itself as the clock because in extracting the required oscillatory information from the oscillator one will produce an unacceptably large back action on $\hat{X}_1$. However, before the experiment begins one can check the frequency of the clock against that of the oscillator. In principle they can be made to agree perfectly, and in principle the clock can be made fully classical so its outputs are real numbers, $\cos\omega t$ and $\sin\omega t$, rather than operators (*10, 15*). In practice, frequency drifts and quantum features of the clock need not cause serious experimental problems (*15, 20*).

A perfect measurement of $\hat{X}_1$, which lasts a finite time $\bar{\tau}$, requires infinitely strong coupling in the transducers ($K \to \infty$) in order to give a signal that overwhelms zero-point noise in the QRS. If one has only finite coupling, then the zero-point noise accompanying the signal gives rise to a limit (Eqs. 16 and 17 with $x \to X_1$ and $N\tau \to \bar{\tau}$) (*13-15*)

$$\Delta X_1 \gtrsim (\hbar/2m\omega)^{1/2}(\beta\omega\bar{\tau})^{-1/2} \quad (31)$$

Here β is the dimensionless coupling constant (Eqs. 16). Thus, whereas stroboscopic measurements with limited coupling can beat the amplitude-and-phase limit by a factor of only $(\beta\omega\bar{\tau})^{-1/4}$ (Eq. 18b with $N = \omega\bar{\tau}/\pi$), continuous back-action-evading measurements of X_1 can beat it by $(\beta\omega\bar{\tau})^{-1/2}$. Stroboscopic measurements are worse because of their smaller duty cycle.

In the realistic case of weak coupling, $\beta < 1$, one must average over many cycles ($\omega\bar{\tau} >> 1/\beta$) in order to substantially beat the amplitude-and-phase limit. In this case one can make use of a trick analogous to measuring the energy by coupling to $\hat{x}^2$ and averaging: one can perform a "single-transducer, back-action-evading measurement" (*13-15*) by coupling to

$$\hat{x}\cos\omega t\cos\Omega t = \frac{1}{2}(\hat{X}_1 + \hat{X}_1\cos 2\omega t + \hat{X}_2\sin 2\omega t)\cos\Omega t \quad (32)$$

(that is, $\hat{H}_I = 2K\hat{x}\hat{q}\cos\omega t\cos\Omega t$) and then sending the signal through a filter (the QRS) with band pass at frequency $\Omega >> \omega$ and bandwidth $\Delta f = 1/2\tau_* << \omega/2\pi$. The filter will "average the X_2 signal away" until its amplitude has fallen by $1/2\omega\tau_*$ relative to that of the X_1 signal. Since the initial r.m.s. X_2 signal strength is $1/\sqrt{2}$ that of the X_1, this corresponds to $\hat{Q}_R = f(\hat{X}_1 + \hat{X}_2/2\sqrt{2}\omega\tau_*)$ in Eq. 6a, which together with the uncertainty relation in Eq. 5c and the argument of Eqs. 6 tells us that (*20*)

$$\Delta X_1 \gtrsim (\hbar/2m\omega)^{1/2}(2\sqrt{2}\omega\tau_*)^{-1/2} \quad (33)$$

This is the error in X_1 due to back action from measurement of X_2. The additional error due to zero-point noise accompanying the X_1 signal into the amplifier is (Eqs. 16 and 17 with $x \to X_1$ and $N\tau \to \bar{\tau}$) (*14, 15, 20*)

$$\Delta X_1 \gtrsim (\hbar/2m\omega)^{1/2}(\beta\omega\bar{\tau})^{-1/2} \quad (34)$$

Here $1/2\bar{\tau}$ is the bandwidth of the experiment ($\bar{\tau}$ is the larger of the QRS averaging time τ_*, and the averaging time in subsequent electronics). The ultimate quantum limit on the sensitivity is Eq. 33 if $\beta > 2\sqrt{2}\tau_*/\bar{\tau}$, and Eq. 34 if $\beta < 2\sqrt{2}\tau_*/\bar{\tau}$. Note that Eq. 34 is the same limit (to within factors of order unity) as in the case of exact coupling to $\hat{X}_1 = \hat{x}\cos\omega t - (\hat{p}/m\omega)\sin\omega t$. Thus, when $\beta < 2\sqrt{2}\tau_*/\bar{\tau}$ and $\omega\bar{\tau} >> 1$, one can abandon the momentum transducer without any serious loss of accuracy.

This type of single-transducer, time-averaged, back-action-evading measurement of X_1 appears today to be the most viable technique for beating the amplitude-and-phase limit (Eq. 14) in gravitational-wave detection. In place of Eq. 14 one will face the limiting measurable force

$$F_0 \gtrsim (2/\bar{\tau})(m\omega\hbar)^{1/2} \times \text{Max}\,[(\beta\omega\bar{\tau})^{-1/2}, (2\sqrt{2}\omega\tau_*)^{-1/2}] \quad (35)$$

Thermal Noise in the Oscillator and Amplifier

The quantum limits derived above are not achievable in the laboratory today because thermal noise exceeds quantum mechanical noise.

Ignore for the moment thermal (Nyquist) noise in the oscillator. Then if the resistors in the QRS are cooled sufficiently, the dominant nonquantum noise will be that in the amplifier (first classical stage). The amplifier, which we assume to be linear, can be characterized by its power gain G and its noise temperature T_n. The QRS feeds the amplifier a signal at frequency $f = \Omega/2\pi$, to which the amplifier adds a noise power per unit bandwidth

$$\frac{dP_n}{df} = \frac{\hbar\Omega}{\exp(\hbar\Omega/kT_n) - 1} \quad (36)$$

Here k is Boltzmann's constant. If the incoming signal has power P_s, then the amplified signal and noise have power (23)

$$GP_s + \left(G\,\frac{\hbar\Omega}{\exp(\hbar\Omega/kT_n) - 1} + \frac{\hbar\Omega}{2}\right)\Delta f \quad (37)$$

Here Δf is the bandwidth, and the $\hbar\Omega/2$ is a zero-point energy that accompanies the signal throughout its trek through the amplifier and other electronics, but does not get amplified (23). The quantum limits of previous sections of this article are attributable to this zero-point energy. In the presence of a real linear amplifier, with nonnegligible noise temperature $T_n >> \hbar\Omega/k$ and large gain $G >> 1$, the signal power P_s must fight not $(\hbar\Omega/2)\Delta f$, but rather

$$\left(\frac{\hbar\Omega}{\exp(\hbar\Omega/kT_n) - 1} + \frac{1}{G}\,\frac{\hbar\Omega}{2}\right)\Delta f \simeq kT_n\Delta f \quad (38)$$

(see Eq. 37). Consequently, it is reasonable to expect that the amplifier noise will modify our quantum limits (Eqs. 6c, 13-15, 17-19, 21, 22, 25, 31, and 33-35) by replacing $\hbar$ with (12)

$$\hbar \rightarrow \frac{2kT_n}{\Omega} \quad (39)$$

These modified quantum limits are sometimes called amplifier limits. It will never be possible, even in principle, to reduce these amplifier limits below the corresponding quantum limits (19, 24, 25).

The best linear amplifiers that have been built are parametric amplifiers and maser amplifiers, which operate at microwave frequencies and have (kT_n/Ω) as small as $\sim 10\hbar$. With such amplifiers one can only hope to get within a factor 20 or $\sqrt{20}$ of our quantum limits ($\hbar \rightarrow 20\hbar$). And even to achieve this one must design a QRS which upconverts the oscillator's signal frequency (kilohertz in the gravitational-wave case) to the microwave (gigahertz) region.

Any physical oscillator (such as the fundamental mode of a gravitational-wave bar antenna) is weakly coupled to a thermal bath of dynamical systems (sound waves in the bar). This coupling produces a frictional damping of large-amplitude motions, and it also produces a thermal-buffeting random walk of the oscillator's amplitude (Nyquist noise). The r.m.s. random-walk change of the oscillator's amplitude during time $\bar{\tau}$ is

$$(\Delta x_0)_{Nyq} = (\Delta p_0/m\omega)_{Nyq} = (\Delta X_1)_{Nyq} = (\Delta X_2)_{Nyq} \simeq (kT/m\omega^2)^{1/2}(\omega\bar{\tau}/Q)^{1/2} \quad (40)$$

Here T is the temperature of the thermal bath (the bar's temperature), and Q is the oscillator's quality factor (number of radians of oscillation required for frictional damping of large-amplitude oscillations by a factor e in energy). The corresponding r.m.s. energy change is

$$(\Delta E)_{Nyq} \simeq (2E_0kT)^{1/2}(\omega\bar{\tau}/Q)^{1/2} \quad (41)$$

These Nyquist noises must not exceed the amplifier limits (quantum limits with $\hbar \rightarrow 2kT_n/\Omega$) if one is to achieve the amplifier limits in real experiments. Some numbers will be given below.

Prospects for Stroboscopic Measurements

One possible scheme for stroboscopic measurements of a mechanical oscillator (gravitational-wave antenna with mass $m \simeq 10$ kg and frequency $\omega \simeq 3 \times 10^4$ sec^{-1}) is shown schematically in Fig. 2. The mass of the oscillator is physically attached to the central, movable plate of a capacitor (capacitance between outer plates = C), which plays the role of transducer. The capacitor resides in the QRS—a high-frequency LC circuit [frequency $\Omega = (LC)^{-1/2} \simeq 10^{10}$ sec^{-1}], which has small losses [amplitude damping time $\tau = 2(RC\Omega^2)^{-1} << 0.1/\omega$] and which is driven at its resonant frequency Ω by an external generator. In practice this circuit would be a microwave cavity (26). At the measurement times $\omega t = 0, \pi, 2\pi, \ldots$ the generator is turned on for a time $\tau/2$ and then turned off, and in an additional time $\tau/2$ the excitations in the circuit die out. During the brief on-time τ, the amplifier sees a voltage signal $V_s = (V_0/d)\,\Omega\tau x \cos \Omega t$, where V_0/d is the amplitude of the oscillating electric field between the capacitor plates. The experimenter averages the amplitude of this signal (with alternating sign) over N measurements to determine the position x of the oscillator.

It is straightforward to analyze the noise performance of this system by standard circuit theory. Alternatively, one can invoke the general formulas of Eqs. 15 to 19 for stroboscopic measurement schemes. Assuming that the resistor's physical temperature is less than the amplifier's noise temperature $T_n \simeq 10$ K, the amplifier noise dominates and in Eq. 18b we must replace $\hbar \rightarrow 2kT_n/\Omega$. Assuming that the amplifier is properly impedance-matched to the circuit, the measurement will achieve the limiting precision (Eq. 18b)

$$\Delta x \simeq \left(\frac{2kT_n/\Omega}{m\omega}\right)^{1/2}(\beta N)^{-1/4} \quad (42)$$

Comparison of the voltage signal with Eq. 16a reveals that $K|g_{21}| = (V_0/\sqrt{2}d)(\Omega\tau)$; scrutiny of Fig. 2 reveals that the QRS output impedance, as seen by the amplifier, is $g_{22} = 2\tau/C$; consequently the dimensionless coupling constant of Eq. 16b is $\beta = (V_0/d)^2C\Omega\tau/(4m\omega^2)$. Combining this with the required pulse time $\tau = (\beta N\omega^2)^{-1/2}$ (Eq. 18a), we find

$$\beta N = \left[\frac{(V_0/d)^2CN\Omega}{4m\omega^3}\right]^{2/3} \quad (43)$$

To avoid voltage breakdown in the capacitor, its electric field amplitude should not exceed $(V_0/d) \simeq 10^6$ volt/cm. Assuming other reasonable parameters $C \simeq 1$ pf, $\Omega \simeq 10^{10}$ sec^{-1}, $N \simeq 1000$, $T_n \simeq 10$ K, $m \simeq 10$ kg, and $\omega \simeq 3 \times 10^4$ sec^{-1}, we find

$$\beta N \simeq 20 \qquad \Delta x \simeq 1 \times 10^{-17} \text{ cm} \quad (44)$$

Thus this system can achieve a sensitivity that is a factor $(20)^{1/4} \simeq 2.1$ below the amplitude-and-phase amplifier limit; but this is still an order of magnitude worse than the amplitude-and-phase quantum limit $(\hbar/2m\omega)^{1/2} \simeq 1 \times 10^{-18}$ cm.

Nyquist noise in the antenna (Eq. 40 with $\omega\bar{\tau} = \pi N$) will be less than the measurement precision $\Delta x \simeq 1 \times 10^{-17}$ cm if the antenna is cooled to 4 K and has a quality factor $Q \simeq 4 \times 10^9$. This is comparable to the best mechanical Q that has been achieved (27) for a sapphire crystal at 4 K.

Prospects for Single-Transducer Back-Action-Evading Measurements

The configuration of Fig. 2 can also be used in a single-transducer, back-action-evading measurement of $\hat{X}_1$. In this case the circuit's amplitude damping time $2(RC\Omega^2)^{-1}$ becomes the averaging time τ_* of the QRS filter (previously it was the stroboscopic pulse length), and we require $\tau_* >> 1/\omega$ (previously it was $<< 1/\omega$). Instead of being pulsed, the generator's modulating voltage has the steady-state form $V_m = U_0 \sin \Omega t \sin \omega t$, which produces an electric field $(V_0/d) \cos \Omega \cos \omega t$ in the capacitors ($V_0 = U_0\Omega/2\omega$). That electric field, interacting with the motions $x = X_1 \cos \omega t + X_2 \sin \omega t$ of the mechanical oscillator, produces a signal voltage

$$V_s = (V_0/d)(\Omega\tau_*/2)\,[X_1 \sin \Omega t + (2\omega\tau_*)^{-1}X_1 \sin \Omega t \sin 2\omega t - (2\omega\tau_*)^{-1}X_2 \sin \Omega t \cos 2\omega t] \quad (45)$$

at the output of the QRS. Amplification of this signal produces information about X_1 and X_2 with relative accuracies $\Delta X_1 = (2\sqrt{2}\omega\tau_*)^{-1}\,\Delta X_2$.

Assuming that the resistor noise is negligible compared to amplifier noise (which it will be if the physical temperature of the resistors is somewhat less than the noise temperature $T_n = 10\ K$ of the amplifier), we can compute the noise performance of this system from Eqs. 16, 33, and 34 with $\hbar \rightarrow 2kT_n/\Omega$. The best quality factor that has been achieved (26) for a superconducting microwave resonator (our QRS circuit) with a narrow capacitive gap is $Q_e = \Omega\tau_*/2 \simeq 10^7$, corresponding to $\tau_* \simeq 10^{-3}$ second. Consequently back-action forces (Eq. 33) limit the sensitivity to

$$\Delta X_1 \simeq \left(\frac{kT_n/\Omega}{m\omega}\right)^{1/2} \frac{1}{(2\sqrt{2}\omega\tau_*)^{1/2}} \simeq 2 \times 10^{-18} \text{ cm} \quad (46)$$

a factor 9 below the amplitude-and-phase amplifier limit and approximately twice the amplitude-and-phase quantum limit. (Here we use $T_n = 10$ K, $\Omega = 10^{10}$ sec^{-1}, $m = 10$ kg, and $\omega = 3 \times 10^4$ sec^{-1} as before.) In order that Nyquist noise in the mechanical oscillator (Eq. 40) not exceed this sensitivity, the averaging time must not exceed $\bar{\tau} \simeq 0.01$ sec. (Here we use the same oscillator temperature and Q as before, $T = 4$ K and $Q = 4 \times 10^9$.) To achieve the limit in Eq. 46 we also require a coupling constant $\beta \simeq 2\sqrt{2}\tau_*/\bar{\tau} \simeq 0.3$ (Eq. 34). To compute β, first derive $K|g_{21}| = (V_0/d)(\Omega\tau_*/2\sqrt{2})$ from Eqs. 16a and 45 with $x \rightarrow X_1$; then evaluate the impedance seen by the amplifier in Fig. 2 at the X_1 signal frequency $\Omega = (LC)^{-1/2}$, $g_{22} = 2\tau_*/C$; then evaluate Eq. 16b

$$\beta = (V_0/d)^2 C\Omega\tau_*/(16m\omega^2) \quad (47)$$

The required β of 0.3 can be achieved with the same electric field in the capacitive gap as we used before: $V_0/d = 1 \times 10^6$ V/cm.

This example and that of the last section confirm that it is easier to achieve a given level of sensitivity by continuous, single-sensor, back-action evasion than by stroboscopic techniques. However, along the route toward realization of such experiments there remain a series of difficult experimental problems—not least of which is the frequency stability of the clock that regulates the voltage generator.

On the Limiting Frequency Stability of a Generator

Although current technology can achieve the frequency stability required by the above examples, it is of longer term interest to know ultimate quantum mechanical limits on the stabilities of clocks.

At present the world's most stable clocks are the superconducting cavity stabilized oscillator (SCSO) (28) and the hydrogen maser (29). Both involve self-excited electromagnetic oscillations inside a cavity. In the SCSO the clock frequency Ω is regulated by the cavity's normal mode, and a change Δl of a typical dimension l of the cavity will produce a frequency change

$$\Delta\Omega/\Omega \simeq \Delta l/l \quad (48)$$

In the maser, if the electromagnetic quality factory Q_e of the cavity (Teflon bubble) exceeds $\frac{1}{2}\Omega \times$ (mean time hydrogen atoms spend in cavity) $\equiv \Omega_a$, then Eq. 48 will be true. Otherwise, $\Delta\Omega/\Omega \simeq (\Delta l/l)(Q_e/Q_a)$, and the limit derived below is correspondingly modified.

A quantum limit on the frequency stability of any electromagnetic oscillator satisfying Eq. 48 is derived in (30). The source of the limit is quantum fluctuations in the deformation of the cavity walls by electromagnetic stresses. Since the stresses in the electromagnetic field are equal to its energy density $\hat{H}_e/l^3$ (with $\hat{H}_e$ the Hamiltonian of the electromagnetic oscillator), the force on the walls is $\hat{H}_e/l$, and this deforms the walls by $\delta l = \hat{H}_e/lk$, where k is the mechanical spring constant of the walls. The electromagnetic field is in a thermalized coherent state with n_0 quanta, which possesses quantum fluctuations $\Delta H_e \geq n_0^{1/2}\hbar\Omega$; consequently, $\Delta l \geq n_0^{1/2}\hbar\Omega/kl$, which leads to frequency fluctuations (Eq. 48)

$$\Delta\Omega/\Omega \gtrsim n_0^{1/2}\hbar\Omega/kl^2 \quad (49)$$

This electromagnetic back-action limit must be contrasted with the limiting precision for measurements of Ω during an averaging time $\bar{\tau}$: $\Delta\Omega \gtrsim \Delta\psi/\bar{\tau}$, where $\Delta\psi \gtrsim n_0^{-1/2}$ is the quantum uncertainty in the phase of the oscillator's coherent state

$$\Delta\Omega/\Omega \gtrsim n_0^{-1/2}(\Omega\bar{\tau})^{-1} \quad (50)$$

(Townes-Schawlow limit). These two limits lead to an optimal number of quanta n_0 and an ultimate quantum limit

$$\frac{\Delta\Omega}{\Omega} \gtrsim \left(\frac{\hbar}{kl^2\bar{\tau}}\right)^{1/2} \quad (51)$$

For a cavity with wall thickness comparable to cavity dimensions l, or for a "cavity" made by coating the outside of a dielectric crystal with superconducting material (31), the spring constant k is related to the Young's modulus E_M of the cavity walls by $k \simeq E_M V/l^2$, where V is the cavity volume. Then

$$\Delta\Omega/\Omega \gtrsim (\hbar/E_M V\bar{\tau})^{1/2} \quad (51')$$

In practice $E_M \lesssim 10^{13}$ dyne/cm^2, $V \simeq 1$ cm^3, so $\Delta\Omega/\Omega \gtrsim 10^{-20}$ $(\bar{\tau}/1 \text{ sec})^{-1/2}$. This limit is achievable in principle, but current technology is far from it.

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Arctic Oceanic Climate in Late Cenozoic Time

Yvonne Herman and David M. Hopkins

Summary. Faunal and lithologic evidence is used to reconstruct paleoceanographic events over the last 4.5 million years. The inception of perennial sea-ice cover is dated at about 0.7 million years.

Several significant global climatic and tectonic events affected the evolution of Arctic oceanic climates during late Cenozoic time. The terminal Miocene temperature decline, culminating in the major expansion of the Antarctic ice sheet, and the concomitant worldwide lowering of sea level that resulted in the isolation of the Mediterranean Sea about 5 million years ago (*1*) may have been synchronous with the onset of glaciation at high latitudes in the Northern Hemisphere. At that time, the Bering land bridge completely isolated the Arctic from the Pacific Ocean (*2-5*), and because circulation in the world ocean was primarily latitudinal, there was probably little interchange between Atlantic and Arctic waters (*6*).

The sudden appearance of a flood of Pacific mollusks in Iceland indicates that the Bering land bridge was breached about 3.5 million years ago (*7, 8*). These events coincided approximately with the emergence of the Isthmus of Panama, completed about 3 million years ago (*9*), which resulted in a salinity contrast between salty North Atlantic and somewhat fresher North Pacific surface waters (*10*) and in the reorientation of oceanic circulation to a more vigorous south-north current. The Gulf Stream as we know it today may have evolved at about this time (*6*): The overall consequence of these events was the persistent influx into the western part of the Arctic Ocean basin of low-salinity North Pacific water through the Bering Strait (*11*) and the influx of a much larger volume of salty Atlantic water into the eastern part of the Arctic basin by way of the Norwegian Sea (*10*).

This change in oceanic circulation led to intensified atmospheric circulation. Increased transport of moist air over an open and relatively warm North Atlantic Ocean to adjoining subpolar highlands evidently resulted in the episodic development of local ice sheets large enough to lower sea level as much as 40 meters as early as 3.4 million years ago (*12*). The situation is somewhat analogous to the abortive high-latitude glaciation recorded about 115,000 years ago during an early phase of the last glaciation (Wisconsin/Würm) (*13*). Continental ice sheets at least two-thirds as large as those of the late Pleistocene developed about 2.4 million years ago, shortly after the beginning of the Matuyama reversed epoch (*12, 14*). Latitudinal temperature gradients increased gradually, and by late Pliocene time the modern marine faunal provinces were established (*15*).

Sedimentary Record in the Arctic Basin

The continuous sedimentary record representing roughly the last 4.5 million years is preserved in deep-sea cores raised from bathymetric highs by the Lamont-Doherty Geological Observatory (LDGO) from ice platforms drifting over the central part of the Arctic basin (Fig. 1 and Table 1). Despite certain ambiguities, the radiometric dates (*16*) and magnetic stratigraphy (*17*) of these cores together with biostratigraphic and lithologic correlations (*18-20*) provide control points for the time framework of this Pliocene and Pleistocene sequence (Fig. 2). Three major climatic regimens, here represented by three stratigraphic units, can be recognized within this time interval. Rates of sedimentation were very low (1 to 3 millimeters per 1000 years) in all three units.

The oldest unit (unit III) comprises sediments deposited between approximately 4.5 and 2.5 million years ago and consists of fairly well sorted red clays containing manganese and micronodules. The botryoidal micronodules, which constitute up

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Quantum Nondemolition Measurements

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