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Call market book information and efficiency

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Abstract

What are the consequences of making bids and offers in the book available to traders in a call market? This is a problem in market design. We employ a computational mechanism design methodology to attack this problem and find that allocative efficiencies are higher in a closed book design. We validate our computational approach by running a series of tests with human subjects in exactly the same environments.

JEL classification: D83; C63; C92; D44

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1. Introduction

In designing markets, one must make many choices. Some are large, such as: should one deploy a continuous auction or a call market? Some are small, such as: if one deploys a call market should the book be open or closed? There are three basic scientific approaches one might use to answer this type of question: theoretical, experimental, or computational. What differentiates these approaches is the basic

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model of the participant. In theoretical market design, the rational agent model is used and much of the research has relied on equilibrium game theoretic models. An example of this type of research applied to call markets can be found in Satterthwaite and Williams (1993). In experimental market design, the model is itself a human – the subject in the experiments.[1] Examples of this type of research applied to call markets can be found in McCabe et al. (1993) and in Friedman (1993). Here, we use the third approach, computational mechanism design, where the model of the agent is a computer program, a 'smart agent.'

We study call markets. A call market accumulates bids and offers from traders for a period of time and then, in a batch process, clears them at a uniform price. We study such a market institution in a repeated environment. That is, traders participate in a series of call markets in the same environment, developing experience and learning as they go. We are interested in the effect that different information treatments have on allocative efficiency. We study two extreme cases. In one, which we call the *open book*, traders are provided information after each call about all the bids and offers that were processed in that call. In the other, which we call the *closed book*, traders are provided information after each call only about the clearing price and whether they traded or not. In both cases, no information is provided during the accumulation of bids and offers before a call.

We were able to find two previous studies that considered the effect of market information on a call market. In both cases, subjects interacted repeatedly but the environment was changed (new values and costs were provided) after each call. Also both studies simultaneously varied other features of the call market design and so their results were not as clear cut as one might wish. McCabe et al. (1993) provided information to subjects as bids accumulated before each call. In particular, they posted a standing bid Pb and standing ask Pa. In their closed book design, no other information was provided to subjects either before or after the call. In their open book design, subjects saw all the bids and offers as bids accumulated for the call. They found (1) when using the 'other side update rule',[2] a closed book produces greater trading efficiency than the open book, (2) when using the 'both sides update rule', open book auctions produce greater trading efficiency than closed book auctions, and (3) when using the 'other side updater rule' with the endogenous close, experienced subjects produce higher trading efficiencies with the open book than without it, but inexperienced subjects produce the opposite result. Friedman (1993) is a comprehensive, multi-dimensional study of various continuous double auction and call-market designs. Assets (commodities with random values) are traded. This is similar to randomizing payoffs in each new period. One of his designs involves a call market with exactly our closed market design. No information is provided either

[1]There are at least two reasons why lab subjects are only a 'model' of those who would actually end up using the markets being designed. First, those using the mechanism might be more experienced or focused than the subjects. Second, in the lab one is testing scaled down versions of the mechanisms and environments so that the subjects could have cognitively easier decisions. Of course, these differences work in opposite directions.

[2]Updater rules have to do with whether a new bid is accepted into the book during the accumulation phase of the market. The details of these are not important for this paper.

during accumulation or between calls. Another design is a call market in which a continuously updated summary description of the book was visible during the accumulation period – five near marginal orders on each side were displayed.[3] Among his designs, this is the closest to our open book design. Friedman found efficiencies of 94.4 for the closed book design and 95.3 for the other. The t-statistic for the comparison, however, was only 0.42 suggesting little difference in the institutions.

We keep our comparison simple. Using a computational mechanism design methodology, we discover that providing information about the book to traders between calls lowers allocative efficiency. In a very real sense, information creates noise. One might argue that the reason for our findings is the particular computer agents we are using and not the result of the alternative designs of the markets. The only a priori constraint on ones choice of possible computer agents is computational complexity and memory. Thus, it would be pretty easy to first decide what result one wanted and then to build agents that would produce that result. This is neither scientific nor informative. So, we ran series of economics experiments that replicated both the environments and the market designs. The patterns of bids and offers, the behavior of price volatility, and the allocative efficiencies produced in the laboratory experiments were similar to those produced by our agents, thereby validating our computational approach.

Our investigation into call markets is part of a larger research effort that is attempting to create a valid computational testbed for mechanism design, a collection of agents that will provide results similar to those from laboratory experiments with human subjects. We began in Arifovic and Ledyard (2003) by considering a public goods environment and a class of Groves–Ledyard mechanisms. We were able to create a valid testbed for that situation. But if a testbed is to be at all useful, it must match laboratory performance over a wide range of environments and a wide range of mechanisms. In this paper, we test the limits of our previous testbed by using it, without modification, to evaluate market designs in several private goods environments. The results we report here suggest it passes this test.

We begin by describing a class of environments and how the basic call market operates in these environments. We then introduce our testbed. The rest of the paper reports on, and compares, the results of simulations in the testbed and laboratory experiments.

2. Environments and call markets

We describe two call market designs and the simple private goods environments we use for testing the performance of those designs. The environments are kept as simple as possible.

[3]Friedman calls the first design with no information Book 0 and the second design Book 2. Book 1 provided a continuously updated provisional price, the price that would obtain if the market were called now, during the accumulation.

2.1. The environments

The environments have a fixed number of buyers, N, and sellers, N. Sellers each own one unit of a commodity and buyers each want to consume one unit of the commodity. Sellers must pay a cost if they sell, buyers receive a value if they buy. Each buyer's valuation of a good is given by $V_i \in [0, \eta]$, where $i = \{1, \dots, N\}$. Each seller's cost of a good is given by $C_j \in [0, \eta]$, where $j = \{1, \dots, N\}$. Each vector of values and costs $e = (V_1, \dots, V_N, C_1, \dots, C_N)$ is called an environment. It is assumed throughout that each buyer i knows V_i and N and each seller j knows C_j and N. This is common knowledge.

Since we are interested in repeated play, buyers and sellers engage in a series of trades using a call market over a number of periods. T, where T is also common knowledge. They are allowed to trade at most only one unit each period.

In these environments, we test two versions of a call market.

2.2. The simple call market

There are many possible microstructures for call markets which can be found by varying parameters such as the number of provisional calls before a trade, the information about bids and offers submitted to the call, how many bids can be deleted and re-bid before the call, etc. We focus on two very basic versions. There is only one call before trade occurs and there is no information revealed about submitted bids or offers before the call is made.[4] Bids can be deleted and re-submitted but there is no information gained by doing so.[5]

The simple call market, SCM, is a sealed-bid auction in which buyers submit bids, b^i, and sellers submit offers, o^i, to a 'market'. When all bids and offers are collected, the market is 'called'. The market computes a clearing price as follows. It first ranks all bids and offers. Without loss of generality, let $b^1 \geqslant b^2 \geqslant \cdots \geqslant b^N$ and $o^1 \leqslant \cdots \leqslant o^N$. Let k be the highest number such that $b^k \geqslant o^k$. Let

$$Z = \min\{b^k, o^{k+1}\} \tag{1}$$

and

$$z = \max\{o^k, b^{k+1}\}. \tag{2}$$

Then

$$P(b^1, \dots, b^N, o^1, \dots, o^N) = (Z + z)/2$$

is the clearing price.[6] Every buyer whose bid is at or above that price receives a unit at that price and every seller whose offer is at or below that price sells a unit at that

[4] This is often referred to as 'blind bidding' and is equivalent to a sealed-bid, two-sided auction.

[5] For theoretical analyses of call markets, see Satterthwaite and Williams (1993) and Friedman and Ostroy (1993). For experimental analyses, see Smith et al. (1982), McCabe et al. (1993), Friedman (1993), and Cason and Friedman (1996).

[6] There may be many clearing prices. Any P such that $Z(k) \geqslant P \geqslant z(k)$ is such a price. But we only need select one for our purposes.

price. If $b^i \geqslant P$, (i.e., $i \geqslant k$), then i trades (i.e., is given a unit of the good) and receives a payoff of $V^i - P$. If $o^i \leqslant P$, (i.e., $i \geqslant k$), then i trades (i.e., gives up a unit of the good) and receives a payoff of $P - C^i$. All others do not trade and receive a payoff of 0.[7] We define the *market outcome function* to be

$$W(b,o) = [P(b,o), h_1(b,o), \ldots, h_N(b,o), f_1, \ldots, f_N(b,o)],$$

where $h_i(b,o) = 1$ if buyer i receives a unit and $h_i(b,o) = 0$ otherwise. Also, $f_i(b,o) = 1$ if seller i delivers a unit and $= 0$ otherwise.[8]

Two market designs will be tested which differ only in their feedback to the agents. In the *open book design*, each agent is given full information about all bids, offers and prices from the previous round. At the start of period $t+1$, each agent knows m_t and $P(m_t)$. So $s^i_{t+1} = [m_t, P(m_t)]$. In the *closed book design*, agents are informed only about the price $P(m_t)$ in the previous round. So $s^i_{t+1} = P(m_t)$.

3. The testbed

We think of a testbed for mechanism (or market) design as a computational substitute for the economics laboratory.[9] In mechanism design theory, a mechanism is (M, g) where M is the set of messages that will be recognized and g is an outcome function. This mechanism operates in an environment, $e = (e^1, \ldots, e^N)$ where e^i is what i knows about the environment. In a market situation, e^i includes information about the fundamentals that i knows as well as any common knowledge information. The idea is that each agent i, knowing (M, g) and e^i, chooses an $m^i \in M$ and then is rewarded according to $g(m) = g(m^1, \ldots, m^N)$. In a mechanism used iteratively, an additional feature of the mechanism is the information fed back to each agent between iterations. We let that be $s^i(m)$ for each agent i. Thus an iterative mechanism is (M, g, s). In applications, the designer builds the mechanism (M, g, s) and then it is operated with a set of human agents in the environment, whatever it actually is. In the lab, both e and (M, g, s) are controlled, as much as possible, by the experimenter and then the mechanism is operated with a set of human subjects. In order for a testbed to be helpful we must be able to plug in environments, e, and mechanisms, (M, g, s), and then have the computational agents produce a sequence of messages $m^* = (m_1, m_2, \ldots, m_T)$ that are similar to those that would be produced were an experimenter to use the same mechanism and environment in a lab.

[7] There is one case we have not dealt with – when the number of bids such that $b \geqslant P$ and the number of offers such that $o \leqslant P$ are unequal. In this case, we fill the orders lexicographically first in order of size (from highest to lowest for buyers and in the opposite direction for sellers) and then in order of time received (from earliest to latest). We do not make this procedure explicit here so as to avoid notational complexity and because it never occurred in our empirical work.

[8] For those interested in mechanism design, if we identify b and o as messages sent by the traders to the market, then $W(m) = g(m)$ is the outcome function, or game form, for this mechanism. The messages are required to be in the set $M = [0, \infty)$.

[9] Just as computational aircraft design has replaced the wind-tunnel as a source of design data, we strive to find a testbed in which computational mechanism design can replace the economics lab as a source of design data.

A testbed, therefore, consists of a collection of computer agents ready to respond to a variety of mechanism designs in a variety of environments. It must be flexible enough to take in whatever information we provide about e and (M, g, s) and produce whatever messages are required. Such flexibility is crucial. For obvious reasons, we do not want to have to redesign the agents every time we change the environment or alter the mechanism. It is not necessary or desirable to create agents that are smarter or faster than humans. We need to create agents who produce outcomes similar to those produced by human agents when placed in the same environment. The test of whether we have succeeded in this goal will occur when we compare the outcomes from our testbed simulations with the outcomes of the same experiments done in the economics lab.

3.1. Individual evolutionary learning

In this section, we describe a learning model that we call individual evolutionary learning (IEL). This is a model we first introduced in Arifovic and Ledyard (2003). There we were interested in the performance of a class of Groves–Ledyard mechanisms in a public good environment. Here we use the same testbed in a significantly different environment with private goods. One reason for doing this is to see whether the testbed is robust enough to deal with significant variations in environment and mechanism.

At the beginning of round $t \in \{1, 2, \ldots, T\}$, each agent $i \in [1, \ldots, N]$ has a collection A_t^i of possible alternative messages consisting of J alternatives,[10] $a_{j,t}^i \in A_t^i$, $j \in \{1, \ldots, J\}$. At each t, an agent selects an alternative randomly from A_t^i using a probability density [11] π_t^i on A_t^i. The alternative selected is her message m_t^i. The outcomes and feedback are then determined using the outcome function, $g(m_t)$, and the feedback functions, $s_{t+1}^i = s^i(m_t)$. Finally, using the information in s_{t+1}, each agent then computes a new A_{t+1}^i and π_{t+1}^i. This computation is the heart of our behavioral model and consists of three pieces: foregone utilities, replication, and experimentation.

To update A_t^i and π_t^i, the first step is to calculate the *foregone utility* for each alternative in the set. The calculation uses the utility function, part of e^i, as well as any other information the agent might have. The foregone utility for a message is the (expected) payoff, given the signal s_t^i, that the alternative $a_{j,t}^i$ would have received had it been actually used, taking the behavior of other agents as given. For now we use the notation $U^i(a_{j,t}^i | s_t^i)$ to represent this utility. We will have to be more precise below.

We construct A_{t+1}^i as follows. First, *experimentation* takes place. For each entry in A_{t+1}^i and for each $j = 1, \ldots, J$, with probability ρ we select one message at random from M and let $a_{j,t+1}^i$ equal that message.

[10] J is a free parameter of the behavioral model and, as such, can be varied in the simulations. J can be loosely thought of as a measure of the processing capacity or memory of the agent. We do not consider variations of J in this paper.

[11] The pair (A_t^i, π_t^i) is a mixed strategy for i at t.

After experimentation, *replication* occurs in a way that reinforces messages that would have been good choices in previous rounds. Better paying (using their foregone payoffs at t) alternatives replace those that might pay less. For $j = 1, \ldots, J$, pick two members of A_t^i randomly (with uniform probability) with replacement. Let these be $a_{k,t}^i$ and $a_{l,t}^i$. Then

$$a_{j,t+1}^i = \left\{ \begin{array}{c} a_{k,t}^i \\ a_{l,t}^i \end{array} \right\} \quad \text{if} \quad \left\{ \begin{array}{c} U(a_{k,t}^i|s_t) \geqslant U(a_{l,t}^i|s_t) \\ U(a_{k,t}^i|s_t) < U(a_{l,t}^i|s_t) \end{array} \right\}.$$

Finally, given A_{t+1}, we *update* the selection probabilities by

$$\pi_{k,t+1}^i = \frac{U(a_{k,t+1}^i|s_t)}{\sum_{j=1}^J U(a_{j,t+1}^i|s_t)} \tag{3}$$

for all $i \in \{1, \ldots, N\}$ and $k \in \{1, \ldots, J\}$. In case there are negative foregone payoffs in a set, payoffs are normalized by adding a constant to each payoff that is, in absolute value, equal to the lowest payoff in the set.

3.2. Some remarks

We could initialize our agents in a number of ways in order to represent what subjects bring to the lab with them or to represent what thinking hard about the problem they face might accomplish. In this paper, we forego any attempts at sophisticated initializations and construct the initial set A_1^i by randomly selecting, with replacement, J messages from the set of possible messages, M. We construct the initial probability π_1^i by letting $\pi_1^i(a_{j,1}^i) = 1/J$.

Replication for $t+1$ favors alternatives with a lot of replicates at t and alternatives that would have paid well at t if they had been used. So it is a process with a form of averaging over past periods – if the actual messages of others have provided a favorable situation for an alternative $a_{j,t}^i$ on average then it will tend to accumulate replicates in A_t^i and thus be more likely to be actually used.

Over time, alternatives that consistently earn higher foregone payoffs receive more replicates and their prominence in the set increases. On the other hand, alternatives with consistently low foregone payoffs receive smaller and smaller number of replicates. Eventually, they disappear from the set. Thus, the potentially successful alternatives are remembered and reinforced while the less successful ones are forgotten. Over time, sets become more homogeneous as most alternatives become replicates of the best performing alternative.

Experimentation introduces new alternatives that may be tried out, in spite of their prior evaluations. This insures that a certain amount of diversity is maintained. Experimentation is not as random as it may look. While it is true that an alternative is selected at random from M, that alternative must have a reasonably high foregone utility relative to the last period or future periods to have any chance of ever being used. A newly generated alternative has to increase in frequency in order to increase its selection probability. This can happen only if it proves successful over several periods.

The testbed then generates over time a sequence of choices m_t, strategies, outcomes and signals in the following way:

$$(A_1, \pi_1), [g(m_1), s(m_1)], \ldots, (A_t, \pi_t), [g(m_t), s(m_t)], \ldots \ .$$

4. The call market in the testbed

In this section we explain how we insert an environment e and a mechanism, $(g, M.s)$, into our testbed by describing how we do this for the private goods environments and the two call market designs we have called open and closed book.

4.1. Implementation

The only thing we need to specify in order to insert the environment and the mechanism into the testbed is how the foregone utilities are computed for a given $e = (V_1, \ldots, V_N, C_1, \ldots, C_N)$. Because the information agents have is different in the two designs, what they can compute is also different. We have taken the naive approach and implicitly assumed that agents treat the signals they see as if those signals will be repeated exactly in the next period. There are no sophisticated expectations models or collection of historical signals. All we assume is 'best' responses.[12]

4.1.1. Open book

In the open book design, each trader knows all the bids and offers from the previous period. That is, $s_t^i = m_t$. Thus they can compute $W(m_t/a_{jt}^i)$ for each bid (offer) $a_{jt}^i \in A_t^i$. Then they can compute $U^i(a_j^i|s_t^i) = [V^i - P(m_t/a_j^i)]h_i(m_t/a_j^i)$ for a buyer.[13] For a seller, $U^i(a_j^i|s_t^i) = [P(m_t/a_j^i) - C^i]f_i(m_t/a_j^i)$.

More explicitly, without loss of generality, take the bids and offers in m_t and rank and renumber them so that $b^{t,1} > b^{t,2} > \cdots > b^{t,N-1}$ and $o^{t,1} < o^{t,2} < \cdots < o^{t,N}$. Let k be the maximum number such that $b^{t,k} \geqslant o^{t,k}$. We need to compute what price would occur and whether we would trade, if we added a bid of b to these lists. It is easy to see that the hypothetical utility of a bid b would be:

if $b^{t,k} > o^{t,k+1}$

$$U(b|\cdot) = \left\{ \begin{array}{c} 0 \\ V - \dfrac{b + o^{t,k+1}}{2} \\ V - \dfrac{b^{t,k} + o^{t,k+1}}{2} \end{array} \right\} \quad \text{if} \quad \left\{ \begin{array}{c} b \leqslant o^{t,k+1} \\ o^{t,k+1} \leqslant b \leqslant b^{t,k} \\ b^{t,k} \leqslant b \end{array} \right\}$$

[12] This is also exactly what we did for public goods mechanisms in Arifovic and Ledyard (2004).

[13] We use the notation (m/a^i) in the standard way to represent the vector m with the ith component replaced by a^i.

and if $b^{t,k} < o^{t,k+1}$

$$U(b|\cdot) = \left\{ \begin{array}{c} 0 \\ V - \dfrac{b + o^{t,k+1}}{2} \\ V - \dfrac{b^{t,k-1} + o^{t,k+1}}{2} \end{array} \right\} \text{ if } \left\{ \begin{array}{c} b \leqslant b^{t,k} \\ b^{t,k} \leqslant b \leqslant b^{t,k-1} \\ b^{t,k-1} \leqslant b \end{array} \right\}.$$

We should note that, by this method of computation of foregone utilities, we are modeling our traders as if they care about manipulating the equilibrium prices on the margin. That is, they take into account the fact that their bids or offers may cause prices to change. As is noted in Friedman (1993, p. 443), 'In the clearinghouse …detailed order flow information may encourage attempts to manipulate price.' [14] Here we are explicitly taking the possibility of manipulation into account.

4.1.2. Closed book design

In the closed book design, each trader knows only the price from the previous period. That is, $s_t^i = P(m_t)$. They do not know what the m_t were that led to this price. There are many ways to use sequences of price data to form expectations about others' strategies and thus to 'predict' the potential prices in the next periods.

Given $P_t = P(m_t)$, each buyer can compute for each bid $a_{jt}^i \in A_t^i$

$$U(a_{j,t}^i, s_t^i) = \left\{ \begin{array}{c} V - P_t \\ 0 \end{array} \right\} \text{ if } \left\{ \begin{array}{c} a_{jt}^i \geqslant P_t \\ a_{jt}^i < P_t \end{array} \right\}.$$

Given $P_t = P(m_t)$, each seller can compute for each offer $a_{jt}^i \in A_t^i$

$$U(a_{j,t}^i, s_t^i) = \left\{ \begin{array}{c} P_t - C^i \\ 0 \end{array} \right\} \text{ if } \left\{ \begin{array}{c} a_{jt}^i \leqslant P_t \\ a_{jt}^i > P_t \end{array} \right\}.$$

We should note that, by this method of computation of foregone utilities, we are modeling our traders in the closed book design as if they are price takers. That is, they act as if their bids or offers will not cause prices to change. Their is little else a naive agent can do in this situation.

Once we have specified how each agent computes $U(a_{j,t}^i, s_t^i)$, there is nothing left to specify. The test bed is ready to go.

5. Results of simulations

We are interested in the performance of the two call market designs – the sequences of outcomes $g(m_t)$ – across different environments, e.

[14] 'Clearinghouse' is Friedman's name for call markets. He also goes on to say 'I am not aware of any theoretical literature which addresses this point.' We have also not found any.

Table 1
Values of parameters used in IEL simulations

Parameter set	1	2	3	4
Buyer 1	1.00	0.90	1.00	2.00
Buyer 2	0.93	0.70	0.95	1.80
Buyer 3	0.92	0.50	0.90	1.60
Buyer 4	0.81	0.30	0.80	1.40
Buyer 5	0.50	0.022	0.15	1.20
Seller 1	0.66	0.05	0.0	0.00
Seller 2	0.55	0.25	0.05	0.20
Seller 3	0.39	0.45	0.10	0.40
Seller 4	0.39	0.65	0.20	0.60
Seller 5	0.30	0.85	0.85	0.80

5.1. The parameters used

We used the following parameter values. For the environment, we had $N = 5$ buyers and $N = 5$ sellers for each simulation. The four sets of values, e, used in the simulations are in Table 1. These provide a reasonable variation in the size of the margins, $Z - z$, computed in Eqs. (1) and (2) and in the potential gains from trade. For the testbed we chose the following.[15] Each trader's mixed strategy set had $J = 100$ messages. The probability of experimentation was set to 0.033. We drew the new strategies that result from the experimentation from the normal distribution with the mean value equal to the value of the previous bid (offer) and standard deviation equal to 1.

5.2. Performance measures

For performance measures, we look at efficiency, trading prices, and the values of individual bids and offers over the course of our simulations. Efficiency is the ratio of the gains from trade in a call to the maximum possible gains from trade. Formally, that measure of the efficiency E_t in the trading period t is

$$E_t = \frac{\sum_{i=1}^{N} V_t^i h^i(m_t) - \sum_{j=1}^{N} C_t^j f^i(m_t)}{\sum_{i=1}^{N} V_t^i h^i(V,C) - \sum_{j=1}^{N} C_t^j f^i(V,C)}.$$

5.3. Results

Table 2 contains the average efficiency (and variance) reached over 100 iterations for a given parameter set and given book design. Each iteration lasted for 20

[15]To check for the robustness of the IEL model, we chose these values to be consistent with those we used in Arifovic and Ledyard (2003) with the Groves–Ledyard mechanisms, a very different class of mechanism.

Table 2
Average efficiency across four parameter sets

Set	Mechanism	Mean	Variance
1	Closed	0.9164	0.0400
	Open	0.8116	0.0456
2	Closed	0.9227	0.0328
	Open	0.8537	0.0470
3	Closed	0.9405	0.0273
	Open	0.8980	0.0349
4	Closed	0.9256	0.0313
	Open	0.7740	0.0257

periods.[16] We chose this length of an iteration in order to be able to compare our simulation data with human data. It is important to realize that we do no 'training' of our testbed agents. We neither estimate parameters based on data sets from experiments nor put the agents through long periods of social learning prior to beginning the simulations. Each new iteration starts with a new initialization – a random seeding of the 'memory' A_1^i and a uniform density for the selection probabilities π_1^i. So it is not a surprise that first period efficiencies are low. What may be a surprise is that the testbed agents seem to learn pretty fast.

Our main finding is that the average efficiency levels are higher for the closed than for the open book design. In particular, averaging across all iterations and parameter values, the closed book design exhibited an efficiency of 92% while the open book design exhibited an efficiency of 83%, a significant difference of 0.09. Across the four parameter sets, the difference in efficiency between the closed and open book design ranged from 0.05 to 0.15.

But, let us look at the dynamics of the efficiencies. We show, in Figs. 1–8, the time series of the efficiency and the trading price in one representative iteration of the closed and open book design, for each set of parameter values 1–4, respectively.

Simulations with the closed book design reach higher levels of efficiency faster than with the open book. In the closed book design, simulations always converged to the efficiency level 1.00, and remained there. But, at the beginning, while learning and adaptation is taking place, efficiency levels tend to be quite low for the first several periods, leading to the lower averages. The closed book design efficiencies also exhibits stability once convergence to the maximum efficiency occurs.

In the case of open book simulations, we observe three different types of patterns. The first pattern is characterized by convergence to the maximum efficiency with occasional departures away from that maximum level. The departures are followed by a return to the maximum efficiency level. The second pattern is characterized by

[16]Each calculation of average efficiency involves 2000 calls, one for each of the 20 periods of the 100 iterations. Since we used four different parameter sets for each of the two market designs, the average efficiency across all iterations and all parameter values involves 8000 calls.

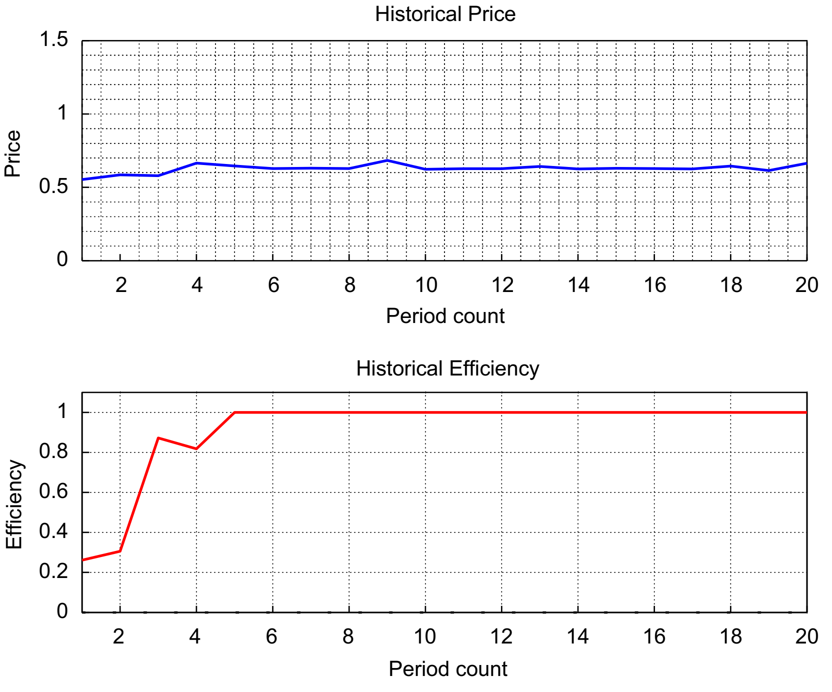

Fig. 1. Price and efficiency for parameter set 1–closed book.

convergence to the maximum level of efficiency followed by a drop in the level that then remains permanently lower. Finally, the third pattern is characterized by occasional jumps to the maximum efficiency level followed by departures, resulting in persistent fluctuations in its level. These patterns vary across different parameter sets, and it is quite likely that there is a link between the model's valuations and costs, the learning process and the open book design. Fig. 2, parameter set 1, corresponds to what we characterized above as pattern 2 behavior. Figs. 4, (parameter set 2) and 6 (parameter set 3), correspond to pattern 1 behavior, while Fig. 8 (parameter set 4), is closest to pattern 3 type of behavior.

In contrast to efficiencies, prices exhibit more stable behavior in both open and closed book designs. Prices eventually settled down to something around the Walrasian equilibrium price, P(V,C), even when efficiencies kept spiking. This difference in the behavior of efficiencies and prices can be directly attributed to the differences in the values to which traders bids and offers converge. In the closed book design, buyers' bids converge to their true valuations and sellers' offers converge to their true costs. In the open book design, bids and offers of those traders, whose bids are actually executed, converge to the values close to the equilibrium price. This bidding behavior leads to the significantly different performance in efficiency and price. In the open book design, the tight constellation of bids and offers around the equilibrium price, means that slight variations in their

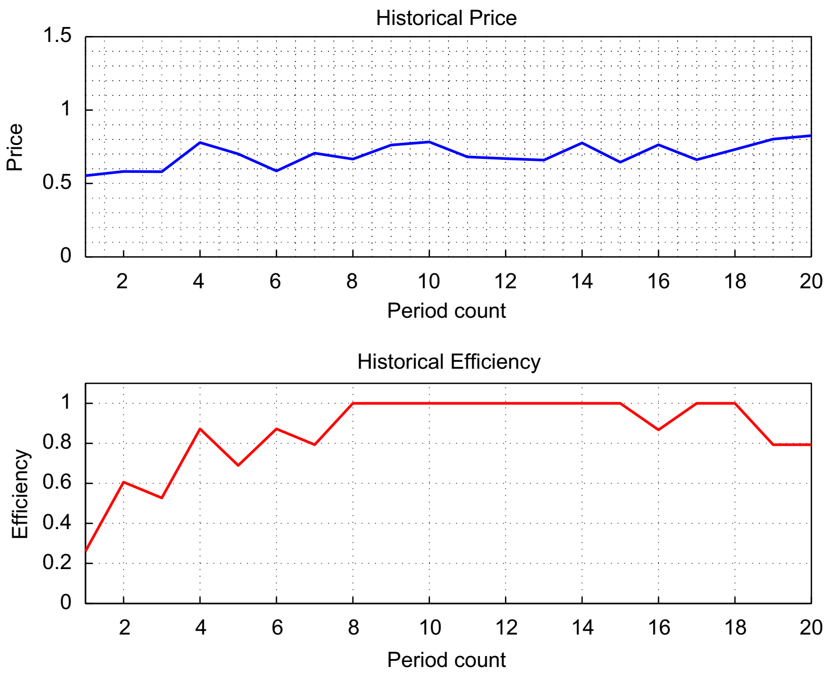

Fig. 2. Price and efficiency for parameter set 1–open book.

values can cause some desirable trades not to occur, even though the price may not move much, which in turn implies lower efficiency. This does not happen in the closed book design since small variations in bids and offers away from the true valuations will not change who trades with whom unless there was little to gain in the first place.

The convergence of bids and offers to values in the closed book treatment also provides insurance against unwarranted experimentation. In our world, there is no need for experimentation once some stability is attained because none of the fundamentals ever change. But our agents, and humans, will occasionally try something new and mechanisms must be robust against this type of occasional experimentation. In the closed book design, only rarely does experimentation lead to inefficient outcomes.

The results of the simulations indicate that the design details of the market matter. The speed of convergence of prices and volume to the market equilibrium and the volatility of prices and volume depend on the amount of information available to the traders. Equilibrium prices and volume are reached faster in the low information design where traders receive information only about the trading price. The volatility of volume is smaller in the low information design which means that average efficiency is higher for that design.

A natural conclusion from our simulations in the IEL testbed is that a closed book design is superior to an open book design. But that would be warranted only if we

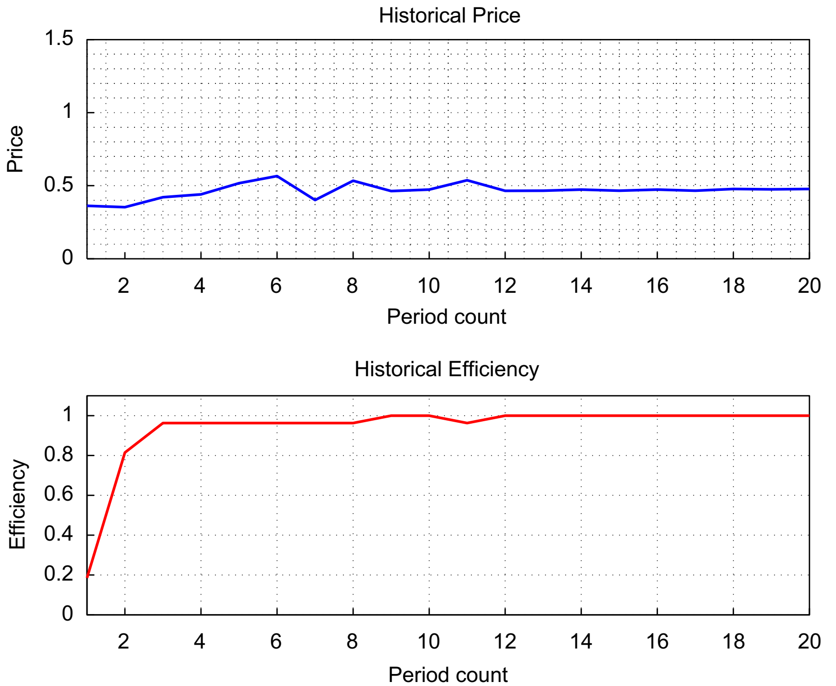

Fig. 3. Price and efficiency for parameter set 2–closed book.

can be sure the IEL testbed yields similar results to what we would observe with humans. To see whether that is true, we ran a series of experiments with human subjects and compared those results to these simulations.

6. Results of experiments

In this section of the paper, we describe our experimental design for testing the open and closed book call markets with human subjects.[17] We set up the experiments to exactly match the environments and mechanisms used for the simulations. Each observation comes from a group of 10 subjects: 5 buyers and 5 sellers. Each group of subjects participated in a sequence of 8 phases. Each phase is identified with a particular environment, *e*, and a particular market design. At the start of a phase, each subject was given a value (for buyers) or a cost (for sellers) that was fixed throughout the phase. In each phase only one type of call market (open or closed book) was used. The market was repeated 20 times in each phase.

The first part of Table 3 provides details about the parameter values, *e*, for each of the 8 phases. The second part of Table 3 provides details about the type of market

[17]We provide a copy of the instructions which the subjects were given at the start of each session in the Appendix.

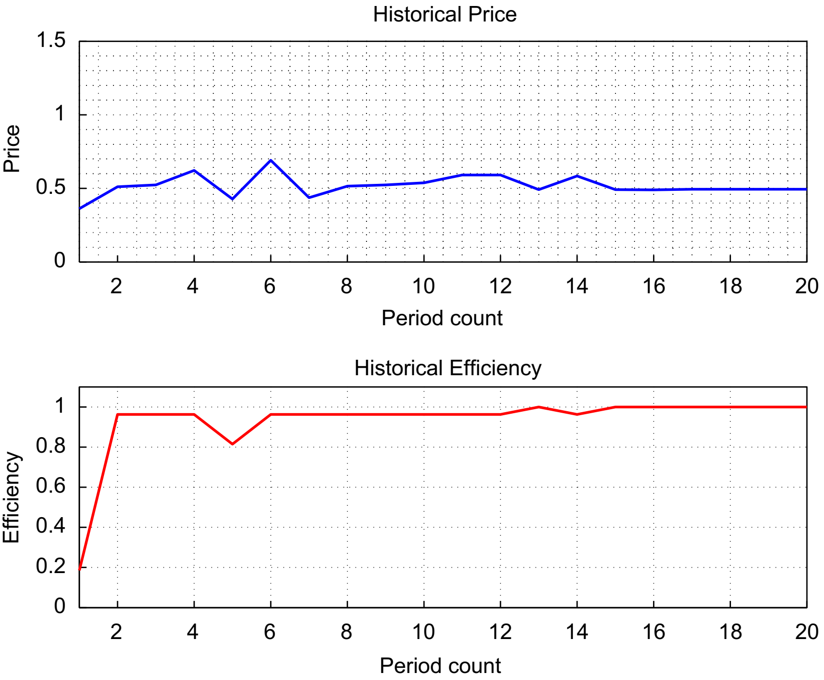

Fig. 4. Price and efficiency for parameter set 2–open book.

(open or closed) that was implemented in each of the phases. The values for phases 1–4 are the values used for the simulations in Table 1 multiplied by 100. The values for phases 5–8 are essentially the same as those in phases 1–4 from the point of view of the market.[18] We added 100 to the values in 1–4 and then permuted these new values among buyers or sellers. The goal was to get a number of comparable observations without the subjects knowing the equilibrium price at the start of a phase.

There are two basic observations,[19] one of which ended early due to a software problem. The first group of subjects got through the sixth round of the sixth phase. The experiment ended at that point. The second group of subjects got all the way to the end of all 8 phases without any problems. As a result, we have data from 13 complete phases. There is a total of 266 call markets in the data. Of these, 140 are run under the closed book rules and 126 are under the open book rules. In further text, we will refer to the first experimental session, as session A, and to the second, as session B.

[18]Phases 1 and 6 go together as do, 2 and 5, 3 and 4, and 7 and 8.

[19]A basic observation is one experimental session in which one group of 10 subjects participates in the sequence of phases. Two observations are not really sufficient to draw any definitive conclusions. Our research plan is to conduct a larger number of the experiments. For now, we view these data as instructive but not conclusive.

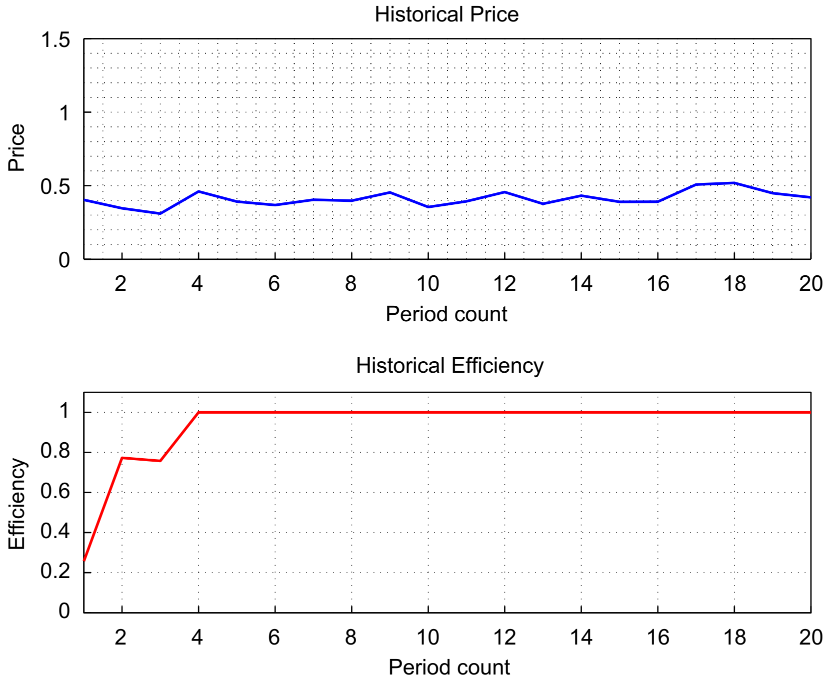

Fig. 5. Price and efficiency for parameter set 3–closed book.

6.1. The results

We group the data according to the parameter values. Thus we will group together data from each of phase 1 and 6, phase 2 and 5, phase 3 and 4, and phase 7 and 8. In each pair, one of the phases uses a closed book design and the other uses an open book design. This allows for an easy comparison of the behavior observed for the same parameter values, but for different call market designs.

6.1.1. Efficiencies

Table 4 lists the efficiencies for open and closed book markets by the different parameter sets.

We begin our analysis of the efficiencies by averaging over all observations. The open book average efficiency is 0.90 with a standard deviation of 0.1. The closed book average efficiency is 0.93 with a standard deviation of 0.13. While this may seem to suggest that there is little difference between the two, that would be a premature conclusion.[20] If we compute a *t*-statistic (for comparing two means with

[20]In fact if it were not for the (surprising to us) result of 0.76 for the fifth phase of the second group, the closed design efficiencies would be clearly higher than those of the open book design. Without the 0.76, the average efficiency of the closed book design is 0.96. This is 0.6 higher than the open book efficiency and, with a standard deviation of 0.1, would imply that the closed book yields higher efficiencies than the open

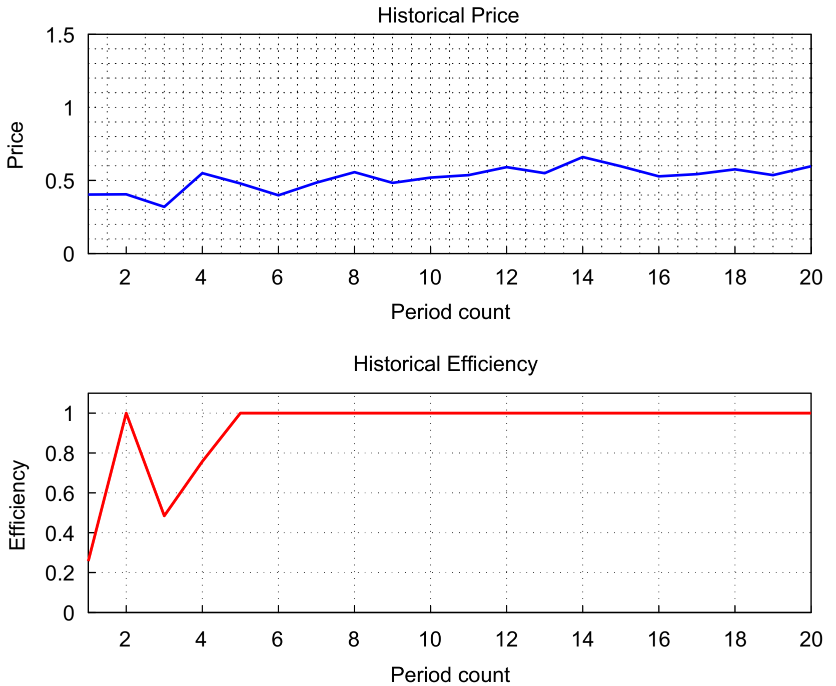

Fig. 6. Price and efficiency for parameter set 3–open book.

unknown variance) we get $t = 2.09$ with 264 degrees of freedom. We can reject the hypothesis that the open book efficiencies are equal to the closed book efficiencies at a 0.025 level of significance.

Let us drill down a little to see if we can find out more. One can see that there are differences across the parameter sets. There is no difference in the efficiencies of the open or closed design for parameter sets 3 and 4. There is for sets 1 and 2. The main difference in these two cases is the size of the margin – the scope for variation in the theoretical equilibrium price. For parameter set 1 this is $11 = 66 - 55$. For set 2, it is 5, for set 3 it is 60, and for 4 it is 40. This difference in margins is important because low efficiencies occur when a high valued buyer or low valued seller does not trade because they were out bid by a lower valued buyer or higher valued seller who should not trade. The tighter the margin is, the more likely this is to happen.[21]

(*footnote continued*)
book. We need more experiments to determine whether the 0.76 is an anomaly we can ignore or one that must be taken seriously.

[21]It is no accident that in those sessions in which efficiencies are low, prices tend to move outside of the margin.

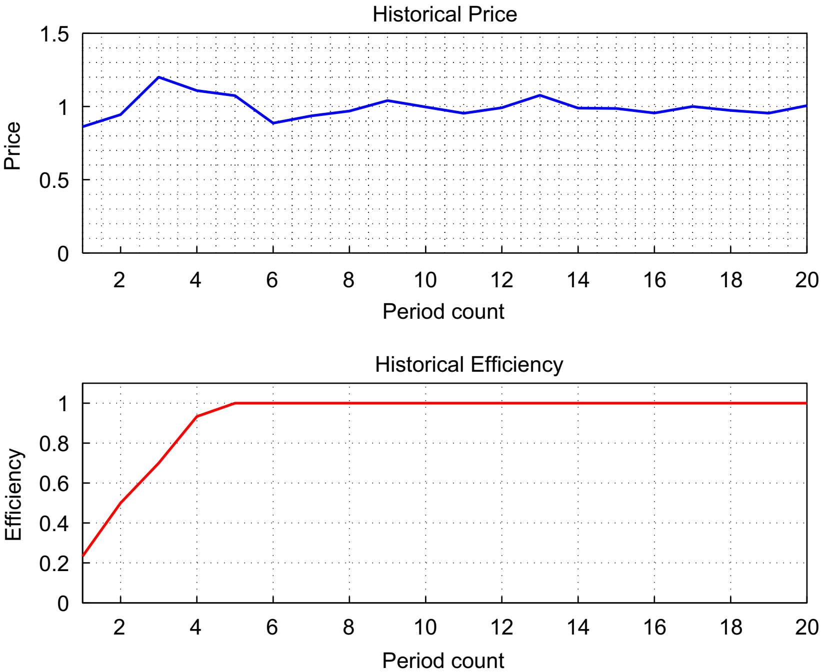

Fig. 7. Price and efficiency for parameter set 4–closed book.

It is also instructive to look at the time series of the efficiencies. We provide that information in Figs. 9–12, grouped by parameter set. Each of the four charts provides data from two relevant phases. The legends are to be interpreted as follows. The first letter, A or B, indicates the experimental session in which the data are generated. The next number indicates the phase for that data, and the last letter, O or C, indicates whether this was an open or closed market design.

One observation we can make from the time series: learning occurs over time. The efficiencies in the early rounds are significantly different than those of the later rounds. The average efficiency in the first 5 rounds was 0.82 for the open book and 0.89 for the closed book, a significant difference. In the last 5 rounds (of 20), the average efficiency was 0.95 for the open book and 0.96 for the closed book, not a significant difference. A second observation is that there are clear differences in the volatility of efficiencies over time across the parameter values. But we have not been able to discern any systematic relationship between the volatility and either the design, open or closed, or the parameter values, such as the size of the margin. There is a relationship between the observed volatility and the bidding behavior of the subjects which we explore in the next sub-section.

6.1.2. Prices and bids

We have provided Figs. 13–20) describing the prices and bids for each of the four parameter sets. The data are again grouped according to the phases that go together.

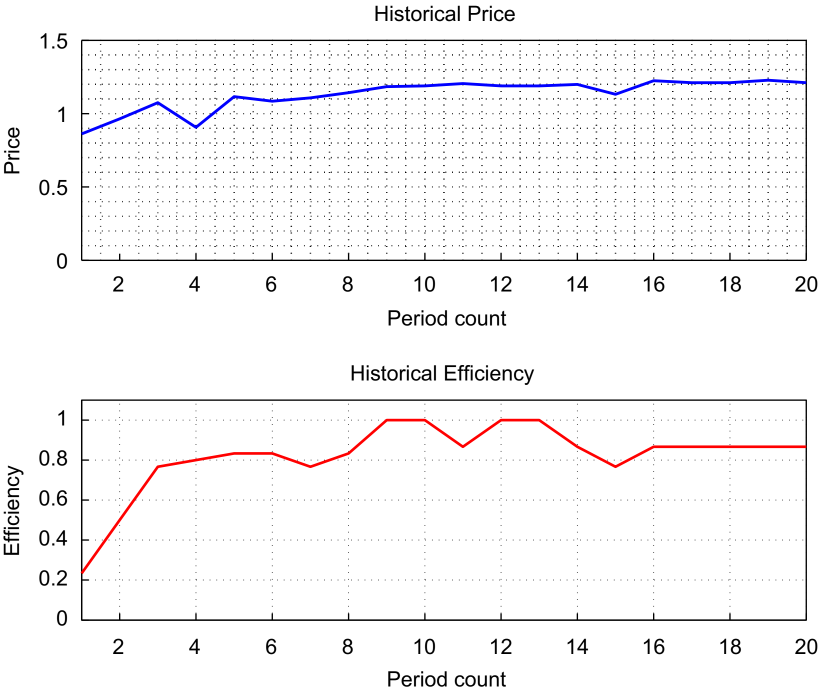

Fig. 8. Price and efficiency for parameter set 4–open book.

Thus, Fig. 13 presents the time series of prices and Fig. 14 provides information on bids and offers for phases 1 and 6 (parameter set 1). Fig. 15 provides data on prices and Fig. 16 provides data on bids and offers for phases 2 and 5. Fig. 17 presents price data and Fig. 18 presents bids and offers data for phases 3 and 4. Finally, Fig. 19 shows time series of prices, and Fig. 20 presents data on bids and offers for phases 7 and 8.

We need to explain the bids – offers figures. For each phase, we computed the average bid or offer of each bidder for the last 10 rounds of that phase. By the 10th round, bids and offers have pretty much settled down so this is a good indication of the basic bidding behavior of the participants.[22] We then ranked the bids from high to low and ranked the offers from low to high. When plotted, the bids and offers provide, respectively, a 'revealed demand curve' and a 'revealed supply curve'. We have displayed these curves from left to right. The legend provides information on which phase is plotted where. For example, for Fig. 14 we have plotted from left to right data from experimental session A, phase 1 and phase 6, and then from experimental session B, phase 1, and phase 6 again. Phase 1 is a closed market design

[22]The discerning reader will notice that we have not plotted all five bidders in most of the charts. The reason is that in many cases the owners of extra marginal units, with significantly high or low values, simply do not bid or bid really silly numbers. Since they quickly learn they will not be able to trade, they either did not bid and submit bids or, to avoid boredom, they submitted bids that are not serious. We chose not to plot these.

Table 3
Experiment parameter sets

Phase	p1	p2	1	2	3	4	5	6	7	8
Buyers										
1	400	400	50	90	100	180	130	193	120	180
2	400	400	92	50	80	190	170	150	180	160
3	400	400	93	70	15	195	190	181	160	140
4	400	400	100	30	95	115	159	200	140	200
5	400	400	81	10	90	200	110	192	200	120
Sellers										
6	300	300	66	5	10	100	185	139	60	40
7	300	300	30	65	20	110	145	166	0	60
8	300	300	39	25	85	120	105	130	80	20
9	300	300	55	85	0	105	125	139	20	80
10	300	300	39	45	5	185	165	155	40	0
Type	C	O	C	O	O	C	C	O	C	O
Seconds/round	25	25	25	25	25	25	25	25	25	25
Interim delay	5	5	5	5	5	5	5	5	5	5
Startup delay	120	120	120	120	120	120	120	120	120	120
# rounds	10	10	20	20	20	20	20	20	20	20

In this table, p1, and p2 are practice phases.
Phase parameters (C = closed book), (O = open book).
The exchange rate was \$0.006/franc.
The total time of one session is 108 min.
The average payout/subject is \$30 plus a \$5 showup fee.

Table 4
Efficiencies across four parameter sets–human data

Parameter set	Phases	Open	Closed
1	1,6	0.89, 0.79	0.98, 0.98
2	2,5	0.92, 0.86	1.0, 0.76
3	3,4	0.95, 0.9	0.9, 0.95
4	8,7	0.97	0.98

and phase 6 is an open market design. We have also plotted, as reference lines, the highest possible value of the Walrasian price, the high margin value, Z, identified as MD, and the lowest possible Walrasian price, z, identified as MS.

We look first at prices. If demand – supply analysis has any force then we should observe prices between the margins Z and z which are defined in (1) and (2). It can be seen from the charts that this is true with a couple of notable exceptions. In phase two of the experimental session A, there were some early prices outside the margins which is understandable since subjects are still learning. In the experimental session B, however, there were three significant violations of the margins: Phases 2, 5, and 6. Two of these, 2 and 6 were open book. In each case, the seller on the margin kept

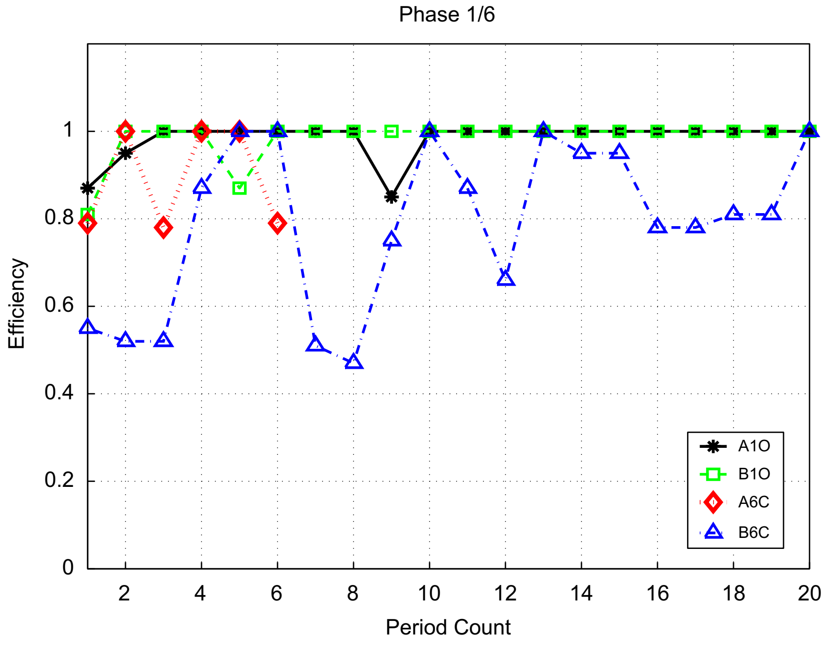

Fig. 9. Efficiency for phases 1 and 6.

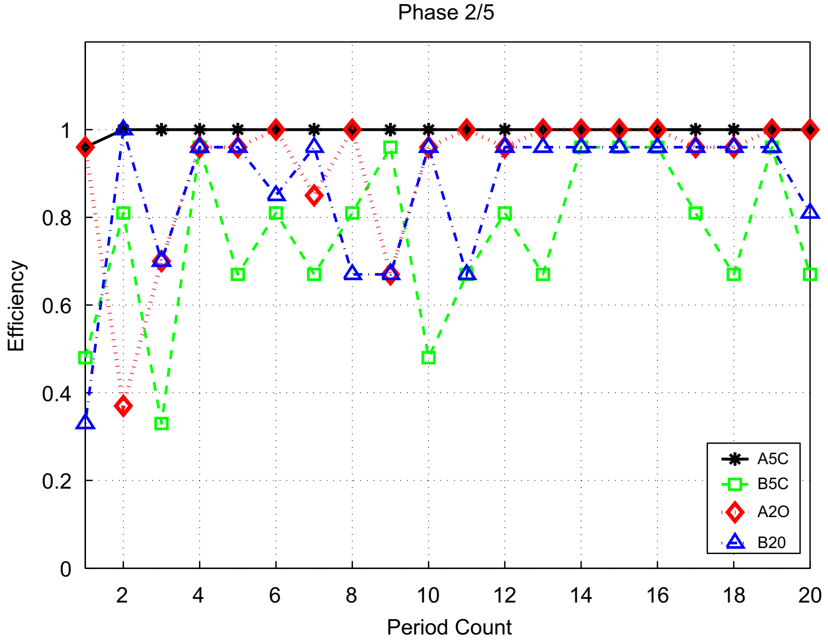

Fig. 10. Efficiency for phases 2 and 5.

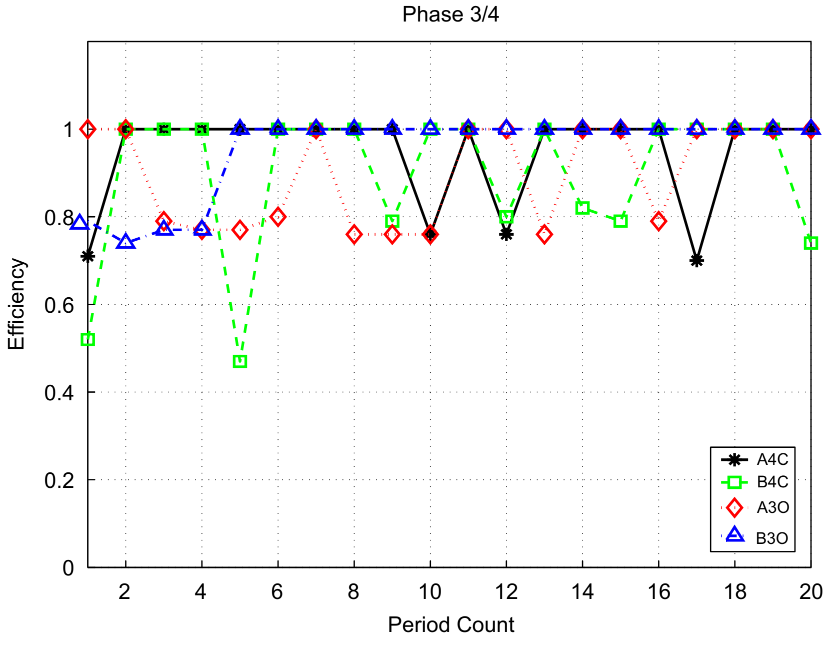

Fig. 11. Efficiency for phases 3 and 4.

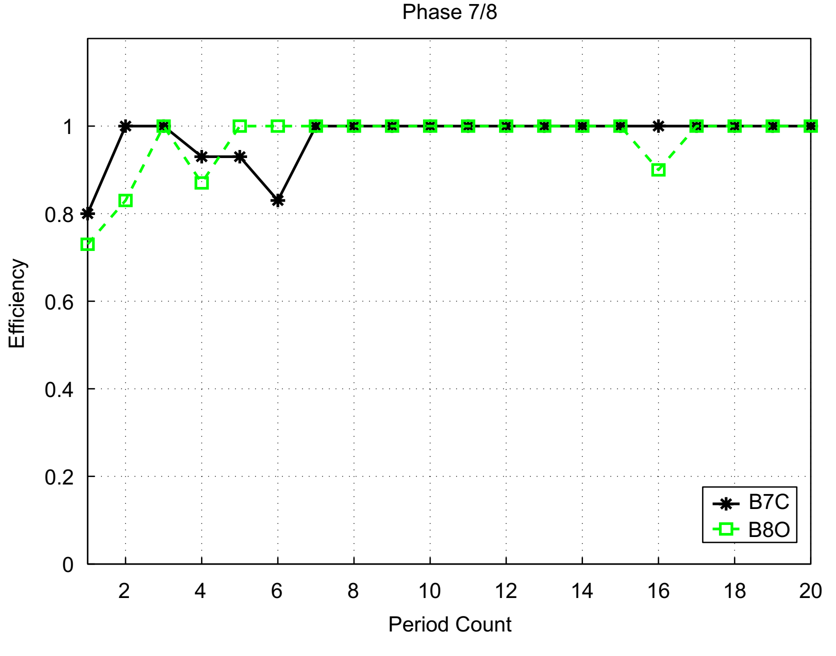

Fig. 12. Efficiency for phases 7 and 8.

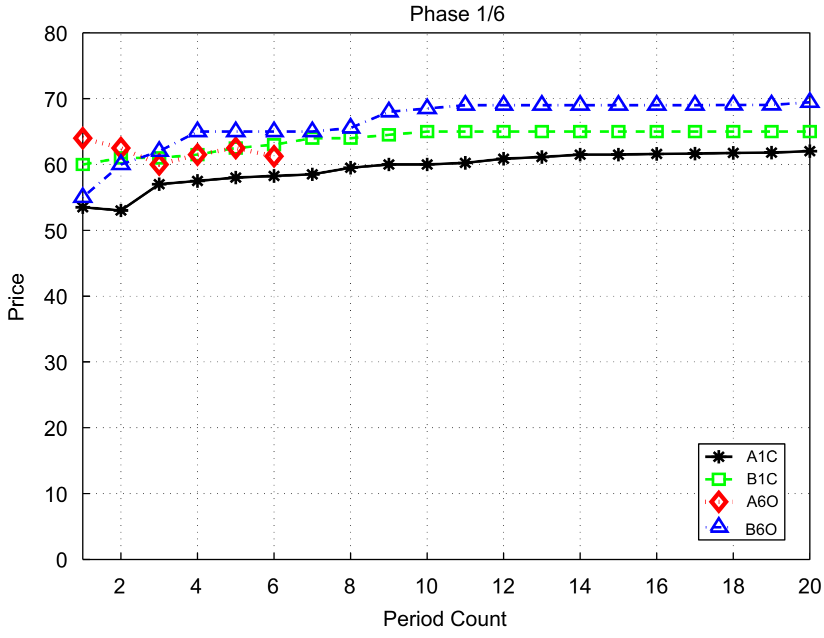

Fig. 13. Prices for phases 1 and 6.

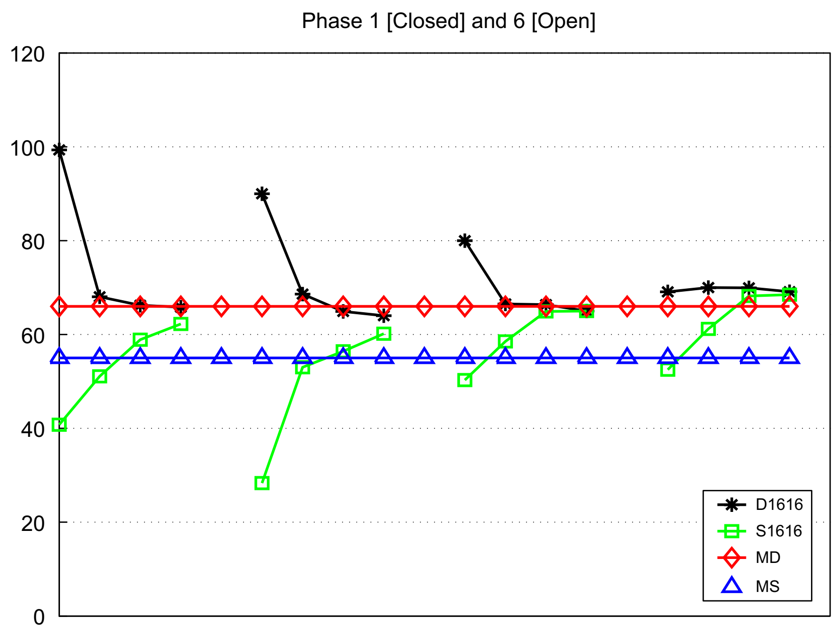

Fig. 14. Submitted bids and offers. From left to right: session A, phases 1 and 6; session B, phases 1 and 6.

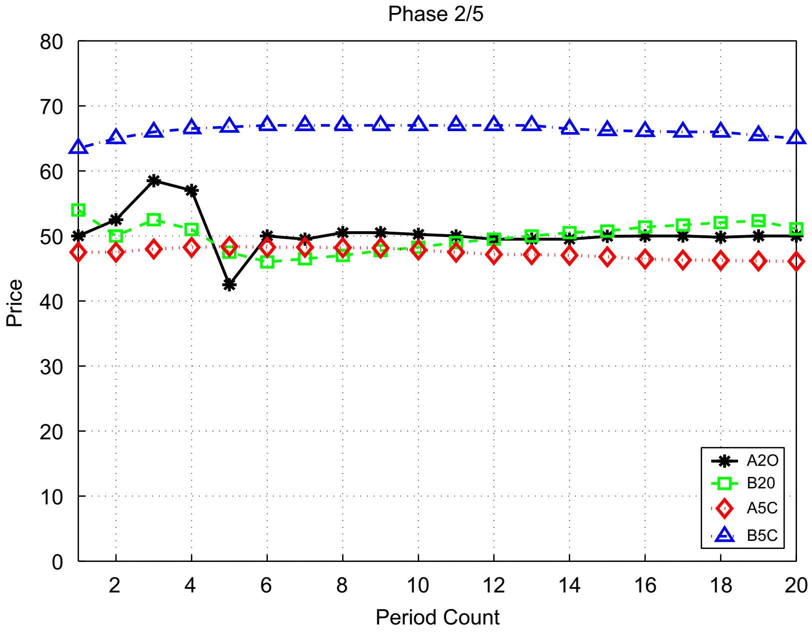

Fig. 15. Prices for phases 2 and 5.

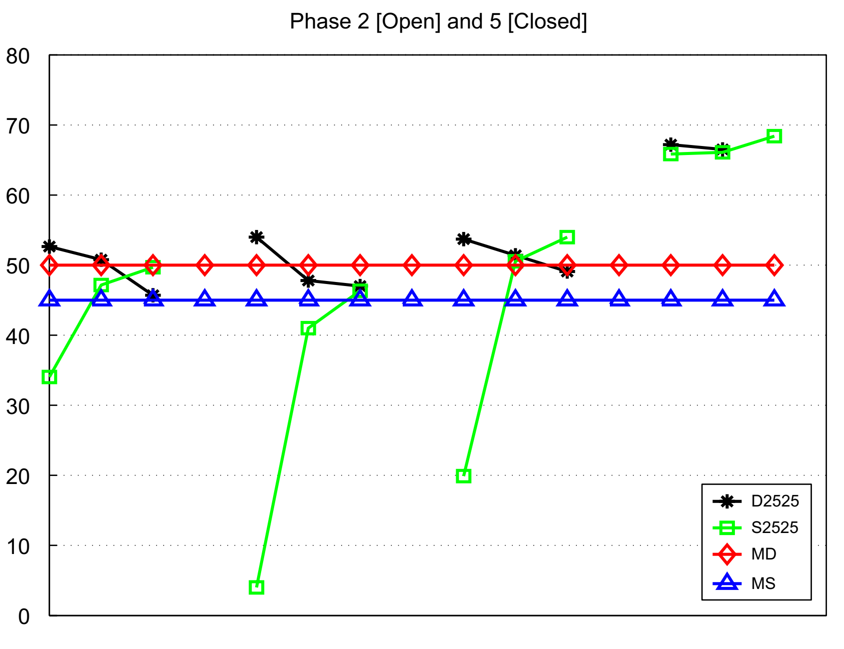

Fig. 16. Submitted bids and offers. From left to right: session A, phases 2 and 5; session B, phases 2 and 5.

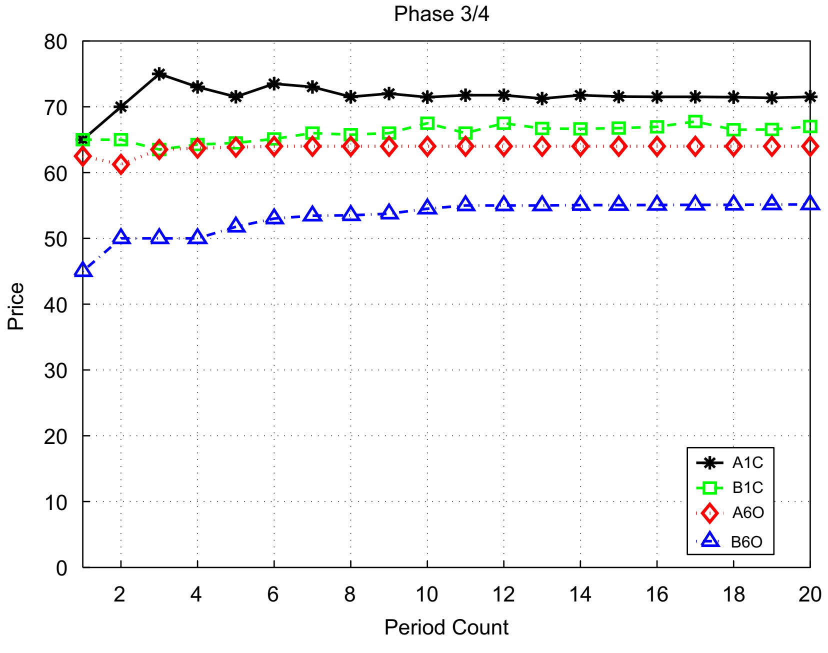

Fig. 17. Prices for phases 3 and 4.

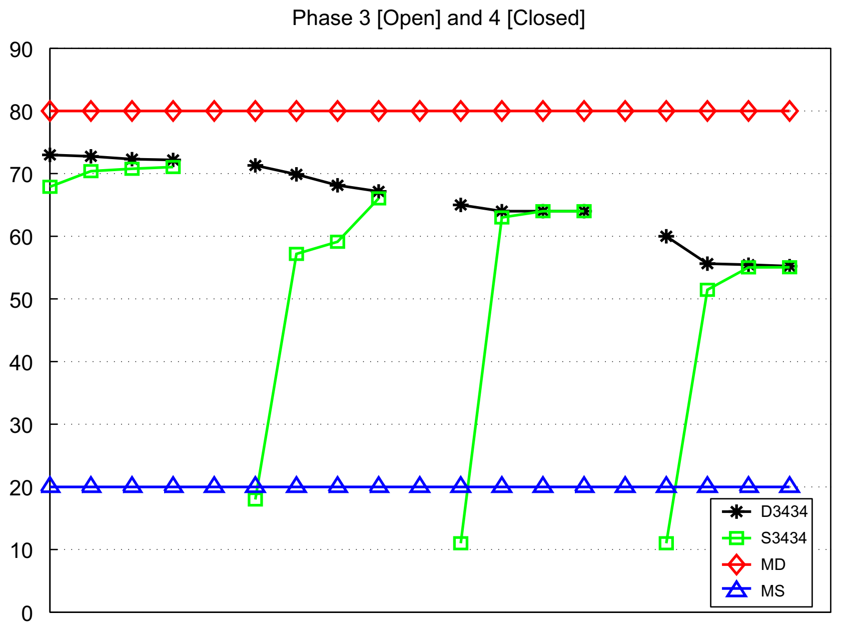

Fig. 18. Submitted bids and offers. From left to right: session A, phases 3 and 4; session B, phases 3 and 4.

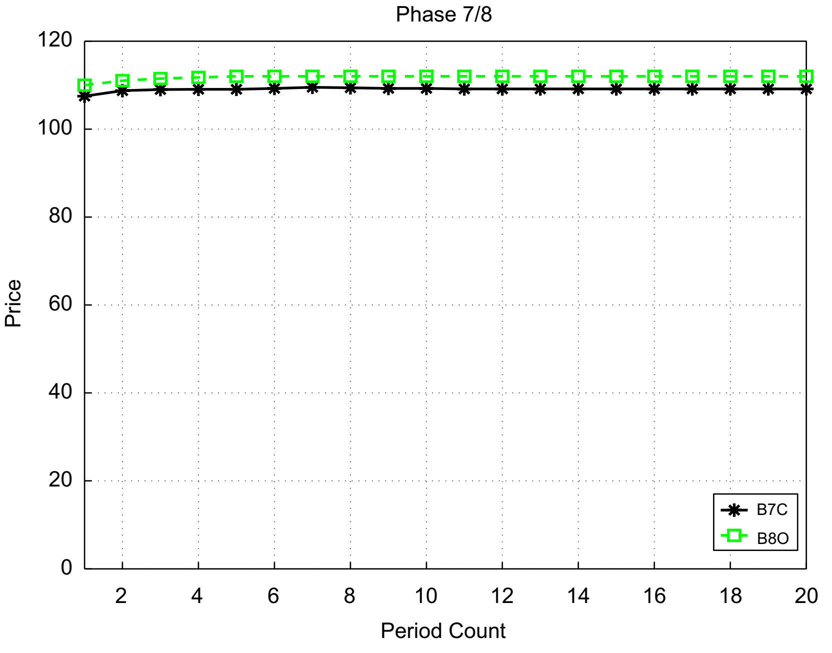

Fig. 19. Prices for phases 7 and 8.

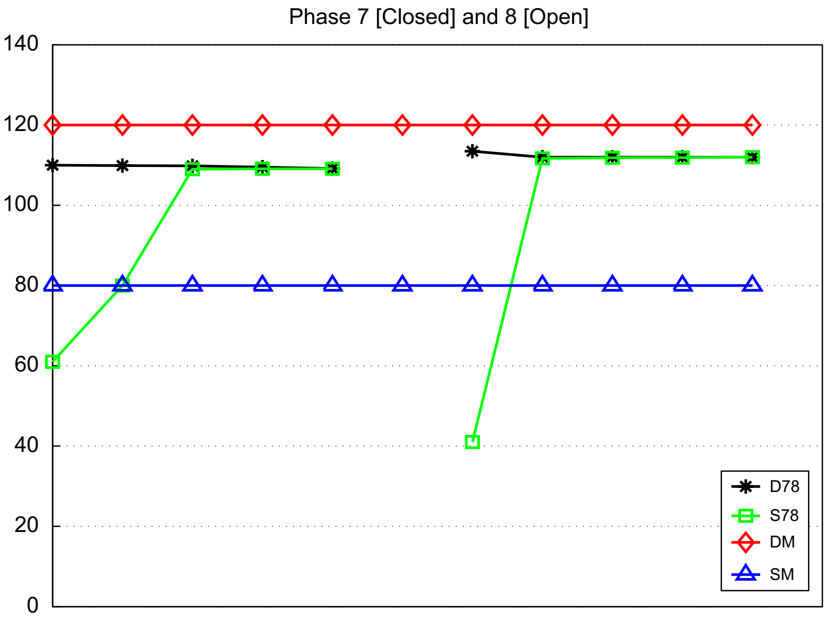

Fig. 20. Submitted bids and offers. From left to right: session A, phases 7 and 8; session B, phases 7 and 8.

bidding 5–10 francs higher than their value, even though they were trading very little. In phase 2, seller 10 with value 45 bid consistently from 50–55 and only traded 3 times. In phase 6, seller 7 with a value of 66 did a similar thing. Either these sellers were aggressive (trying hard to manipulate the price upward, at which they succeeded) or very passive (setting a profit margin and not caring about losing 5 francs/round). In either case, they did affect the price outcomes.

In phase 5, where there was a closed book and therefore less obviously subject to manipulation, buyers 1, 4, and 5 with values of 10, 30, and 50 followed strategies that took them out of the picture.[23] Recomputing the margins for this configuration of values yields 45 and 25. But prices ranged from 63 to 67 for the entire period. Somehow sellers 6, 7, and 8 settled on a sequence of bids that were low enough to keep seller 10 with a value of 65 out and high enough to be usually accepted (except 3/20 times) by the two active buyers. Each seller sold exactly 12/20 times in the process. Total profits for this phase for 7, 8, and 9 were, respectively, approximately, 240, 720, and 480. Had the price been bid down to 45, seller 7 would have gotten 0 and the other two would have traded 20/20 times getting, respectively, approximately, 800 and 400. This is the only time such implicit collusion seemed to happen in the experiments. One wonders why the two buyers, 2 and 3, did not try to force the price lower. With one exception in round 1, neither ever bid below 65, quietly acquiescing to the demands of the sellers.

We think that bidding behavior explains the price patterns. In the experiments, bidding seems to be qualitatively different in the open and closed market designs. With a couple of exceptions,[24] the average of the later bids and offers in the open book phases tend to be flatter than those in the closed book. What we mean by this is that the bids in the experiments using the closed book design are, in many cases, further apart from the offers than in the experiments using the open book design. Another way of thinking of this is that the bidders in the experiments using the open book design are bidding closer to the clearing prices while bidders in the experiments with the closed book design are more likely to bid nearer their true values. Bidding close to the clearing prices means one is attempting to manipulate those prices. Bidding closer to one's true value is a less strategic strategy. As we pointed out in the section on the simulations with the IEL testbed, it is the flatness of the open book bids that leads to volatility in efficiencies even if prices are not volatile.

7. Comparison of simulations and experiments

If the use of simulations is to be helpful in market design, one must have testbeds that behave similarly to humans. Can we say that our IEL testbed compares well to humans? We are reluctant to conclude too much at this point with the small number of

[23]Buyer 4 with a value of 50 did not bid in the first round, bid 45 in rounds 2–11 (never trading) and did not bid in rounds 12–20. Buyer 1 with a value of 20 did not bid in any round. Buyer 5 with a value of 10 did not bid after round 3.

[24]Two exceptions are in Fig. 14, the open book design second from the left, and in Fig. 16, the closed book design fourth from the left.

experimental observations that we have, but there are some significant similarities. In both experiments and simulations, efficiencies are significantly higher in the closed book designs than in the open book designs. Also, both simulations and experiments show a lot more variation of the efficiency levels in the first periods (rounds) than towards the end which indicates the effects of learning. Both experiments and simulations exhibit fairly stable prices, even though efficiencies are volatile. The reason for this has to do with the bidding behavior. In both simulations and experiments, there is a tendency in the open book design for bidders to try to be strategically aggressive by bidding near the clearing price to try to manipulate that price. There is much less tendency to do this in the closed book design and bids, especially of those holding intra-marginal units, tend to be nearer the true values or costs of the bidder.

It is worth pointing out again that the simulations are done in 'real time', i.e., the number of simulation periods is exactly equal to the number of experimental periods. Also, we start the simulations by choosing a random selection of initial strategies. Nevertheless, the agents in IEL are very quickly producing patterns of bidding, prices and efficiencies similar to those from human experiments. We know of no other learning model able to do this.

8. Concluding remarks

In this paper, we examined a very simple market microstructure question: Is there difference in behavior exhibited in the closed and open book design? In other words, does the amount of information available to the traders affect the outcomes? Our answer is yes. The closed book, which provides less information, leads to more efficient and less volatile outcomes. This answer is based on both simulations with our IEL testbed and experiments with humans subjects.

A secondary purpose of the paper was to test whether the IEL simulations produce data that are similar to those produced by human subjects. We found that there was close agreement on the basics. Both experiments and simulations generate higher efficiencies for the closed book design than for the open book. Both generate stable prices and volatile efficiencies. And both generate similar bidding behavior with bids being flatter in the open book than that in the closed book.

Acknowledgments

We thank Shyam Sunder for very helpful comments and Michael Maschek for excellent research assistance. We gratefully acknowledge the financial support of the National Science Foundation, Grants SES-0338732 and SES-0121478.

Appendix. Experiment instructions

Experiment instructions: This is an experiment in trading. You have a very simple role to play. You are either a buyer or a seller. The instructions for each are below.

You will be buying and selling through an online call market, the details of which will also be provided below. Trading will occur in rounds. Items and trades are valued in SSEL Francs (SF). Earnings will be cumulated and paid at the end of the experiment.

Your SF will be converted to $US at the rate of 0.6 cents/franc.

The buyers: Each Buyer will potentially be able to buy one unit each round. You will be given a value for the item. This is the amount the experimenter will pay you if you successfully buy an item in that round. How you buy an item is explained in the section on the market below. The price you pay for the item will be computed by the market. This is also described below. You will keep the difference between the value and the price. So, for example, if your value is 100 SF and the market price is 40 SF in round 2 then you will have earned 60 SF in round 2.

The sellers: Each seller will potentially be able to sell one unit per round. You will be given a cost of the item. This is the amount the experimenter will charge you if you successfully sell an item in the round. How you sell an item is explained in the section on the market below. The price you receive for the item will be computed by the market. This is also described below. You will keep the difference between the price and the cost. So, for example, if your cost is 1 SF and the market price is 40 SF in round 2 than you will have earned 30 SF in round 2.

The market: The market is organized as a 'call' market, which means trades do not actually occur until the market closes. The market will be open once each round. After the market opens, you can submit a bid to either buy or sell. You are the only one who will see your bid until the market closes. When time runs out, the market closes, and the computer determines whether your bid is accepted (i.e., whether you trade) and at what price. Everyone who trades will trade at the same price.

The market computation first ranks all buyers' bids from high to low $[b(1) > b(2) > \cdots > b(n)]$. Then it ranks all sellers' bids from low to high $[s(1) < s(2) < \cdots < s(n)]$. It then finds the largest number such that the buyer's bid is greater than or equal to that seller's offer. (That is the number k for which $b(k) > s(k)$ and $b(k+1) < s(k+1)$.) Only the buyers and sellers whose bids are numbered below $k+1$ will trade.

The price will be

$$(1/2)[\min\{b(k), s(k+1)\} + \max\{s(k), b(k+1)\}].$$

In economics terms, we create a demand curve out of the buyers' bids and a supply curve out of the sellers' offers. We complete the trades that maximize the gains from trade (the demand – supply equilibrium) and then set the price at the mid-point between the bids where the curves intersect. At this price, all accepted buyer bids are at least as much as the price and all rejected buyer bids are no higher than the price. Similarly, no accepted seller bids will be greater than the price and no rejected seller bids will be less than the price.

Example 1. Five buyers bid 75, 45, 50, 80, and 20. Five sellers offer 60, 5, 55, 25, and 30. Buyers with values 80, 75, and 50 trade. Sellers with values 5, 25 and 30 trade. The price is $47.5 = (1/2)(45 + 50)$.

Example 2. Buyer 1 bids 50, buyer 2 bids 35, seller 1 bids 40, seller 2 bids 10. Buyer 1 and seller 2 trade. The price is $37.5 = (1/2)(40 + 35)$.

The experiment: There will be eight phases in which you can earn money. There will also be two practice phases.

Each practice phase will consist of 10 rounds. The rest of the phases will consist of 20 rounds each. There are two types of phases: A and B.

At the start of a phase you will be assigned a value (if a buyer) or a cost (if a seller) for this phase. In each round, when the market opens, you will be able to submit a bid. You create and submit a bid with the 'submit' button. You can modify your bid by first canceling it (on the right side of the screen) and then creating a new bid. Once you are satisfied with your bid, you can hit the 'all done' button. If all players have pressed this button before time runs out, the round will end early.

If the phase is type A, when the market closes, you will be told whether your trade is accepted or not and at what price. You will be able to access this 'historical' information throughout the experiment.

If the phase is type B, when the market closes you will not only be told the market price but also all the bids submitted during that round. You will be able to access this 'historical' information throughout the experiment.

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