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## *Information and Dynamics: Sequences of Call Markets*

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**Abstract.** *This paper is part of a wider research project with the objective of creating computational testbeds for designing and testing new mechanisms—new economic and political institutions. Here we illustrate the power of such an approach by testing two call market designs in a repeated demand-supply environment. We find there to be significant differences in performance depending on the information provided to the traders between calls. In particular, we find that both dynamic and static performance is better, less volatility and higher gains from trade, if traders receive less information between calls.*

**Key Words.** *call markets, adaptive behavior*

### **1. Introduction**

The field of experimental economics has enabled the building and use of experimental *testbeds* in which scaled down versions of institutions can be evaluated in the presence of humans interacting through the system—much in the spirit of wind-tunnel testing of airplane designs. In this way the *right model of human behavior* is ensured since it is humans themselves making the decisions. Human imperfections in information processing, incomplete optimizing, and irrational expectations are automatically captured in the process. The data generated with experimental testbeds have been invaluable in sorting through competing institutional designs. For example, we now know more about the effect of various market micro-structures (i.e., trading rules, information structures, etc.) on the performance of asset markets than we would if restricted to the use of theory and historically generated data.

But, as with wind tunnels, these experimental testbeds are difficult to scale up, are costly to operate, and have slow turn around times. A thorough search over the space of environment and mechanism para-

meters relevant for a mechanism's performance, using experiments with human subjects, is prohibitively expensive in both monetary and time resources. But if we are to improve our understanding of the details of mechanism design and the effect of various environments on the performance of mechanisms, we will need more data. In some cases, an exhaustive search is required. (See, for example, Arifovic and Ledyard, 2001.) To help in that search, we are developing a computer testbed in which we can reliably test a wide range of mechanisms for a wide range of environments. The details of that testbed are provided below. To make sure we have a computational testbed that is relevant to the job and will provide data similar or, even better, identical to what we would observe in a laboratory experiment with the same environment and mechanism, we need to calibrate the testbed to existing experimental results in as many situations as possible. With each calibration we hope to improve the testbed or to point out additional experiments that might be necessary to improve the calibration.

In Arifovic and Ledyard (2001), we took this approach for a collection of Groves-Ledyard mechanisms in an environment with a public good. In this paper, we describe the implementation of our methodology for two forms of call markets in a private goods environment (the simple, one good, demand-supply world). The environments we look at have a fixed number of buyers and sellers. Sellers each own 1 unit of a commodity and buyers each want to consume 1 unit of the commodity. Sellers must pay a cost if they sell, buyers receive a value if they buy. In this world we test a call market, a sealed-bid auction in which buyers submit bids, willingness-to-pay, and sellers submit offers, willingness-to-accept, to a "market". When all bids and offers are collected, the market is "called".

That is, the market computes a demand-supply equilibrium price. Every buyer whose bid is above that price receives a unit at that price and every seller whose offer is below that price sells a unit at that price. There are many micro-forms of call markets depending on the number of calls before a trade, the information about bids and offers submitted to the call, etc. In this paper we focus on the simplest version. There is only one call before trade occurs and there is no information revealed about submitted bids or offers before the call is made.<sup>1</sup>

## 2. The Environment—The DS Environment

There are  $N$  buyers and  $N$  sellers in the demand-supply, DS, environment. Each seller is endowed with 1 unit of the good in each time period. Each buyer's valuation of a good is given by  $V_i \in [0, 1]$ , where  $i = \{1, \dots, N\}$ . Each seller's cost of a good is given by  $C_j \in [0, 1]$ , where  $j = \{1, \dots, N\}$ . Costs and valuations are drawn randomly from the  $[0, 1]$  interval.

We are interested in the Walrasian equilibrium of a market operating in this environment. We find that equilibrium by first ranking all values and costs. Without loss of generality let  $V^1 > V^2 > \dots > V^N$  and  $C^1 < \dots < C^N$ . Let  $k$  be the highest number such that  $V^k \geq C^k$ . Let  $M(k) = \min\{V^k, C^{k+1}\}$  and  $m(k) = \max\{C^k, V^{k+1}\}$ . Let  $P(V^1, \dots, V^N, C^1, \dots, C^N) = (M(k) + m(k))/2$ .  $P$  is the Walrasian equilibrium price.<sup>2</sup> If  $V^i \geq P$ , (i.e.,  $i \geq k$ ), then  $i$  trades (i.e., is given a unit of the good) and receives a payoff of  $V^i - P$ . If  $C^i \leq P$ , (i.e.,  $i \geq k$ ), then  $i$  trades (i.e., gives up a unit of the good) and receives a payoff of  $P - C^i$ . All others do not trade and receive a payoff of 0.

We define the *Walrasian outcome function* to be

$$g(V, C) = [P(V, C), h_1(V, C), \dots, h_N(V, C), f_1, \dots, f_N(V, C)]$$

where  $h_i(V, C) = 1$  if buyer  $i$  receives a unit and  $h_i(V, C) = 0$  otherwise. Also  $f_i(V, C) = 1$  if seller  $i$  delivers a unit and  $= 0$  otherwise.

## 3. The Mechanism—SCM

The mechanism we will examine in this world is called a sealed-bid call market, SCM. The SCM works in

the following way. Buyers submit bids  $b_1, \dots, b_N$  and sellers submit offers  $o_1, \dots, o_N$ ; to the market-maker. The market-maker then computes a Walrasian equilibrium as if those bids and offers were the true values of the traders. That is, the market maker computes  $g(b, o)$ .  $P(b, o)$  is the price at which all transactions take place.  $h_i(b, o) = 1$  if buyer  $i$  receives a unit and  $h_i(b, o) = 0$  otherwise. Also  $f_i(b, o) = 1$  if seller  $i$  trades a good and  $= 0$  otherwise.

If we think of  $b$  and  $o$  as messages sent by the traders to the market, then  $g(m)$  is the outcome function, or game form, for this mechanism. The messages are selected from the set of possible messages  $M = [0, 1]$ .

## 4. The Testbed

The testbed is created by implementing a particular behavioral model. Our goal is to have the testbed be independent of the mechanisms we want to test. So we will use the same model we used in (Arifovic and Ledyard 2001) for public goods mechanisms. In each round, agents will send messages to the mechanism based on random selection from a set; that is, they use a mixed strategy. The mechanism will pick outcomes and then inform the agents about them and (as part of the mechanism design) other information in the form of a signal. The agents then adjust the set they are selecting from and the probability density that determines their selection.

At the beginning of round  $t \in \{1, 2, \dots, T_{\max}\}$ , each agent  $i \in [1, \dots, 2N]$ , (where  $2N$  includes  $N$  buyers and  $N$  sellers), has a collection  $A_t^i$  of possible alternative messages at time  $t$ . A collection  $A_t^i$  consists of  $J$  alternatives,<sup>3</sup>  $a_{j,t}^i \in A_t^i$ ,  $j \in \{1, \dots, J\}$ . At each  $t$ , an agent selects an alternative randomly from  $A_t^i$  using a probability density<sup>4</sup>  $\pi_t^i$  on  $A_t^i$ . This alternative is her message  $m_t^i$  (her bid or offer) to the market-maker. We construct the initial set  $A_1^i$  by randomly selecting, with replacement,  $J$  messages from the set of possible messages.<sup>5</sup> We construct the initial probability  $\pi_1^i$  by letting  $\pi_1^i(a_{j,1}^i) = 1/J$ .

The price and transactions are then determined by the market-maker using the Walrasian outcome function,  $g(m_t)$ . Two designs will be tested which differ only in their feedback to the agents. In the *Open Book Design*, each agent is given full information about all bids, offers and prices from the previous round. At the start of period  $t + 1$ , each agent knows  $m_t$  and  $P(m_t)$ . So  $s_{t+1}^i = [m_t, P(m_t)]$ . In the *Closed Book Design*,

agents are informed only about the price  $P(m_t)$  in the previous round. So  $s_{t+1}^i = P(m_t)$ .

Using the information in  $s_{t+1}$ , each agent then computes a new  $A_{t+1}^i$  and  $\pi_{t+1}^i$ . This computation is the heart of our behavioral model and consists of three pieces: foregone utilities, replication, and experimentation.

#### 4.1. Foregone utility

To update  $A_t^i$  and  $\pi_t^i$ , the first step is to calculate what we call *foregone utilities* for each alternative in the set. This is the (expected) payoff, given the signal  $s_t^i$ , that the alternative  $a_{j,t}^i$  would have received if it had been actually used, taking the behavior of other agents as given. We use the notation  $U^i(a_{j,t}^i | s_t^i)$  to represent this utility. As the computation of foregone utilities depends on the signal received, we describe these for each design.

**4.1.1. Open book design.** In the open book design, each trader knows all the bids and offers from the previous period. That is,  $s_t^i = m_t$ . Thus they can compute  $g(m_t/a_{j,t}^i)$  for each bid (offer)  $a_{j,t}^i \in A_t^i$ . Then they can compute  $U^i(a_{j,t}^i | s_t^i) = [V^i - P(m_t/a_{j,t}^i)] h_i(m_t/a_{j,t}^i)$  for a buyer.<sup>6</sup> For a seller,  $U^i(a_{j,t}^i | s_t^i) = [P(m_t/a_{j,t}^i) - C^i] f_i(m_t/a_{j,t}^i)$ . We provide some detail for these computations in the appendix.

**4.1.2. Closed book design.** In the closed book design, each trader knows only the price from the previous period. That is,  $s_t^i = P(m_t)$ . While there are many ways to use sequences of price data to “predict” the potential prices in the next periods, we take the simplest way out and assume that each agent just uses the current price to compute foregone utilities.

Given  $P_t = P(m_t)$ , each buyer can compute for each bid  $a_{j,t}^i \in A_t^i$

$$U(a_{j,t}^i, s_t^i) = \begin{cases} V - P_t \\ 0 \end{cases} \quad \text{if } \begin{cases} a_{j,t}^i \geq P_t \\ a_{j,t}^i < P_t \end{cases}.$$

Given  $P_t = P(m_t)$ , each seller can compute for each offer  $a_{j,t}^i \in A_t^i$

$$U(a_{j,t}^i, s_t^i) = \begin{cases} P_t - C^i \\ 0 \end{cases} \quad \text{if } \begin{cases} a_{j,t}^i \leq P_t \\ a_{j,t}^i > P_t \end{cases}.$$

#### 4.2. Replication

We construct  $A_{t+1}^i$  in a way that reinforces messages that would have been good choices in previous rounds.

First we allow potentially better paying (using their foregone payoffs at  $t$ ) alternatives to replace those that might pay less. For  $j = 1, \dots, J$ , we let  $a_{j,t+1}^i$  be chosen as follows. Pick two members of  $A_t^i$  randomly (with uniform probability) with replacement. Let these be  $a_{k,t}^i$  and  $a_{l,t}^i$ . Then

$$a_{j,t+1}^i = \begin{cases} a_{k,t}^i \\ a_{l,t}^i \end{cases} \quad \text{if } \begin{cases} U(a_{k,t}^i | s_t) \geq U(a_{l,t}^i | s_t) \\ U(a_{k,t}^i | s_t) < U(a_{l,t}^i | s_t) \end{cases}.$$

#### 4.3. Experimentation

Experimentation takes place after replication. For each collection  $A_{t+1}^i$  and for each  $j = 1, \dots, J$ , with probability  $\rho$  we select one message at random from  $M$  and let  $a_{j,t+1}^i$  equal that message.

#### 4.4. Updating of $\pi(t)$

Given  $A_{t+1}$ , we now update the selection probabilities. Let

$$\pi_{k,t+1}^i = \frac{U(a_{k,t+1}^i | s_t)}{\sum_{j=1}^J U(a_{j,t+1}^i | s_t)} \quad (4.1)$$

for all  $i \in \{1, \dots, N\}$  and  $k \in \{1, \dots, J\}$ . In case there are negative foregone payoffs in a set, payoffs are normalized by adding a constant to each payoff that is, in absolute value, equal to the lowest payoff in the set.

#### 4.5. Some remarks

Replication for  $t+1$  favors alternatives with a lot of replicates at  $t$  and alternatives that would have paid well at  $t$  if they had been used. So it is a process with a form of averaging over past periods—if the actual messages of others have provided a favorable situation for an alternative  $a_{j,t}^i$  on average then it will tend to accumulate replicates in  $A_t^i$  and thus be more likely to be actually used in the mechanism.

Over time, alternatives that consistently earn higher foregone payoffs receive more replicates and their prominence in the set increases. On the other hand, alternatives with consistently low foregone payoffs receive smaller and smaller number of replicates. Eventually, they disappear from the set. Thus, the potentially successful alternatives are remembered and reinforced while the less successful ones are forgotten. Over time, sets become more homogeneous as most alternatives become replicates of the best performing alternative.

However, experimentation introduces new alternatives that might be tried out independent of their prior evaluation. This insures that a certain amount of

diversity is maintained. Experimentation is not as random as it looks. While it is true that an alternative is selected at random from  $M$ , that alternative must have a reasonably high foregone utility relative to the last period or future periods to have any chance of ever being used. A newly generated alternative has to increase in frequency in order to increase its selection probability. This can happen only if it proves successful over several periods.

The testbed then is driven over time by the sequence of

$$(A_1, \pi_1), g(m_1), \dots, (A_t, \pi_t), g(m_t), \dots$$

## 5. Results of Simulations

We are interested in the performance of the mechanisms—the sequences of outcomes  $g(m_t)$ .

### 5.1. The parameters used

For each informational treatment, we used the following parameter values. We had  $N = 5$  buyers and  $N = 5$  sellers for each simulation. Each trader's mixed strategy set had  $J = 100$  messages. The probability of experimentation was set to 0.0033. We drew the new values that result from the experimentation in two different ways. In the first one, we drew bids (offers) from

the uniform distribution within the range of  $[0, V^i]$  ( $[C_i, 1]$ ). In the second one, we drew the values from the normal distribution with the mean equal to the value of the previous bid (offer) and standard deviation equal to 1.<sup>7</sup>

### 5.2. Performance measures

For performance measures, we look at efficiency, trading prices, and the values of individual bids and offers over the course of our simulations. Efficiency is the ratio of the gains from trade in a call to the maximum possible gains from trade. Formally, that measure of the efficiency  $E_t$  in the trading period  $t$  is:

$$E_t = \frac{\sum_{i=1}^N V_i^i h^i(m_t) - \sum_{j=1}^N C_j^j f^j(m_t)}{\sum_{i=1}^N V_i^i h^i(V, C) - \sum_{j=1}^N C_j^j f^j(V, C)}$$

### 5.3. Results

We show, in Fig. 1, the time series of the efficiency and the trading price in one simulation of the closed book design.<sup>8</sup> In Fig. 2 are shown the efficiency and price time series for one simulation of the open book mechanism.<sup>9</sup> Each is highly representative of the simulations for that design. In all of our simulations with the *closed book* design, both price and efficiency converged rapidly to the predictions of the Walrasian equilibrium model. Efficiencies were above 80% after a few

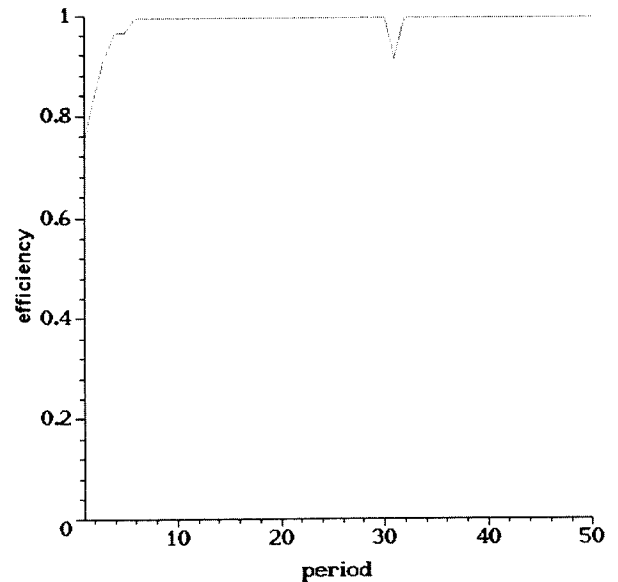
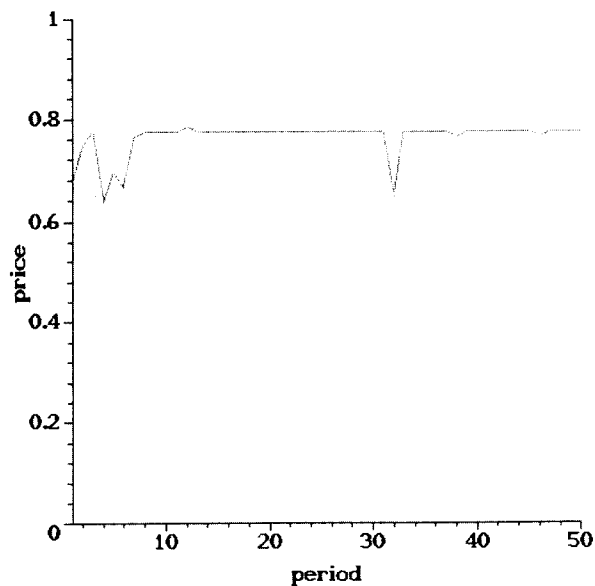


Fig. 1. Closed book treatment.

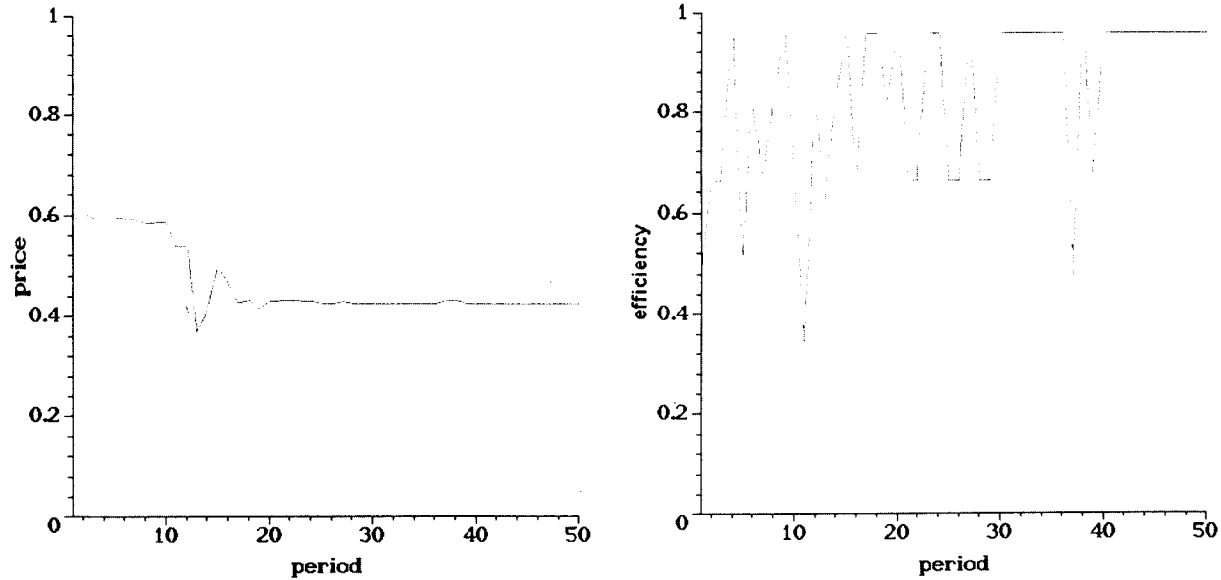


Fig. 2. Open book treatment.

rounds and, at 100% by the 10th round, but in some cases even sooner. Prices converged to  $P(V, C)$  in the same period of time. The story for the *open book* design is significantly different. In only one of our simulations with the open book design was convergence rapid. In that case it looked a bit like the closed book data. However, in all other simulations prices and efficiencies took from 20 to 40 rounds to settle down but, even then, each experienced continued events of spikes. In most cases, as in Fig. 2, prices eventually settled down to something around the Walrasian equilibrium price,  $P(V, C)$ , but efficiencies kept spiking. These differences are a direct consequence of the differences in the values to which traders bids and offers converge.

In the closed book design, buyers' bids converge to their true valuations and sellers offers converge to their true costs. In the open book design, bids and offers of those traders whose bids are actually executed converge to the values close to the equilibrium price.<sup>10</sup> This bidding behavior leads to the significantly different performance in efficiency and price. In the open book design, the tight constellation of bids and offers around the equilibrium price, means that slight variations in their values can cause some desirable trades not to occur, even though the price may not move much, which in turn implies lower efficiency. This does not

happen in the closed book design since small variations in bids and offers away from the true valuations will not change who trades with whom unless there was little to gain in the first place. So this is why we can see price stability in either design but efficiency, and volume, stability in the closed book design and not the open book design.

Finally, the convergence of bids and offers to values in the closed book treatment also provides insurance against unwarranted experimentation. In our world, there is no need for experimentation once some stability is attained because none of the fundamentals ever change, but our agents will occasionally try something new. Mechanisms should be robust against this type of occasional experimentation. In the closed book design, only rarely does experimentation lead to inefficient outcomes. Once that happened in the simulation reported in Fig. 1 at around period 30 but the market quickly recovered to 100% efficiency. It was the buyer with the value of 0.93 who bid around 0.7 in period 30, significantly lower than the value and, as a result, an inefficient trade occurred. However, that buyer quickly returned to bidding near 0.93 and continued to do so for the rest of the simulation. In the open book design we see much different performance. Here, even minor experimentation will lead to inefficiencies just as minor variations in the bids will.

## 6. Conclusion

In this paper, we described the implementation of our computational testbed for a call market mechanism in demand-supply environments. The results of the simulations indicate that the design details of the market matter. The speed of convergence by prices and volume to the market equilibrium and the volatility of those prices and volume depend on the amount of information available to the traders. Equilibrium prices and volume are reached faster in the low information design where traders receive information only about the trading price. Volatility of volume is much less in the low information design which means that average efficiency is higher for that design. The bottom line is that less information feedback yields higher performance.

## Appendix

In the *open book* call market mechanism, the public information includes all bids and offers. We model the computation of foregone utilities  $U^i(b|s_r^i)$  as  $U^i[g(m_i/b)]$ . Without loss of generality, take the bids and offers in  $m_t$  and rank and renumber them. So  $b^{t,1} > b^{t,2} > \dots > b^{t,N-1}$  and  $o^{t,1} < o^{t,2} < \dots < o^{t,N}$ . Let  $k$  be the maximum number such that  $b^{t,k} \geq s^{t,k}$ .

We need to compute what price would occur and whether we would trade, if we added a bid of  $b$  to these lists. It is easy to see that the hypothetical utility of a bid  $b$  would be:

if  $b^{t,k} > o^{t,k+1}$

$$U(b|\cdot) = \begin{cases} 0 \\ V - \frac{b + o^{t,k+1}}{2} \\ V - \frac{b^{t,k} + o^{t,k+1}}{2} \end{cases} \text{ if } \begin{cases} b \leq o^{t,k+1} \\ o^{t,k+1} \leq b \leq b^{t,k} \\ b^{t,k} \leq b \end{cases}$$

and if  $b^{t,k} < o^{t,k+1}$

$$U(b|\cdot) = \begin{cases} 0 \\ V - \frac{b + o^{t,k+1}}{2} \\ V - \frac{b^{t,k-1} + o^{t,k+1}}{2} \end{cases} \text{ if } \begin{cases} b \leq b^{t,k} \\ b^{t,k} \leq b \leq b^{t,k-1} \\ b^{t,k-1} \leq b \end{cases}.$$

## Notes

1. For theoretical analyses of call markets, see Satterthwaite and Williams (1993) and Friedman and Ostroy (1995). For experimental analyses, see Smith et al. (1982) and Cason and Friedman (1998).
2. Technically speaking there may be many Walrasian prices. Any  $P$  such that  $M(k) \geq P \geq m(k)$  is such a price. But we only need select one for our purposes.
3.  $J$  is a free parameter of the behavioral model and, as such, could be varied in the simulations.  $J$  can be loosely thought of as a measure of the processing capacity of the agent. We do not consider such variations in this paper.
4. In essence the pair  $(A_t^i, \pi_t^i)$  is a mixed strategy for  $i$  at  $t$ .
5. For the call market mechanism, we select from  $[0, V^i]$  for buyer  $i$  and from  $[C^i, \bar{V}]$  for seller  $i$ .
6. We use the notation  $(m/a^i)$  in the standard way to represent the vector  $m$  with the  $i$ th component replaced by  $a^i$ .
7. For each combination of informational treatment and the experimentation process we conducted four simulations with different seed value for the pseudo random number generator.
8. In this closed book simulation, the buyers' values were 1.00, 0.93, 0.92, 0.81, 0.01. The sellers' costs were 0.30, 0.39, 0.39, 0.55, 0.66. The theoretical demand-supply equilibrium price is any number in [0.66, 0.81]. Maximum efficiency occurs if all items trade except for the buyer with value 0.01 and the seller with cost 0.66.
9. In this open book simulation, the buyers' values were 0.90, 0.70, 0.50, 0.30, 0.10. The sellers' costs were 0.05, 0.25, 0.45, 0.65, 0.85. The theoretical demand-supply equilibrium price is any number in [.45, .50]. Maximum efficiency occurs if only the three highest value buyers and three lowest cost sellers trade.
10. In the open book simulations in Fig. 2, the bids of buyers with 0.5, 0.7 and 0.9 converge to 0.47 or 0.48. The buyers with values of 0.1 and 0.3 seem to remain random throughout. But since the low valued bidders rarely trade, their foregone utility is usually 0 for all bids less than or equal to their values so one should expect random selection to occur for them.

The sellers are a bit more confusing. 0.85 converges to 0.87 (and is not random). 0.65 remains random in the range [0.65, 0.85]. 0.45 converges to 0.6 for 25 periods and then becomes random in the range [0.45, 0.82]. 0.25 seems to be random in the range [0.25, 0.4]. The seller with the lowest cost of 0.05 converges to 0.46.

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