
Cost-benefit analysis as a statistical hypothesis test: an example from urban transportation †

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Received 1 April 1970

Abstract. A method to evaluate the estimated social benefits and costs of many urban transportation decisions is presented, based on the techniques of statistical hypothesis testing. After a slight critique of current practice in cost-benefit analyses, a general equilibrium model is formulated that includes many of the relevant variables and decisions which interact with an urban transportation system. The model is based on individual behavior assumptions (as opposed to macro-behavior) and uses the concept that commodities provide attributes. The significant omissions are mentioned and discussed. It is next shown that if the model is true, then hypotheses involving the Pareto-superiority (and Pareto-optimality) of various decisions with respect to transportation systems imply hypotheses restricting the values of parameters of a single linear equation. This allows the testing (i.e. falsification) of the original hypotheses by using the familiar *T* and *F* tests of regression analysis. These tests are based on a single-equation regression only using observations on the rates of use of, the attributes provided by, and the inputs to, the transportation system.

The simplicity of the tests seems to indicate that revisions of the model towards reality would be very productive.

1 Introduction

Cost-benefit analysis has evolved as a technique used to answer questions concerning the social desirability of projects which are not correctly evaluated by the market system. The 'benefit/cost ratio' has become to the public production sector what profits have long been to the private production sector: a number which determines whether, and to what extent, production of a commodity should take place. The rationale for requiring private producers to increase, or begin, production of any commodity which increases profits is based on the theorems of welfare economics. In particular it is known that if (a) there are no externalities, (b) there are no monopolistic decision makers, (c) there is at least one desirable divisible commodity, and (d) there is a complete set of contingency markets⁽¹⁾, both present and future, then any change in production which increases profits, evaluated at market equilibrium prices, leads to a Pareto-superior reallocation of resources. Also, Pareto-optimality is achieved by profit maximization. The rationale for requiring public producers to increase, or begin, production of any commodity for which the benefit/cost ratio is greater than one is that this also leads to a Pareto-superior reallocation of resources. In fact, if (a) to (d) hold, then an increase in profits evaluated at equilibrium prices occurs if and only if the benefit/cost ratio is greater than one. However, if one of the hypotheses (a) to (d) fails to hold, then neither implication need be true, and positive profits no longer imply, necessarily, a

⁽¹⁾ By contingency markets is meant a market for each commodity in each possible state of the world. Contracts in such markets would read: "The purchaser agrees to accept delivery from the seller, who agrees to deliver, x units of commodity l if event e takes place. If event e does not occur, no delivery occurs".

† This paper was completed as part of a joint project for the Institute of Physical Planning in the School of Urban and Public Affairs and for the Transportation Research Institute, both of Carnegie-Mellon University.

Pareto-superior reallocation. The public production rule has been used to overcome this drawback, usually called 'market failure', of the private production rule.

Of particular interest, in the applications of both the private and public decision rules, is the revision of these rules, necessary to achieve Pareto-optimality, when assumption (d) does not hold. One area of current research in welfare economics is an attempt to present revisions which, in the presence of (a) to (c), will retain the desirable outcome of profit maximization when (d) does not hold (see, for example, Stigum, 1970). The problem is that, although there is a large literature involving decisions of consumers under uncertainty, which takes into account their tastes and subjective probabilities (see, for example, Savage, 1958; Fishburn, 1968), it is not clear, with limited contingency markets, that it is possible to assign a 'utility' function to each producer (one variable of which may be present discounted profits) such that maximization of the expected value of utility will imply, in equilibrium under (a) to (c), Pareto-optimal allocation of resources.

This problem with the private production rule also arises in the public production rule. One method of dealing with the problem has been to compute a number, called the present discounted (expected) value of benefits minus costs, in which the discount rate is set higher than the market rate to account for risk and uncertainty. While this, or some related approach, might be sufficient to account for attitudes towards risk, it is doubtful that the reporting of a single number includes all the information necessary to make a decision under uncertainty. In fact, the expected value of benefits minus costs is not a measure of the true social gains but simply a statistic which may, or may not, be near the true value. Obviously, the methods of statistical decision theory could be used to evaluate the reliability of the numbers derived in any cost-benefit study.

Two things underlie all cost-benefit studies: a set of data and a model of individual behavior (usually implicit). The data are used to *estimate* future behavior, and then these estimates are used to compute the impact of the proposed project on society measured by the statistic, *present discounted expected value of social benefits minus costs* (called, from now on, *expected net social gain*). Consider the following three hypotheses:

- H0: the actual social benefits minus costs of a proposed project are positive,
- H1: the model of individual behavior and its interaction is correct, and
- H2: the data available accurately measure those variables in the model to which they are applied.

The statistic, expected net social gain, tests the joint truth of H0 to H2. If, for example, H1 and H2 are both false, then it is possible that either the expected net social gain is negative and H0 is true or the expected net social gain is positive and H0 is false. In either case, the assumption, that the expected net social gain correctly measures the actual gains, is false. The current practice in reporting the results of cost-benefit studies obscures these considerations.

One approach, which clearly distinguishes assumptions from derived results, clarifies the relationship of the data to the model, and precisely indicates the probable truth of the hypothesis H0, is contained in the theory of statistical hypothesis testing. To illustrate this approach, an example is presented which is based on a model of an urban transportation system. Although the model does not seem to be a very accurate description of reality (its drawbacks are discussed later), it does incorporate some new ideas and represents a first step in this direction. Perhaps the most surprising result of the whole paper is that, although the model is of a general equilibrium type, the tests of the hypotheses, concerning the social costs and benefits of proposed changes in the transportation system, are based only on a single-equation regression. That is, a complicated set of hypotheses is reduced to hypotheses which restrict the

parameters of a single equation. If for no other reason, the simplicity of the application of the procedure outlined in this paper is an indication of the potential use of the approach.

2 A model of an urban transportation system

The model used in this section is based, in part, on the model of consumer behavior in Lancaster (1966). Consumers are assumed to receive various attributes (such as comfort, time, safety) from usage of a transportation system. The exact amount received depends on the rate of use by the consumer, on the rate of use by others, and on the product provided by the transportation sector. Some significant omissions are discussed at the end of this section.

2a Notation

$i = 1, \dots, I$ are consumers.

$n = 1, \dots, N$ are nodes in the transportation network (n may be an actual location or a transfer point).

$j = 1, \dots, J$ are links in the transportation network (j is described by the nodes it connects as well as the mode it represents).

$k = 1, \dots, K$ are attributes received from transportation use.

$X^i = (X_1^i, \dots, X_J^i)$ is the number of trips per time period made by i on each j .

$y^i = (y_1^i, \dots, y_N^i)$, where y_n^i is a vector of commodities purchased by i , at location n , per period.

$Z^i = (Z_1^i, \dots, Z_K^i)$ is the amount of attributes received by i per period.

$T^i = (T_1^i, \dots, T_N^i)$ is the number of trips per period by i to n .

$P = (P_1, \dots, P_J)$ is the price of a trip on j paid to the producer.

$P^* = (P_1^*, \dots, P_J^*)$ is the private price of a trip on j (the cost of gasoline, for instance).

$A = ((a_{kj}))$ is a K by J matrix where a_{kj} is the amount of attribute, k , received per trip on j .

$E^i = ((e_{rn}^i))$ is an R by N matrix where $e_{rn}^i = 1$ if i must traverse the physical link r to reach N , and $e_{rn}^i = 0$ otherwise. [$R = (N)(N-1)$].

$D = ((d_{rj}))$ is an R by J matrix where $d_{rj} = 1$ if link j allows movement along the physical link r and $d_{rj} = 0$ otherwise.

$q = (q^1, \dots, q^N)$ where q^n is the price vector of commodities, y_n , at n .

Q^i is the amount of taxes paid by i .

Since, in general, peak-period and non-peak-period travel are different, we will distinguish the peak-period values of all the above variables by placing a $\hat{\cdot}$ over the appropriate letter.

2b Individual behavior (consumers)

In this section, the first assumption of our model is specified. It is that consumers maximize utility subject to their budget constraints plus some travel constraints.

Behavioral hypothesis B1: Each consumer, $i = 1, \dots, I$ chooses $X^i, Z^i, T^i, \hat{X}^i, \hat{T}^i \geq 0$, and y^i to maximise

$$U^i \left(Z^i, \sum_{n=1}^N y_n^i \right) \tag{1}$$

subject to

$$Z^i - AX^i - \hat{A}\hat{X}^i = 0 \tag{1a} \quad DX^i - E^i T^i = 0 \tag{1b} \quad D\hat{X}^i - E^i \hat{T}^i = 0 \tag{1c}$$

$$F^i(y^i, T^i, \hat{T}^i) \leq 0 \tag{1d} \quad (p^* + p)X^i + (\hat{p}^* + \hat{p})\hat{X}^i + qy^i + Q^i = 0, \tag{1e}$$

where $A, \hat{A}, D, E^i, p^*, p, \hat{p}^*, \hat{p}, q$, and Q^i are given and F^i is a continuous vector-valued function describing what trips are needed to purchase and/or sell y^i . One component of y^i is i 's own labor⁽²⁾.

Technological hypothesis T1: The set of constraints (1a)–(1e) describe a closed non-empty convex set which satisfies a constraint qualification. U^i is a continuously differentiable, strictly concave function.

Of use later, are the first order necessary conditions of hypothesis B1. Letting the Lagrangian associated with problem (1) be

$$L^i = U^i\left(Z^i, \sum_{n=1}^N y_n^i\right) - \theta^i(Z^i - AX^i - \hat{A}\hat{X}^i) - \hat{\theta}^i F^i(y^i, T^i, \hat{T}^i) + \eta^i(DX^i - E^i T^i) \\ + \hat{\eta}^i(D\hat{X}^i - E^i \hat{T}^i) - \lambda^i[(p^* + p)X^i + (\hat{p}^* + \hat{p})\hat{X}^i + qy^i + Q^i]$$

where $\theta^i, \hat{\theta}^i, \eta^i, \hat{\eta}^i$, and λ^i are vectors of the appropriate dimension, we have: for each $i = 1, \dots, I$,

$$\frac{\partial L^i}{\partial Z_k^i} \equiv \frac{\partial U^i}{\partial Z_k^i} - \theta_k^i \leq 0, \quad (k = 1, \dots, K),$$

$$\frac{\partial L^i}{\partial y_n^i} \equiv \frac{\partial U^i}{\partial y_n^i} - \hat{\theta}^i \frac{\partial F^i}{\partial y_n^i} - \lambda^i q_n = 0, \quad (n = 1, \dots, N),$$

$$\frac{\partial L^i}{\partial X_j^i} \equiv \sum_{k=1}^K \theta_k^i a_{kj} + \sum_{r=1}^R \eta_r^i d_{rj} - \lambda^i (P_j + P_j^*) \leq 0, \quad (j = 1, \dots, J),$$

$$\frac{\partial L^i}{\partial \hat{X}_j^i} \equiv \sum_{k=1}^K \theta_k^i \hat{a}_{kj} + \sum_{r=1}^R \hat{\eta}_r^i d_{rj} - \lambda^i (\hat{P}_j + \hat{P}_j^*) \leq 0, \quad (j = 1, \dots, J),$$

$$\frac{\partial L^i}{\partial T_n^i} \equiv -\hat{\theta}^i \frac{\partial F^i}{\partial T_n^i} - \sum_{r=1}^R \eta_r^i e_{rn} \leq 0, \quad (n = 1, \dots, N),$$

$$\frac{\partial L^i}{\partial \hat{T}_n^i} \equiv -\hat{\theta}^i \frac{\partial F^i}{\partial \hat{T}_n^i} - \sum_{r=1}^R \hat{\eta}_r^i e_{rn} \leq 0, \quad (n = 1, \dots, N),$$

and the constraints (1a) to (1e).

At this point, it is useful to consider three drawbacks to this model of consumer behavior. The most obvious is the apparent lack of time as a dimension other than for distinguishing between peak and non-peak periods. There are two alternatives. The first is that each variable is described by its date (e.g. year 1968), increasing the dimensionality of the problem by the number of variables. This approach creates problems for the statistical analysis which follows since it implies there is only one observation. Alternatively one can assume that the consumer solves problem (1) each period and is myopic, thus increasing the number of observations and reducing the dimensionality while sacrificing some reality. It is my opinion that this reduction in reality is insignificant in comparison with errors in data measurement, and that shifts in the observed values over time of, for example, $\sum_{i=1}^I X^i$ are due more to exogenous changes in I than to changes in the X^i themselves.

The second drawback is less obvious but also less important. Constraint (1e) contains P^* without indicating its source. In fact P_j^* is derived from q plus some

⁽²⁾ The form of (1b)–(1d) can be arbitrary to a certain extent. For example, $f^i(y^i, X^i, \hat{X}^i) < 0$. However, their current form was chosen to indicate the inter-relationship between location, transportation, and commodity demands.

technological constraints which indicate the personal consumption purchases required to use link j (see Lancaster, 1966). However, this complicates unnecessarily the presentation without changing the results to follow. I leave it to the reader to verify this.

The third drawback is probably the most damaging. We have assumed that the consumer takes E^i as given. In fact, this is determined by i 's choice of his home location. Thus we are assuming that the locational distribution of consumers' homes is given. This is not unrealistic for, say, a period of one year. However, it neglects the fact that, over a longer time period, E^i is a decision variable of i and exogenous changes in the transportation system will alter the locational distribution of consumers⁽³⁾. It is hoped that this drawback will be adjusted for in future models.

2c Individual behavior (private producers)

We treat the private production sector as if it were one firm acting like a perfect competitor. That is, we assume that the private production sector is efficiently organized. As far as the analysis of the transportation system is concerned, it is relatively unimportant whether this is a 'correct' treatment⁽⁴⁾.

Behavioral hypothesis B2: The production sector acts as if it chooses y to maximise

$$qy \quad (2)$$

subject to

$$G(y) \leq 0 \quad (2a)$$

where G is a continuous vector-valued function describing the technological possibilities.

Technical hypothesis T2: The set of y such that $G(y) \leq 0$ is a closed, convex, non-empty set. The solution of problem (2) is unique.

As with the consumer, the first order implications of H2 are of interest. Letting $L \equiv qy - \epsilon G(y)$ be the Lagrangian of problem (2), we have

$$\frac{\partial L}{\partial y_n} \equiv q_n - \epsilon \frac{\partial G}{\partial y_n} = 0 \quad (n = 1, \dots, N)$$

and the constraint (2a).

The drawbacks of this hypothesis include those of H1 plus an explicit exclusion of the direct impact of the transportation sector on production decisions. Because shipments of both inputs and outputs employ the transportation system, the technological possibilities and costs of such utilization should be included in problem (2). They are excluded solely for expository purposes. Again, it is hoped that this drawback will be rectified in future work with this model.

2d Individual behavior (public production of 'transportation')

It is assumed for the present that public decisions affecting the matrices A , \hat{A} , and D as well as p , \hat{p} , and Q^1, \dots, Q^I are made arbitrarily (that is, there is no fixed decision rule), and are given by the data. It is the purpose of this analysis to indicate whether the given decision variables are set 'optimally' or whether there are changes which will move the economy to a Pareto-superior allocation of resources. We can, however, still describe the technological nature of public decisions. A peculiarity of transportation systems is that the matrices A and \hat{A} depend not only on the physical inputs of the transportation authority but also on the number of consumers using the system.

(3) I am unaware of any locational theory based on individual decision rules.

(4) See hypothesis T4 in Section 4.

Behavioral hypothesis B3: The public transportation sector purchases y^t , a vector of commodities, from the private sector at prices q , and uses these on the transportation network so that, letting β_j be the variable inputs used on j at non-peak hours, $\hat{\beta}_j$ be the variable inputs used on j at peak hours, and γ_j be the capital inputs used,

$$y^t = \sum_j (\beta_j + \hat{\beta}_j + \gamma_j) . \quad (3a)$$

The public sector also sets the prices P, \hat{P} and taxes Q^1, \dots, Q^I so that it has no deficit: that is:

$$P \sum_{i=1}^I X_i + \hat{P} \sum_{i=1}^I \hat{X}_i - qy^t + \sum_{i=1}^I Q^i \leq 0 . \quad (3b)$$

2e Equilibrium assumptions

The following hypothesis concerns the operation of the allocation process itself. We assume that private prices, P^*, \hat{P}^* , and q respond to excess demands. It is also assumed that the choices of consumers' trips are made by playing an I -person non-cooperative game. This makes the equilibrium allocation a Nash-equilibrium point.

Assuming that D, F^i, E^i , and G remain fixed and letting a_k be the k th row of A , then hypotheses B1 and T1 imply there exist, for each $i = 1, \dots, I, j = 1, \dots, J$, and $n = 1, \dots, N$, continuous demand functions⁽⁵⁾:

$$X_j^i = X_j^i(a_{1.}, \dots, a_{K.}, \hat{a}_{1.}, \dots, \hat{a}_{K.}, P, P^*, \hat{P}, \hat{P}^*, Q^i, q) ,$$

$$\hat{X}_j^i = \hat{X}_j^i(a_{1.}, \dots, q) ,$$

and

$$y_n^t = y_n^t(a_{1.}, \dots, q) .$$

We also assume there exist $2KJ$ continuous technological relations, for $k = 1, \dots, K$, and $j = 1, \dots, J$ ⁽⁶⁾:

$$a_{kj} - a_{kj}(X_1, \dots, X_J, \beta_1, \dots, \beta_J, \gamma_1, \dots, \gamma_J) \equiv 0 , \quad (4a)$$

and

$$\hat{a}_{kj} - \hat{a}_{kj}(\hat{X}_1, \dots, \hat{X}_J, \hat{\beta}_1, \dots, \hat{\beta}_J, \gamma_1, \dots, \gamma_J) \equiv 0 , \quad (4b)$$

where

$$X_j = \sum_{i=1}^I X_j^i .$$

It is tempting to assume, for example, that

$$a_{kj} = a_{kj}(X_j, \beta_j, \gamma_j) .$$

However, this is possible only if j is a separate right-of-way. Otherwise, X_j^i for $j' \neq j$ will have an effect on a_{kj} .

We let

$$E(a_{1.}, \dots, q, \beta, \hat{\beta}, \gamma) \equiv \sum_{j=1}^J (\hat{\beta}_j + \beta_j + \gamma_j) + \sum_{i=1}^I y^i(a_{1.}, \dots, q) - y(q)$$

be the excess demand function for private commodities, where $y(q)$ is the continuous 'supply curve' of the private production sector derived from hypotheses B2 and T2. The following hypothesis concerns the relation of the behavioral hypotheses and the observed values of the variables.

⁽⁵⁾ See Quandt and Baumol (1966) for similar functions.

⁽⁶⁾ See Mohring (1968) for similar functions.

Equilibrium hypothesis E1: At any point, the observations of the variables $X, \hat{X}, A, \hat{A}, q, \beta, \hat{\beta}, \gamma, \gamma^i$, and y satisfy

$$X_j - \sum_{i=1}^I X_j^i(a_1, \dots, q) = 0 \tag{5a}$$

$$\hat{X}_j - \sum_{i=1}^I \hat{X}_j^i(a_1, \dots, q) = 0 \tag{5b}$$

$$E(a_1, \dots, q, \beta, \hat{\beta}, \gamma) = 0 \tag{6}$$

and relations (4a) and (4b), and (4) to (6) have continuous first partials.

That is, shifts in the values of the public choice variables, $(p, \hat{p}, \beta, \hat{\beta}, \gamma)$, will cause shifts in the *observed* values⁽⁷⁾ of X, \hat{X}, A , and \hat{A} which are not predicted directly by Equations (4) or (5). The exact shifts can be determined by calculating the reduced form of Equations (4) to (6).

We let

$$H \equiv \frac{\partial(a, \hat{a}, X, \hat{X}, E)}{\partial(a, \hat{a}, X, \hat{X}, q)}$$

be the Jacobian of the system (4) to (6) where

$$H \equiv \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & \frac{\partial(a)}{\partial(X)} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial(\hat{a})}{\partial(\hat{X})} & 0 \\ \frac{\partial(X)}{\partial(a)} & \frac{\partial(X)}{\partial(\hat{a})} & 0 & 0 & \frac{\partial(X)}{\partial(q)} \\ \frac{\partial(\hat{X})}{\partial(a)} & \frac{\partial(\hat{X})}{\partial(\hat{a})} & 0 & 0 & \frac{\partial(\hat{X})}{\partial(q)} \\ \frac{\partial E}{\partial(a)} & \frac{\partial E}{\partial(\hat{a})} & 0 & 0 & \frac{\partial E}{\partial q} \end{bmatrix}$$

and, for example,

$$\frac{\partial(a)}{\partial(X)} = \begin{bmatrix} \frac{\partial a_{11}}{\partial X_1} & \dots & \frac{\partial a_{11}}{\partial X_J} \\ \vdots & & \vdots \\ \frac{\partial a_{KJ}}{\partial X_1} & \dots & \frac{\partial a_{KJ}}{\partial X_J} \end{bmatrix}$$

To assure the existence of the reduced-form equations, we assume

Equilibrium hypothesis E2: For any observable values of the variables, $|H| \neq 0$.

Combining hypotheses E1 and E2, we have, by the implicit function theorem, that there exist continuously differentiable vector-valued functions, g^1, g^2, g^3, g^4 , and g^5 .

⁽⁷⁾ It is at this point that divergence from the analysis in Quandt and Baumol (1966) occurs.

such that for $k = 1, \dots, K, j = 1, \dots, J$,

$$a_{kj} = g_{kj}^1(\beta, \hat{\beta}, \gamma, p, \hat{p}, Q) \quad (7a) \quad \hat{a}_{kj} = g_{kj}^2(\beta, \dots, Q) \quad (7b)$$

$$X_j = g_j^3(\beta, \dots, Q) \quad (7c) \quad \hat{X}_j = g_j^4(\beta, \dots, Q) \quad (7d) \quad q = g^5(\beta, \dots, Q) \quad (7e)$$

It is these functions which are involved in estimation procedures and which are used to evaluate public sector decisions. Of direct interest in both these tasks, is the matrix of first partials of Equations (7),

$$\Delta \equiv \frac{\partial(g^1, \dots, g^5)}{\partial(\beta, \dots, Q)}$$

By the implicit function theorem,

$$\Delta = H^{-1} \times \begin{bmatrix} \frac{\partial(a)}{\partial(\beta)} & 0 & \frac{\partial(a)}{\partial(\gamma)} & 0 \\ 0 & \frac{\partial(\hat{a})}{\partial(\hat{\beta})} & \frac{\partial(\hat{a})}{\partial(\gamma)} & 0 \\ 0 & 0 & 0 & \frac{\partial(X)}{\partial(s)} \\ 0 & 0 & 0 & \frac{\partial(\hat{X})}{\partial(s)} \\ I & I & I & \frac{\partial(E)}{\partial(s)} \end{bmatrix} \quad \text{where } s \equiv (P, \hat{P}, Q^1, \dots, Q^J).$$

In passing, it should be noted that H , and therefore H^{-1} , is a $2KJ + 2J + \dim(q)$ square matrix and Δ is a $2KJ + 2J + \dim(q)$ by $\dim(\beta) + \dim(\hat{\beta}) + \dim(\gamma) + \dim(s)$ matrix where $\dim(\cdot)$ is the dimension of the vector in the parentheses.

At this point the basic model and its relation to the observations is complete. It is useful to reiterate two of its drawbacks and their impact on cost-benefit calculations. The first is that the impact of the transportation system on the production sector (such as reduced locational disadvantages, or lower shipping costs) is ignored. This obviously leads to an understatement of social benefits due to increased transportation capacity. The second is that the relocation of consumers and producers in response to changes in the transportation system is ignored. The impact of this omission on cost-benefit calculations is subtle but important. It is my feeling that, in most medium to large urban areas, the omission leads to an *overstatement* of the benefits of a CBD-suburb oriented 'corridor' system and an *understatement* of the benefits of a grid system oriented to a diffusion of both work and residential location. This feeling derives from observations that, due to sound economic reasons (such as land acquisition costs in the CBD as opposed to the fringes), new work locations will not be in the CBD but in the perimeter areas of the urban center.

3 Optimality of public decisions

To provide a basis for cost-benefit calculations, the conditions for 'optimal' government transportation decisions are explored. The approach is similar to Mohring's (1968). In this section, it is assumed throughout that hypotheses B1, B2, T1, T2, E1, and E2 always hold. Necessary and sufficient marginal conditions, for government decisions to be Pareto-optimal under these hypotheses, are presented. Under hypotheses T1 and T2, these also indicate a method for evaluation of proposed marginal transportation alterations.

We assume social welfare is measured by a continuously differentiable real-valued function $W(U^1, \dots, U^I)$ where W is increasing in each U^i . The existence of such a function is assured since, corresponding to each Pareto-optimal allocation, there exist I positive constants, Φ^1, \dots, Φ^I , such that maximization of $W(U^1, \dots, U^I) = \sum_{i=1}^I \Phi^i U^i$ will select that Pareto-optimal allocation.

Optimal decisions thus occur if the government, by selecting $\beta, \hat{\beta}, \gamma, P, \hat{P}$, and Q , maximizes

$$W(U^1, \dots, U^I) \tag{8}$$

subject to

$$P \sum_{i=1}^I X_i + \hat{P} \sum_{i=1}^I \hat{X}_i - q \sum_{j=1}^J (\beta_j + \hat{\beta}_j + \gamma_j) + \sum_{i=1}^I Q^i = 0 \tag{3b'}$$

and equilibrium conditions (4), (5), and (6).

Under equilibrium hypotheses E1 and E2, Equations (4), (5), and (6) hold if, and only if, Equations (7) hold. Thus, substituting the appropriate Equations of (7) into (8) and (3b) reduces the problem to maximizing (8) subject to (3) over the same variables.

Technical hypothesis T3: The constraints, (3b), (4), (5), and (6), describe a non-empty, compact⁽⁸⁾, convex set and satisfy a constraint qualification.

Although separate conditions on the a_{kj} functions and the demand and supply curves of the participants could be made to insure hypothesis T3, this approach has been chosen for expository clarity.

The Lagrangian associated with Equation (8) is

$$L^* \equiv W(U^1, \dots, U^I) - \eta^0 [PX + P\hat{X} - qy^t + \sum_{i=1}^I Q^i]$$

where $X, \hat{X}, a, \hat{a}, y^t, y$, and q are all functions of $(\beta, \hat{\beta}, \gamma, P, \hat{P}, Q)$ defined by Equations (7). Letting $\partial a_{kj} / \partial \gamma_e$, for example, be the appropriate entry of Δ (which is a matrix of constrained differentials), we can write some of the first order necessary conditions of the problem (8) as:

for $s_e = \beta$ or $\hat{\beta}$ or γ ,

$$\begin{aligned} \frac{\partial L^*}{\partial s_e} = & \sum_{i=1}^I \frac{\partial W}{\partial U^i} \left\{ \sum_{k=1}^K \frac{\partial U^i}{\partial z_k} \left[\sum_{j=1}^J \left(\frac{\partial a_{kj}}{\partial s_e} X_j^i + \frac{\partial \hat{a}_{kj}}{\partial s_e} \hat{X}_j^i \right) + \sum_{j=1}^J \left(a_{kj} \frac{\partial X_j^i}{\partial s_e} + \hat{a}_{kj} \frac{\partial \hat{X}_j^i}{\partial s_e} \right) \right] \right. \\ & \left. + \sum_{n=1}^N \frac{\partial U^i}{\partial Y_n^i} \left(\frac{\partial Y_n^i}{\partial s_e} \right) \right\} + \eta^0 \left[\sum_{j=1}^J \left(P_j \frac{\partial X_j}{\partial s_e} + \hat{P}_j \frac{\partial \hat{X}_j}{\partial s_e} \right) - \left(\frac{\partial q}{\partial s_e} y^t + q \frac{\partial y^t}{\partial s_e} \right) \right] = 0. \end{aligned}$$

Substituting some of the first-order conditions for individual consumer maximization (see Section 2b) gives:

$$\begin{aligned} \frac{\partial L^*}{\partial s_e} = & \sum_{i=1}^I \frac{\partial W}{\partial U^i} \left\{ \left[\sum_{k=1}^K \frac{\partial U^i}{\partial z_k} \left(\sum_{j=1}^J \frac{\partial a_{kj}}{\partial s_e} X_j^i + \frac{\partial \hat{a}_{kj}}{\partial s_e} \hat{X}_j^i \right) \right] + \left[\sum_{j=1}^J \left(\lambda^i [P_j + \hat{P}_j] - \sum_{r=1}^R \eta_r^i d_{rj} \right) \frac{\partial X_j^i}{\partial s_e} \right. \right. \\ & \left. \left. + \left(\lambda^i [\hat{P}_j + \hat{P}_j^*] - \sum_{r=1}^R \hat{\eta}_r^i d_{rj} \right) \frac{\partial \hat{X}_j^i}{\partial s_e} \right] + \left[\sum_{n=1}^N (\lambda^i q_n) \frac{\partial y_n^i}{\partial s_e} + \hat{\theta}^i \frac{\partial F^i}{\partial y_n^i} \frac{\partial y_n^i}{\partial s_e} \right] \right\} \\ & + \eta^0 \left[\sum_{j=1}^J \left(P_j \frac{\partial X_j}{\partial s_e} + \hat{P}_j \frac{\partial \hat{X}_j}{\partial s_e} \right) - \left(\frac{\partial q}{\partial s_e} y^t + q \frac{\partial y^t}{\partial s_e} \right) \right] = 0. \end{aligned}$$

⁽⁸⁾ This eliminates from consideration some forms of increasing returns-to-scale phenomena which often arise in transportation problems.

But, we have assumed that D , E^i , and F^i are fixed. Therefore,

$$\sum_{r=1}^R \eta_r^i \left(\sum_{n=1}^N e_{rn}^i \frac{\partial T_n^i}{\partial s_v} - \sum_{j=1}^J d_{rj} \frac{\partial X_j^i}{\partial s_v} \right) = 0,$$

$$\sum_{r=1}^R \hat{\eta}_r^i \left(\sum_{n=1}^N e_{rn}^i \frac{\partial \hat{T}_n^i}{\partial s_v} - \sum_{j=1}^J d_{rj} \frac{\partial \hat{X}_j^i}{\partial s_v} \right) = 0,$$

$$\hat{\theta}^i \left(\frac{\partial F^i \partial y_n^i}{\partial y_n^i \partial s_v} + \frac{\partial F^i \partial T_n^i}{\partial T_n^i \partial s_v} + \frac{\partial F^i \partial \hat{T}_n^i}{\partial \hat{T}_n^i \partial s_v} \right) = 0,$$

must be satisfied by the *observable* variables. Substituting these relations gives:

$$\begin{aligned} \frac{\partial L^*}{\partial s_v} = & \left[\sum_{i=1}^I \frac{\partial W}{\partial U^i} \sum_{k=1}^K \frac{\partial U^i}{\partial Z_k} \left(\sum_{j=1}^J \frac{\partial a_{kj}}{\partial s_v} X_j^i + \frac{\partial \hat{a}_{kj}}{\partial s_v} \hat{X}_j^i \right) \right] \\ & + \eta^0 \left[\left(\sum_{j=1}^J P_j \frac{\partial X_j^i}{\partial s_v} + \hat{P}_j \frac{\partial \hat{X}_j^i}{\partial s_v} \right) - \left(\frac{\partial q}{\partial s_v} y^i + q \frac{\partial y^i}{\partial s_v} \right) \right] = 0. \end{aligned}$$

The first bracketed term is the marginal social benefit of ∂s_v . The second bracketed term is the sum of the marginal increase in transportation revenues minus the marginal cost of ∂s_v . Thus, $\partial L^*/\partial s_v$ is the net marginal social gain from ∂s_v .

If we let, for $kj = 1, \dots, KJ$,

$$V_{kj} \equiv \frac{1}{\eta^0} \sum_{i=1}^I \frac{\partial W}{\partial U^i} \frac{\partial U^i}{\partial Z_k} X_j^i,$$

which is the *marginal social* valuation of ∂a_{kj} , and

$$\frac{\partial C}{\partial(\beta, \hat{\beta}, \gamma)} \equiv q \frac{\partial y^i}{\partial(\beta, \hat{\beta}, \gamma)} + \frac{\partial(q)}{\partial(\beta, \hat{\beta}, \gamma)} y^i,$$

which is the *marginal cost of the program* ($\partial\beta, \partial\hat{\beta}, \partial\gamma$), then

$$\frac{\partial L^*}{\partial(\beta, \hat{\beta}, \gamma)} = (V_1, \dots, V_{K..}, \hat{V}_1, \dots, \hat{V}_{K..}, P, \hat{P}, 0) H^{-1} \times \begin{bmatrix} \frac{\partial(a)}{\partial(\beta)} & 0 & \frac{\partial(a)}{\partial(\gamma)} \\ 0 & \frac{\partial(\hat{a})}{\partial(\hat{\beta})} & \frac{\partial(\hat{a})}{\partial(\gamma)} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ I & I & I \end{bmatrix}$$

$$-\frac{\partial C}{\partial(\beta, \hat{\beta}, \gamma)} = 0.$$

Referring to Section 2e and the definition of Δ , we see that if Δ_1 is the matrix composed of the first three groups of columns of Δ , then

$$\frac{\partial L^*}{\partial(\beta, \hat{\beta}, \gamma)} = (V, \hat{V}, P, \hat{P}; 0) \Delta_1 - \frac{\partial C}{\partial(\beta, \hat{\beta}, \gamma)}$$

or

$$\frac{\partial L^*}{\partial(\beta, \hat{\beta}, \gamma)} = (V, \hat{V}, P, \hat{P}, 0) \frac{\partial(g^1, \dots, g^5)}{\partial(\beta, \hat{\beta}, \gamma)} - \frac{\partial C}{\partial(\beta, \hat{\beta}, \gamma)} \tag{9}$$

which equals zero if the optimal decision has been made.

The interpretation of Equation (9) is straightforward. The first term on the right of the equality is the marginal social benefit of $(\beta, \hat{\beta}, \gamma)$, taking account of the fact that equilibrium hypotheses E1 and E2 hold, and the second term is the marginal social cost. Under the hypotheses T1 and T2, especially the convexity and concavity assumptions, if $\partial L^*/\partial s_p$ is positive (negative) then social welfare is increased (decreased) by a marginal increase in s_p .

Now let us consider the effect of changes in transit prices, (P, \hat{P}) , and head taxes, Q^1, \dots, Q^I . Proceeding as above,

$$\frac{\partial L^*}{\partial P_j} = (V, \hat{V}, P, \hat{P}, 0)H^{-1} \times \begin{bmatrix} 0 \\ 0 \\ \frac{\partial(\hat{X})}{\partial P_j} \\ \frac{\partial(X)}{\partial P_j} \\ \sum_{i=1}^I \frac{\partial(y^i)}{\partial P_j} \end{bmatrix} - \sum_{i=1}^I \frac{1}{\eta^0} \frac{\partial W}{\partial U^i} \lambda^i X_i^i + X_j = 0, \quad (10)$$

$$\frac{\partial L^*}{\partial Q^i} = (V, \hat{V}, P, \hat{P}, 0)H^{-1} \times \begin{bmatrix} 0 \\ 0 \\ \frac{\partial(X^i)}{\partial Q^i} \\ \frac{\partial(\hat{X}^i)}{\partial Q^i} \\ \frac{\partial(y^i)}{\partial Q^i} \end{bmatrix} - \frac{1}{\eta^0} \frac{\partial W}{\partial U^i} \lambda^{i+1} = 0. \quad (11)$$

Multiplying Equation (11) by X_i^i , \hat{X}_i^i , and y^i , summing over all i , and subtracting from Equation (10) gives

$$(V, \hat{V}, P, \hat{P}, 0)H^{-1} \times \begin{bmatrix} 0 \\ 0 \\ \frac{\partial(X)}{\partial(P, \hat{P})} - \sum_i \frac{\partial(X)}{\partial Q^i} X^i \\ \frac{\partial(\hat{X})}{\partial(P, \hat{P})} - \sum_i \frac{\partial(\hat{X})}{\partial Q^i} \hat{X}^i \\ \sum_i \frac{\partial y^i}{\partial(P, \hat{P})} - \sum_i \frac{\partial y^i}{\partial Q^i} y^i \end{bmatrix} = 0. \quad (12)$$

Letting :

$$H = \begin{bmatrix} I & -V_a & 0 \\ -V_x & I & -V_{xq} \\ -V_{ea} & 0 & -V_{eq} \end{bmatrix}$$

then

$$H^{-1} = \begin{bmatrix} M & MV_a & -MV_a V_{xq} V_{eq}^{-1} \\ N(V_x - V_{xq} V_{eq}^{-1} V_{ea}) & N & -NV_{xq} V_{eq}^{-1} \\ +QV_{ea}(I - V_a V_x)^{-1} & QV_{ea}(I - V_a V_x)^{-1} V_a & Q \end{bmatrix}$$

where

$$M \equiv [I - V_a V_x + V_a V_{xq} V_{eq}^{-1} V_{ea}]^{-1}$$

$$N \equiv [I - V_x V_a + V_{xq} V_{eq}^{-1} V_{ea} V_a]^{-1}$$

and

$$Q \equiv -[V_{ea}(I - V_a V_x)^{-1} V_a V_{xq} V_{eq}]^{-1}$$

Now, letting

$$S_j^x = \frac{\partial(X)}{\partial P_j} - \sum_i \frac{\partial(X^i)}{\partial Q^i}$$

we have that

$$(V, \hat{V}, P, \hat{P}, 0) \times \begin{bmatrix} MV_a & -MV_a V_{xq} V_{eq}^{-1} \\ N & -NV_{xq} V_{eq}^{-1} \end{bmatrix} \times \begin{bmatrix} S_j^x & S_j^z \\ S_j^z & S_j^z \\ S_j^y & S_j^y \end{bmatrix} = 0 \quad (13)$$

But the concavity of U^i in hypothesis T1 implies that

$$\begin{vmatrix} S^x \\ S^z \\ S^y \end{vmatrix} \neq 0$$

since the matrix is composed of Slutsky-Hicks compensated substitution terms and is negative definite. Therefore, for Equation (13) to hold it must be true that

$$(V, \hat{V})MV_a + (P, \hat{P})N = 0$$

or

$$(P, \hat{P}) = -(V, \hat{V})MV_a N^{-1}$$

But this implies that

$$(P, \hat{P}) = -(V, \hat{V})V_a, \quad (14)$$

which is equivalent to

$$P_j = -V \frac{\partial(a)}{\partial X_j} \quad \text{and} \quad \hat{P}_j = -\hat{V} \frac{\partial(\hat{a})}{\partial \hat{X}_j} \quad \text{for } j = 1, \dots, J.$$

That is, the price per trip on link j should equal the marginal social cost of an additional user on the link. *In extenso*,

$$P_j = -\frac{1}{\eta^0} \sum_{i=1}^I \frac{\partial W}{\partial U^i} \sum_{k=1}^K \frac{\partial U^i}{\partial Z_k} \frac{\partial a_{kj}}{\partial X_j} X_j^i .$$

In passing, we note that

$$P_j = 0 \quad \text{if and only if} \quad \frac{\partial(a)}{\partial X_j} = 0$$

which in general is not true. We can state as

Theorem 1: Under hypotheses B1, B2, T1, T2, E1, and E2, prices and taxes are set optimally, with respect to

$$-W(U^1, \dots, U^I) \quad \text{if, and only if,} \quad (P, \hat{P}) = -(V, \hat{V}) V_a$$

and

$$\lambda^i \frac{\partial W}{\partial U^i} = \eta^0 \quad \text{for } i = 1, \dots, I .$$

This result implies that if $W(U^1, \dots, U^I) = \eta^0 \sum_{i=1}^I \frac{1}{\lambda^i} U^i$, then Equations (14) are both necessary and sufficient for optimal prices, no matter what taxes are charged. This indicates that decisions with respect to Q^i are important only if one is wedded to a particular Pareto-optimum (or distribution of income). However, using the social welfare function, $\eta^0 \sum_{i=1}^I \frac{1}{\lambda^i} U^i$, places a higher social evaluation on those whose 'marginal utility for money', λ^i , is lower; that is, on the relatively rich.

As a final remark, if we substitute Equation (14) into Equation (9), we get after some manipulation that, *at optimal prices and taxes*, resources are allocated optimally, to transportation, if

$$(V, \hat{V}) \frac{\partial(a, \hat{a})}{\partial(\beta, \hat{\beta}, \gamma)} = \frac{\partial C}{\partial(\beta, \hat{\beta}, \gamma)} .$$

That is, if prices remain optimal, we can work directly with the original functions for a_{kj} and ignore the shifts in equilibrium. The weights (V, \hat{V}) occur instead of (P, \hat{P}) since $(\beta, \hat{\beta}, \gamma)$ are 'public' goods in this model.

4 Evaluating proposed projects and current decisions

Although the preceding section is of some interest in its own right, it is of no use in cost-benefit analyses unless the various functions are known with certainty. This type of certain knowledge very rarely exists. The technique proposed in this section is one possible way to overcome the uncertainty facing the decision maker. The technique is based on some known statistical hypothesis tests.

Ignoring the values of the taxes (subsidies), Q^1, \dots, Q^I , needed to ensure no deficit (surplus) in the transportation sector (since this is not a decision which affects 'efficiency'), we propose to test two types of hypotheses. The first asks whether the current parameter choices of the public (transportation) production sector are optimally chosen. A related hypothesis asks whether a particular parameter should be increased, decreased, or left alone. The second type of hypothesis concerns the costs and benefits of a proposed program for change.

To allow the application of tests involving linear models, we are forced to make some additional technical hypotheses. It would, of course, be nice if it were possible to assume a non-linear model; however, since this paper is merely an indication of a

possible methodology, and since the techniques of non-linear estimation are not well developed, we shall resort to linear approximations.

We proceed by referring again to the reduced-form equations of the model, Equations (7). Taking the Taylor series expansion of these, around some point, gives a set of linear equations:

$$W = \Delta\alpha + \delta + \epsilon \quad (15)$$

where

$$W \equiv (a_1, \dots, a_k, X_1, \dots, X_J, q)'$$

Δ is the matrix defined earlier, $\alpha \equiv (\beta, \hat{\beta}, \gamma, P, \hat{P})'$, δ is a column vector of constants, and ϵ is a column vector of error terms. We assume

Technical hypothesis T3: In addition to the previous hypotheses, the reduced-form Equations (7) are related to the Equations (15) through the observations of (W, α) in the following way: let (W_n, α_n) , $n = 1, \dots, N$, be the N observations. Then, W_n is drawn from $N(\Delta\alpha_n + \delta, \Sigma)$, the multivariate normal with mean $\Delta\alpha_n + \delta$ and covariance, Σ . Also, W_n is distributed independently of W_m for $m \neq n$. That is, if $\epsilon_n = \Delta\alpha_n + \delta - g(\alpha_n)$, then ϵ_n is drawn from $N(0, \Sigma)$.

Under hypothesis T3, it is easy to estimate Δ (see Anderson, 1958, chapter 8). Since it is generally infeasible to utilize observations on the vector of private prices, q , due to the implied dimensionality of the model, it will be assumed that

Technical hypothesis T4: In Equation (7e)

$$\frac{\partial g^s}{\partial(\beta, \hat{\beta}, \gamma, P, \hat{P})} \equiv 0.$$

That is, 'marginal' shifts in the transportation parameters do not affect equilibrium prices in the private markets.

Under hypothesis T4, we need only consider

$$W^* = \Delta^*\alpha + \delta + \epsilon \quad (16)$$

where Δ^* is Δ minus the last $\dim(q)$ rows and $W^* = (a, \hat{a}, X, \hat{X})$. That is, we 'separate' the transportation sector from the rest of the economy.

We now consider some null hypotheses which it is desirable to test.

Null hypothesis N1: For each decision variable, $\beta_1, \dots, \beta_L, \hat{\beta}_1, \dots, \hat{\beta}_L, \gamma_1, \dots, \gamma_L, P_1, \dots, P_J, \hat{P}_1, \dots, \hat{P}_J$, that variable should be increased to achieve a Pareto-superior allocation.

Null hypothesis N2: The current values of the decision variables achieve a Pareto-optimal allocation.

Null hypothesis N3: Given a proposed alteration in the transportation decision variables, $(\Delta\beta, \Delta\hat{\beta}, \Delta\gamma, \Delta P, \Delta\hat{P})$, the proposal leads to a Pareto-superior allocation⁽⁹⁾.

Lemma 1: Under hypotheses B1, B2, T1 to T4, E1, and E2:

(a) the hypothesis N1 implies

$$(V, \hat{V}, P, \hat{P})\Delta_s^* - \frac{\partial C}{\partial s} > 0 \quad \text{for } s = \beta_1, \dots, \gamma_L. \quad (17a)$$

⁽⁹⁾ This only allows evaluation of 'marginal' changes in existing transportation parameters. Analysis of, for example, new modes is not amenable to these techniques. Perhaps a reformulation of consumer choice, which would yield demand functions similar to those in Quandt and Baumol (1966), would allow consideration of plans for new modes.

and

$$(V, \hat{V}, P, \hat{P})\Delta_s^* > 0 \quad \text{for } s = P_1, \dots, \hat{P}_J, \tag{17b}$$

where Δ_s^* is the appropriate column of Δ^* ;

(b) the hypothesis N2 implies

$$(V, \hat{V}, P, \hat{P})\Delta_s^* - \frac{\partial C}{\partial s} = 0 \quad \text{for all } s = \beta_1, \dots, \gamma_L \tag{18a}$$

and

$$(V, \hat{V}, P, \hat{P})\Delta_s^* = 0 \quad \text{for all } s = P_1, \dots, \hat{P}_J; \tag{18b}$$

(c) with the additional hypothesis T5, see below, the hypothesis T3 implies

$$(V, \hat{V}, P, \hat{P})\Delta^* \times \begin{bmatrix} \Delta\beta \\ \Delta\hat{\beta} \\ \Delta\gamma \\ \Delta P \\ \Delta\hat{P} \end{bmatrix} - \frac{\partial C}{\partial(\beta, \hat{\beta}, \gamma)} \times \begin{bmatrix} \Delta\beta \\ \Delta\hat{\beta} \\ \Delta\gamma \end{bmatrix} > 0. \tag{19}$$

The additional hypothesis needed in (c) is

Technical hypothesis T5: For the proposed change, $(\Delta\beta, \dots, \Delta\hat{P})$, the values of V, \hat{V} are constant, and either $\partial C/\partial(\beta, \hat{\beta}, \gamma)$ is constant or the cost of $(\Delta\beta, \Delta\hat{\beta}, \Delta\gamma)$, say

$$C(\Delta\beta, \Delta\hat{\beta}, \Delta\gamma) \text{ is known and substituted for } \frac{\partial C}{\partial(\beta, \dots, \gamma)} \times \begin{bmatrix} \Delta\beta \\ \Delta\hat{\beta} \\ \Delta\gamma \end{bmatrix}$$

Given (V, \hat{V}, P, \hat{P}) and $\partial C/\partial(\beta, \hat{\beta}, \gamma)$, the numbers to the left of the equality or inequality signs in Equations (17) and (18) are linear combinations of the elements of Δ^* . Let

$$b_n = (V, \hat{V}, P, \hat{P})W_n^*$$

$$d = (V, \hat{V}, P, \hat{P})\Delta^*$$

and

$$e = (V, \hat{V}, P, \hat{P})\delta$$

where (V, \hat{V}, P, \hat{P}) is the given value of this vector. Then (by Theorem 2.45; Anderson, 1958), b_n is distributed according to $N(d\alpha_n + e, d\Sigma d')$ for $n = 1, \dots, N$, independently of b_m for $m \neq n$. All the null hypotheses can be expressed in terms of d . That is, we have:

(a) Null hypothesis N1 holds for

$$s = \beta_1, \dots, \gamma_L \text{ if } d_s > \frac{\partial C}{\partial s},$$

and for

$$s = P_1, \dots, \hat{P}_J \text{ if } d_s > 0.$$

(b) Null hypothesis N2 holds if

$$d = \left[\frac{\partial C}{\partial(\beta, \hat{\beta}, \gamma)}, 0 \right].$$

(c) Null hypothesis N3 holds if

$$d \times \begin{bmatrix} \Delta\beta \\ \cdot \\ \Delta\hat{P} \end{bmatrix} - \frac{\partial C}{\partial(\beta, \hat{\beta}, \gamma)} \times \begin{bmatrix} \Delta\beta \\ \Delta\hat{\beta} \\ \Delta\gamma \end{bmatrix} > 0.$$

Thus, if our model is true, then it implies that we have reduced the testing procedure to the evaluation of hypotheses concerning the parameters of a *single-equation regression model*. The implications and tests of these hypotheses are well known (see Anderson, 1958, chapter 8; or Johnston, 1963, chapter 4).

In summary, the approach suggested in this paper involves the following steps:

Step 1: Collect data on observations of the variables $a_{11}, \dots, a_{kj}, \hat{a}_{11}, \dots, \hat{a}_{kj}, X_1, \dots, X_J, \hat{X}_1, \dots, \hat{X}_J, \beta_1, \dots, \gamma_L$.

Step 2: For each observation compute, given (V, \hat{V}, P, \hat{P}) , the number

$$b = (V, \hat{V}, P, \hat{P}) \times \begin{bmatrix} a \\ \hat{a} \\ X \\ \hat{X} \end{bmatrix}.$$

Step 3: Utilizing the single equation model

$$b = d \times \begin{bmatrix} \beta' \\ \hat{\beta}' \\ \gamma' \end{bmatrix} + e + \bar{e},$$

where $\bar{e} \sim N(0, \sigma)$, calculate the appropriate test statistics for each null hypothesis. For N1, these are *t*-statistics, for N2 an *F*-statistic, and for N3 a *t*-statistic.

Step 4: Given the appropriate confidence region (a one-tailed region determined by the willingness to reject the null hypothesis when it is true), if the computed statistic lies outside this region, then reject the appropriate null hypothesis.

We thus have a procedure for analyzing questions involving the Pareto-optimality of current public transportation decisions (N2), the Pareto-superiority of proposed decisions (N3, the cost-benefit hypothesis), and directions in which Pareto-superior decisions might occur (N1). Obviously, there are other hypotheses which one might want to consider. The ones in this paper were selected as of critical interest in public decisions.

A word of caution is appropriate at this point. Although the procedure has been presented as a test of the null hypothesis, it logically tests the combination of H_m and the null hypothesis where $H_m = (B1, B2, T1-T5, E1, E2)$. Thus, if H_m is false, rejection of a true null hypothesis or acceptance of a false null hypothesis might occur. It would, therefore, be desirable to explore further some implications of H_m (for instance Test E2) to test the validity of that part of the model. In the next section, various *a priori* rationales for considering H_m false, and possible work to make H_m acceptable are considered.

5 Indicated future work

Several drawbacks to the procedure outlined in this paper have already been mentioned. In particular, these included implicit treatment of time as a relevant dimension, no allowance for locational decisions of either consumers or producers,

the assumption of perfect competition in the private production sector, and that sector's non-use of the transportation system.

Some other considerations are equally capable of creating problems in applying the procedure in this model. One is the assumption that all attributes are measurable. Another is the requirement that (V, \hat{V}) be measured exogenously to the model. Since V and \hat{V} are the current social valuation of the transportation attributes, this problem is not unique to this model. In fact, I am unaware of any model which provides a basis for a cost-benefit analysis which does not contain this drawback. The reason is that these attributes are essentially public goods and there is no known mechanism for forcing consumers to reveal how they value such attributes⁽¹⁰⁾. The two problems in this paragraph will hinder the use of any model, no matter how complicated, until solutions exist.

There are other variations on the model in this paper which need to be considered. One relies on the development of a theory of locational choice and its interaction with transportation, which is based on rational individual decision making, and which is amenable to analytic solution. As far as I know, none exists. Another variation is to treat the consumer's, and producer's, problem as choice under uncertainty. In particular, one could assume that A and \hat{A} are stochastic, since they depend on X_1, \dots, X_J , which are unknown. It is my feeling that this will not significantly change the derived results of this model. I also feel that this is the appropriate approach for analyzing the impact of advertising and other information on consumer choice and its value to society. Work in this direction is already under discussion.

Finally, it should be noted that cost-benefit calculation is only a rule by which public decisions are currently made. If a more efficient decision rule could be devised, as well as an incentive system based on this rule, similar to 'maximizing profits', which leads to Pareto-optimal public choices, then the extensive computations involved in cost-benefit analysis could be foregone. It is my feeling that research in this direction is of more importance than any other mentioned but is also more difficult.

References

- Anderson, T. W., 1958, *Introduction to Multivariate Statistical Analysis* (John Wiley, New York).
 Fishburn, P., 1968, "Utility theory", *Management Science*, 14, 335-378.
 Johnston, J., 1963, *Econometric Methods* (McGraw-Hill, New York).
 Lancaster, K., 1966, "A new approach to consumer theory", *Journal of Political Economy*, 74, 132-157.
 Mohring, H., 1968, "Peak-loads, increasing returns, and the welfare costs of non-optimal pricing and investment policies", mimeographed, University of Minnesota, Minneapolis, Minnesota, USA.
 Savage, J., 1958, *The Foundations of Statistics* (John Wiley, New York).
 Stigum, B., 1970, "Competitive equilibrium under uncertainty", *Quarterly Journal of Economics* (forthcoming).
 Quandt, R., Baumol, W., 1966, "The demand for abstract transport modes: theory and measurement", *Journal of Regional Science*, 6, 13-26.

(10) It is also not clear whether simultaneous observations on $V, \hat{V}, A, \hat{A}, X, \hat{X}, \beta, \hat{\beta}, \gamma, P$, and \hat{P} are feasible. I thank Nancy Schwartz for pointing this out. However, it is possible that data collection methods will improve when a reasonable model exists, as occurred with National Income Accounting.