

# Supplemental data for “Shock tunnel operation and correlation of boundary layer transition on a cone in hypervelocity flow”

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## 1 Introduction

This is a supplemental report for “Shock tunnel operation and correlation of boundary layer transition on a cone in hypervelocity flow” by J.S. Jewell, J.E. Shepherd and I.A. Leyva [9], which is Paper 000300 in the 29th International Symposium on Shock Waves, held in Madison, WI, July 2013. It includes complete data sets and information on the statistical techniques used in data analysis, which were abbreviated in the main body of the paper.

## 2 Experiments

Experimental data are obtained in Caltech’s T5 Hypervelocity Reflected Shock Tunnel (see Hornung[7]). Flow conditions in T5 are calculated from three tunnel measurements: the shock speed, initial shock tube fill pressure and composition, and reservoir pressure at the end of the shock tube during the run time. Only experiments with measured shock speeds that fall within the uncertainty for the adjusted shock speed curve predicted by the shock jump conditions from the primary diaphragm burst pressure, driver gas composition, and initial shock tube conditions are included in the present data set.

Shock speed is measured by two time of arrival pressure transducers positioned 2.402 m apart, with an approximate measurement uncertainty of  $8 \times 10^{-6}$  s. The uncertainty in the shock speed measurement thus increases as the measured time of arrival difference decreases. At a shock speed of 3000 m/s, typical for the present study, the uncertainty is  $\sim 30$  m/s. The shock tube fill pressure uncertainty is  $\sim 0.25$  kPa, and the measured reservoir pressure uncertainty is typically  $\sim 4$  MPa. Uncer-

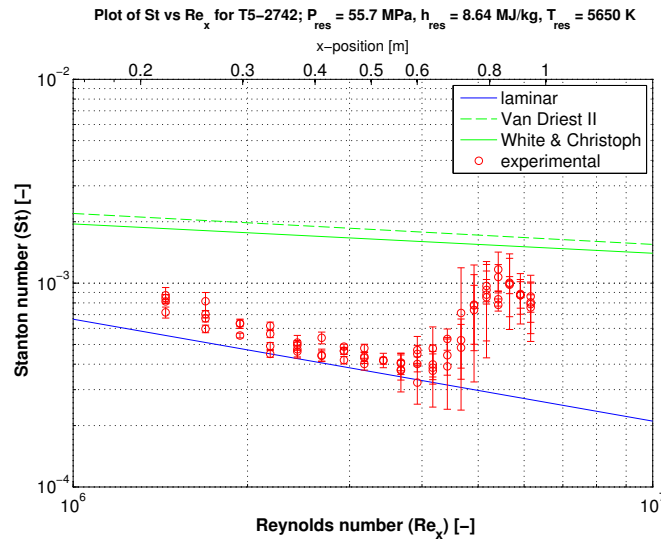
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tainties on the calculated quantities, including those represented by the error bars in Jewell et al.[9], are estimated by perturbing Cantera[6] condition computations[3] within the range of the uncertainties on the measured shock speed, reservoir pressure, and initial shock tube pressure. Only experiments with measured shock speeds that fall within the uncertainty for the adjusted shock speed curve predicted by the shock jump conditions from the primary diaphragm burst pressure, driver gas composition, and initial shock tube conditions are included in the present data set.

There are a number of other potential sources of measurement error, including nonideal gas behavior in the reservoir due to the high pressure, the extrapolation of the shock speed (which decays as it propagates down the shock tube) to the end wall, nonuniformity of reservoir conditions due to nonideal shock reflection, and the method of correcting flow conditions from the ideal reflected-shock pressure to measured reservoir pressure using an isentropic expansion. Furthermore, the 1-D contoured nozzle computation does not account for boundary layer growth within the nozzle, off-design operation conditions that lead to flow nonuniformity, or vibration-translation nonequilibrium and freezing within the nozzle, which is significant for the  $N_2$  cases.

The experimental model is a 1 m smooth 5 degree half-angle cone, with a nominally sharp tip of radius 0.18 mm, instrumented with 80 thermocouples, providing heat transfer measurements from which transition location may be determined. Heat transfer results are normalized by Stanton number and Reynolds number, and the location of transition onset determined as described in Jewell et al.[8]



**Fig. 1** Time-averaged non-dimensional plot of heat transfer results in terms of Stanton number vs. Reynolds number for T5 shot 2742 in air, with the laminar similarity correlation indicated in blue and two common turbulent correlations in green.

Heat transfer results for one experiment, in air at 8.64 MJ/kg and 55.7 MPa, are presented normalized by Stanton number and Reynolds number in Figure 1. Due to the distribution of the sensors around the circumference of the cone, in this representation of the data there are four results at each of 20  $x$ -locations. The circles represent the time-averaged heat transfer result over the  $\sim 1$  ms steady flow time, and the bars represent the root mean squared values from each sensor over the steady flow time. The RMS bars are initially small in the laminar zone as the heat transfer values are consistently at the laminar value, increase in size in the transitional zone as the flow becomes intermittent, and may then decrease in size again as the flow approaches the fully turbulent zone and heat transfer levels are consistently near the turbulent value. A slight drop-off from the maximum heat transfer value is observed in the last rows of thermocouples, as they are positioned near the maximum extent of the T5 test rhombus and may intersect with the expansion fan emanating from the lip of the nozzle. For this experiment, transition onset is observed at 0.59 m from the tip of the cone, and the transitional zone extends at least 0.20 m beyond the observed onset location before fully turbulent flow is achieved.

Datasets of experiments in which transition onset is clearly observed are presented for all  $N_2$  conditions ( $n = 10$ ) in Table 1, and all air conditions ( $n = 24$ ) in Table 2. The calculated boundary layer thickness  $\delta_{.99}$  and the approximate most amplified Mack second mode[11] frequency  $f \approx 0.6U_e/2\delta_{.99}$  at the observed transition location are also reported.

### 3 Previous Results

Datasets in air ( $n = 22$ ) from shots referenced by Adam[1] and Adam and Hornung[2] are presented in Table 3. Some of these experiments were performed by, and first referenced in, Germain and Hornung[5]. Observed transition onset location  $X$  is reproduced here as reported in [1], but the other parameters have been recalculated by the present methods for consistency. In the case of  $Re^*/m$  for Dorrance reference conditions,  $\delta_{.99tr}$ , and  $f_{tr}$ , the parameters have been calculated here for the first time.

**Table 1** Experiments performed in  $N_2$ , with unit Reynolds numbers evaluated at the boundary layer edge and Dorrance reference conditions, observed transition onset location, and  $\delta$  and  $f$  calculated at the transition onset location.

	$h_{res}$ [MJ/kg]	$P_{res}$ [MPa]	$Re/m$ [1/m]	$Re^*/m$ [1/m]	$X_{tr}$ [m]	$\delta_{99tr}$ [mm]	$f_{tr}$ [kHz]
2772	8.00	16.7	$2.31 \times 10^6$	$1.35 \times 10^6$	0.67	2.18	515
2773	8.99	16.7	$2.02 \times 10^6$	$1.25 \times 10^6$	0.67	2.26	522
2775	7.26	17.4	$2.67 \times 10^6$	$1.49 \times 10^6$	0.67	2.07	519
2776	7.17	45.9	$7.09 \times 10^6$	$3.94 \times 10^6$	0.39	0.97	1102
2777	8.91	38.9	$4.67 \times 10^6$	$2.89 \times 10^6$	0.49	1.28	924
2778	10.73	41.4	$3.99 \times 10^6$	$2.72 \times 10^6$	0.55	1.41	910
2779	12.00	42.3	$3.57 \times 10^6$	$2.56 \times 10^6$	0.60	1.53	881
2780	13.64	44.5	$3.23 \times 10^6$	$2.46 \times 10^6$	0.62	1.59	891
2782	14.84	53.9	$3.47 \times 10^6$	$2.76 \times 10^6$	0.61	1.50	980
2783	15.88	53.3	$3.20 \times 10^6$	$2.62 \times 10^6$	0.63	1.56	966

**Table 2** Experiments performed in air, with unit Reynolds numbers evaluated at the boundary layer edge and Dorrance reference conditions, observed transition onset location, and  $\delta$  and  $f$  calculated at the transition onset location.

	$h_{res}$ [MJ/kg]	$P_{res}$ [MPa]	$Re/m$ [1/m]	$Re^*/m$ [1/m]	$X_{tr}$ [m]	$\delta_{99tr}$ [mm]	$f_{tr}$ [kHz]
2714	9.49	67.1	$6.85 \times 10^6$	$5.03 \times 10^6$	0.49	1.00	1183
2715	10.48	68.8	$6.24 \times 10^6$	$4.79 \times 10^6$	0.70	1.22	1004
2716	10.79	67.4	$5.90 \times 10^6$	$4.57 \times 10^6$	0.72	1.27	981
2739	8.03	57.5	$7.28 \times 10^6$	$4.90 \times 10^6$	0.55	1.06	1033
2740	7.97	57.3	$7.32 \times 10^6$	$4.91 \times 10^6$	0.54	1.04	1049
2741	8.34	56.9	$6.85 \times 10^6$	$4.70 \times 10^6$	0.57	1.09	1023
2742	8.64	55.7	$6.54 \times 10^6$	$4.58 \times 10^6$	0.58	1.14	997
2743	9.09	56.3	$6.17 \times 10^6$	$4.43 \times 10^6$	0.64	1.25	926
2744	7.68	60.7	$8.19 \times 10^6$	$5.39 \times 10^6$	0.51	0.98	1094
2753	8.66	52.1	$6.05 \times 10^6$	$4.22 \times 10^6$	0.68	1.27	891
2758	11.07	72.0	$6.11 \times 10^6$	$4.79 \times 10^6$	0.74	1.26	1002
2759	9.62	60.4	$6.12 \times 10^6$	$4.51 \times 10^6$	0.68	1.23	959
2760	6.29	27.2	$4.76 \times 10^6$	$2.72 \times 10^6$	0.51	1.34	730
2761	5.49	28.2	$5.80 \times 10^6$	$3.04 \times 10^6$	0.49	1.24	744
2762	6.06	27.8	$5.11 \times 10^6$	$2.86 \times 10^6$	0.51	1.31	736
2763	6.56	27.2	$4.52 \times 10^6$	$2.66 \times 10^6$	0.53	1.40	716
2764	5.27	16.5	$3.62 \times 10^6$	$1.83 \times 10^6$	0.52	1.65	551
2765	6.47	17.5	$3.03 \times 10^6$	$1.74 \times 10^6$	0.65	1.90	522
2766	7.54	17.0	$2.49 \times 10^6$	$1.55 \times 10^6$	0.64	2.01	526
2769	10.46	60.8	$5.61 \times 10^6$	$4.28 \times 10^6$	0.64	1.23	999
2786	10.03	53.6	$5.22 \times 10^6$	$3.90 \times 10^6$	0.65	1.30	929
2787	10.72	54.7	$4.93 \times 10^6$	$3.76 \times 10^6$	0.61	1.28	971
2789	11.87	56.4	$4.52 \times 10^6$	$3.55 \times 10^6$	0.73	1.44	897
2790	11.55	57.4	$4.77 \times 10^6$	$3.74 \times 10^6$	0.68	1.36	944

**Table 3** Experiments performed in air referenced in Adam[1] and Adam and Hornung[2], with unit Reynolds numbers evaluated at the boundary layer edge and Dorrance reference conditions, observed transition onset location, and  $\delta$  and  $f$  calculated at the transition onset location. Observed transition onset location  $X$  is as reported in [1], but the other parameters have been recalculated by the present methods for consistency, and in the case of  $Re^*/m$ ,  $\delta$ , and  $f$ , calculated here for the first time.

	$h_{res}$ [MJ/kg]	$P_{res}$ [MPa]	$Re/m$ [1/m]	$Re^*/m$ [1/m]	$X_{tr}$ [m]	$\delta_{99tr}$ [mm]	$f_{tr}$ [kHz]
675	10.20	58.5	$5.60 \times 10^6$	$4.23 \times 10^6$	0.54	1.13	1074
683	10.43	65.5	$5.94 \times 10^6$	$4.51 \times 10^6$	0.41	0.96	1280
684	11.10	60.9	$5.25 \times 10^6$	$4.09 \times 10^6$	0.46	1.07	1180
685	10.32	56.9	$5.37 \times 10^6$	$4.06 \times 10^6$	0.43	1.04	1178
686	12.95	55.4	$4.02 \times 10^6$	$3.20 \times 10^6$	0.59	1.38	974
687	13.06	57.3	$4.14 \times 10^6$	$3.33 \times 10^6$	0.59	1.35	1001
688	13.27	62.3	$4.38 \times 10^6$	$3.55 \times 10^6$	0.62	1.33	1018
689	10.65	59.0	$5.34 \times 10^6$	$4.09 \times 10^6$	0.54	1.15	1070
879	11.65	79.1	$6.26 \times 10^6$	$5.00 \times 10^6$	0.49	1.00	1289
888	11.55	79.1	$6.28 \times 10^6$	$4.97 \times 10^6$	0.63	1.14	1124
1113	11.37	77.2	$6.34 \times 10^6$	$5.04 \times 10^6$	0.48	0.98	1293
1115	7.38	68.7	$9.66 \times 10^6$	$6.20 \times 10^6$	0.43	0.83	1275
1151	10.03	45.0	$4.46 \times 10^6$	$3.29 \times 10^6$	0.57	1.32	907
1152	8.88	43.3	$4.97 \times 10^6$	$3.50 \times 10^6$	0.40	1.06	1076
1153	12.19	40.7	$3.30 \times 10^6$	$2.53 \times 10^6$	0.72	1.70	769
1155	8.08	44.3	$5.66 \times 10^6$	$3.80 \times 10^6$	0.45	1.09	1007
1156	7.80	48.0	$6.34 \times 10^6$	$4.18 \times 10^6$	0.39	0.96	1124
1157	5.83	47.2	$8.94 \times 10^6$	$4.90 \times 10^6$	0.23	0.67	1421
1159	11.41	42.5	$3.67 \times 10^6$	$2.80 \times 10^6$	0.72	1.61	788
1160	10.41	41.9	$4.00 \times 10^6$	$2.98 \times 10^6$	0.66	1.49	819
1162	9.04	34.4	$3.91 \times 10^6$	$2.74 \times 10^6$	0.60	1.48	773
1163	11.17	68.1	$5.73 \times 10^6$	$4.47 \times 10^6$	0.59	1.15	1094

## 4 Statistical Analysis

Multivariable linear regression analysis is performed on these data sets with the MATLAB Statistics Toolbox, taking  $P_{res}$  and  $h_{res}$  (normalized by each data set's maximum pressure and enthalpy) as the possible predictor variables and the measured values of  $X_{tr}$  (normalized by the length of the cone, 1 m),  $Re_{tr}^*$  and  $Re_{tr}$  (normalized by the respective maximum unit Reynolds numbers) as the modeled variables. We use a significance level of 5% (i.e. requiring a  $p$ -value less than 0.05 to reject the null hypothesis that a given coefficient is zero). The linear model coefficients that result from this analysis, along with their respective  $p$ -values, are presented in Tables 4, 5 and 6. Note that these tables include the results of statistical analysis for  $Re_{tr}$  in addition to  $X_{tr}$  and  $Re_{tr}^*$ , although only the latter two parameters are referenced in the main paper[9].

Both the present  $N_2$  and air results have a positive dependence on  $h_{res}$  (linear model coefficient 0.56 for  $N_2$ , 0.55 for air) and a negative dependence on  $P_{res}$  ( $-0.45$  for  $N_2$ ,  $-0.15$  for air; note that the air result, with  $p$ -value = 0.054, is marginal). The historical air data of Adam and Hornung[2] are analyzed in the same way, and likewise show a significant positive dependence of  $X_{tr}$  on  $h_{res}$  (0.72) and negative dependence on  $P_{res}$  ( $-0.28$ ).

Both the present  $N_2$  and air results have a positive dependence on  $P_{res}$  (linear model coefficient 0.31 for  $N_2$ , 0.59 for air) for the transition Reynolds number evaluated at Dorrance reference conditions,  $Re_{tr}^*$ , but neither have a dependence on  $h_{res}$  that is statistically significant. The historical air data of Adam and Hornung[2] likewise show a significant positive dependence of  $Re_{tr}^*$  on  $P_{res}$  (0.34), but no statistically significant dependence on  $h_{res}$ .

No statistically significant correlation of  $Re_{tr}^*$  with reservoir enthalpy  $h_{res}$  is observed for any data set, either in the present data or in a statistical re-examination of Adam and Hornung[2], who reported an increase in  $Re_{tr}^*$  with increasing  $h_{res}$ , but did not control for  $P_{res}$ , which varied from 10 to 85 MPa in their air experiments. In both present and past data,  $Re_{tr}^*$  appears to correlate most strongly with  $P_{res}$ .

**Table 4** Multivariable linear regression analysis for  $N_2$  results ( $n = 10$ ) from the present study.  $P_{res}$  and  $h_{res}$  are normalized by their respective maximum values. The coefficients found to be statistically significant ( $p < 0.05$ ) are in bold print.

	$X_{tr}$	$Re_{tr}^*$	$Re_{tr}$
$P_{res}$ coefficient	<b>-0.45396</b>	<b>0.30819</b>	<b>0.29023</b>
$p$ -value	$1.20 \times 10^{-5}$	$5.88 \times 10^{-6}$	$7.89 \times 10^{-7}$
$h_{res}$ coefficient	<b>0.55527</b>	-0.02805	<b>-0.26922</b>
$p$ -value	$2.36 \times 10^{-5}$	0.44139	$1.02 \times 10^{-5}$

**Table 5** Multivariable linear regression analysis for air results ( $n = 24$ ) from the present study.  $P_{res}$  and  $h_{res}$  are normalized by their respective maximum values. The coefficients found to be statistically significant ( $p < 0.05$ ) are in bold print. Note that the  $P_{res}$  coefficient for  $X_{tr}$  in this data, which has  $p = 0.054$ , is of marginal significance.

	$X_{tr}$	$Re_{tr}^*$	$Re_{tr}$
$P_{res}$ coefficient	<b>-0.15238</b>	<b>0.58556</b>	<b>0.54107</b>
$p$ -value	0.05377	$1.72 \times 10^{-9}$	$7.30 \times 10^{-10}$
$h_{res}$ coefficient	<b>0.54802</b>	-0.04254	<b>-0.28748</b>
$p$ -value	$7.39 \times 10^{-5}$	0.62997	0.00117

**Table 6** Multivariable linear regression analysis for historical air results ( $n = 22$ ) referenced in Adam[1].  $P_{res}$  and  $h_{res}$  are normalized by their respective maximum values. The coefficients found to be statistically significant ( $p < 0.05$ ) are in bold print.

	$X_{tr}$	$Re_{tr}^*$	$Re_{tr}$
$P_{res}$ coefficient	<b>-0.28405</b>	<b>0.33725</b>	<b>0.27436</b>
$p$ -value	0.01308	$3.07 \times 10^{-4}$	$3.24 \times 10^{-4}$
$h_{res}$ coefficient	<b>0.72414</b>	0.10048	-0.043046
$p$ -value	$1.23 \times 10^{-5}$	0.28530	0.57148

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