

Effects of subgrid-scale anisotropy on wall-modelled large-eddy simulation of turbulent flow with smooth-body separation

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We examine the role of anisotropic subgrid-scale (SGS) stress in wall-modelled large-eddy simulation of flow over a spanwise-uniform Gaussian-shaped bump, with emphasis on predicting flow separation. The simulations show that eddy-viscosity-based SGS models often yield non-monotonic predictions of the mean separation bubble size on the leeward side under grid refinement, whereas models incorporating anisotropic SGS stress produce more consistent results. To identify where SGS anisotropy is most critical, we introduce anisotropic SGS stress in selected regions of the domain. The results reveal that the windward side, where a strong favourable pressure gradient (FPG) occurs, is crucial in determining downstream separation. Analysis of the Reynolds stress transport equation shows that fluctuations of anisotropic SGS stress modify SGS dissipation and diffusion in this region, thereby altering the Reynolds stress and the onset of separation. Examination of the mean streamwise momentum equation indicates that at coarse resolutions, the mean SGS shear stress dominates, and the differences between the eddy-viscosity-based and anisotropic models remain minor. With grid refinement, resolved Reynolds stresses increasingly govern the near-wall momentum transport, and the influence of SGS stress fluctuations grows as they determine the SGS dissipation and diffusion of Reynolds stresses. Component-wise analysis of the SGS stress tensor further shows that the improvement arises mainly from including significant normal stress contributions. An *a priori* study using filtered direct numerical simulations of turbulent Couette–Poiseuille flow confirms that wall-bounded turbulence under FPG is highly anisotropic, and that anisotropic SGS models provide a more realistic SGS stress representation than eddy-viscosity-based models.

Key words: turbulence simulation, turbulence modelling, turbulent boundary layers

1. Introduction

Complex turbulent flows with separation are common in aerodynamic and hydrodynamic applications, and strongly affect performance and stall behaviour. Accurate prediction of such flows is therefore essential. Wall-modelled large-eddy simulation (WMLES) has emerged as a promising approach (see reviews in Larsson *et al.* 2016; Bose & Park 2018), as it resolves the energy-containing turbulent motions away from the wall while modelling near-wall effects. Compared with wall-resolved LES (WRLES), WMLES significantly reduces grid requirements and computational cost, often by one to two orders of magnitude in practical configurations (Choi & Moin 2012; Yang & Griffin 2021). Owing to these advantages, WMLES is considered a viable tool for high-fidelity simulations of realistic flows, and has been successfully applied to complex engineering problems (Park 2017; Lehmkühl *et al.* 2018; Goc *et al.* 2021, 2024).

Despite this progress, achieving robust and accurate WMLES predictions across a wide range of flow conditions remains challenging. Many wall models rely on equilibrium assumptions that are not valid in separated or strongly non-equilibrium flows. While improved formulations, such as models based on thin boundary layer equations (Wang & Moin 2002; Kawai & Larsson 2013; Park & Moin 2014) or dynamic slip approaches (Bose & Moin 2014; Bae *et al.* 2019), have been developed, recent studies (Lozano-Durán & Bae 2019; Rezaeiravesh, Mukha & Liefvendahl 2019; Zhou & Bae 2024a) show that limitations in subgrid-scale (SGS) modelling play an equally important, if not dominant, role in WMLES performance. In particular, the sensitivity of separated-flow predictions to SGS models has been shown to be substantial (Zhou *et al.* 2023; Zhou & Bae 2024a).

In LES, the effects of unresolved small-scale motions are represented by SGS models, which are often based on the assumption of isotropic turbulence. This assumption underlies classical eddy-viscosity models, originally developed for well-resolved simulations. Although these models provide reasonable energy dissipation, they show weak correlation with the exact SGS stress from direct numerical simulations (DNS) and may fail to capture important statistical properties of turbulent flows (Clark, Ferziger & Reynolds 1979; Kerr, Domaradzki & Barbier 1996; Domaradzki & Saiki 1997; Moser, Haering & Yalla 2021). Their limitations become more pronounced in WMLES, where coarse grids require SGS models to represent more than energy dissipation alone. As a result, eddy-viscosity models often exhibit poor or non-monotonic convergence in separation bubble prediction, and require sufficiently fine resolution to remain reliable (Whitmore *et al.* 2021; Agrawal *et al.* 2022; Zhou & Bae 2024a). This reliance on fine resolution limits their practical applicability and underscores the need for SGS models suited for WMLES that can accurately predict complex turbulent flows.

At coarse resolutions typical of WMLES, SGS models must represent not only energy dissipation but also the contribution of subgrid motions to mean momentum and energy transport. In this regime, the effects of SGS anisotropy become important, particularly at the smallest resolved scales that remain dynamically active. However, classical eddy-viscosity models, with their single degree of freedom, cannot simultaneously represent both SGS stress and dissipation. These limitations highlight the need for SGS models that account for anisotropy and can capture both effects at coarse resolutions. To this end, various anisotropic and nonlinear SGS models have been proposed, including mixed similarity models (Zang, Street & Koseff 1993; Liu, Meneveau & Katz 1994; Vreman, Geurts & Kuerten 1994; Meneveau & Katz 2000; Iyer & Malik 2024), algebraic models (Gatski & Jongen 2000; Marstorp *et al.* 2009; Rasam *et al.* 2017), and other nonlinear

formulations (Abe 2014; Vollandt, Balarac & Corre 2016; Agrawal *et al.* 2022; Uzun & Malik 2025). While these models show improved performance in canonical turbulent flows such as channel flows and turbulent boundary layers (TBLs), their behaviour in complex flow configurations remains insufficiently understood, and the role of SGS anisotropy is not yet fully characterised.

Recent *a priori* analyses based on filtered DNS data indicate that SGS anisotropy strongly influences turbulence dynamics, including the evolution of Reynolds stress, vorticity and enstrophy (Horiuti 2003; Abe 2019; Cimarelli *et al.* 2019; Inagaki & Kobayashi 2023). However, inconsistencies between *a priori* and *a posteriori* evaluations have been widely reported (Vreman, Geurts & Kuerten 1997; Park, Yoo & Choi 2005; Duraisamy 2021; Choi *et al.* 2025). In particular, models that perform poorly in *a priori* tests may yield good *a posteriori* results, and vice versa, highlighting a key limitation of *a priori* analysis. To address this gap, the present study conducts a systematic *a posteriori* investigation of anisotropic SGS stress effects in WMLES of separated turbulent flows.

In particular, this *a posteriori* investigation focuses on the flow over a Gaussian-shaped bump at a relatively high Reynolds number (see figure 1) (Slotnick 2019). This configuration mimics smooth junctions between an aircraft wing and fuselage, where smooth-body separation of a TBL occurs under combined pressure gradients and surface curvature. As a canonical benchmark, it has been extensively studied experimentally (Williams *et al.* 2020; Gluzman *et al.* 2022; Gray *et al.* 2022) and computationally (Balin & Jansen 2021; Uzun & Malik 2022; Arranz, Ling & Lozano-Duran 2023; Agrawal, Bose & Moin 2024; Zhou & Bae 2024*b*; Iyer & Malik 2025), with consistent emphasis on the challenge of accurately predicting the extent and location of leeward-side separation. Recent WMLES studies have demonstrated that separation prediction is highly sensitive to the SGS model, and that anisotropic SGS models markedly improve mean velocity field predictions (Agrawal *et al.* 2022; Iyer & Malik 2022, 2024; Zhou & Bae 2024*a*; Uzun & Malik 2025), making this an ideal test case for the present analysis. To avoid complexities from spanwise variations, this study focuses on a spanwise-uniform bump with periodic spanwise boundary conditions, following the hybrid DNS–WRLES study of Uzun & Malik (2022), which also provides high-fidelity reference data.

Through this study, we aim to improve understanding of how anisotropic SGS models influence the statistics and dynamics of separated flow in WMLES, characterise the properties of anisotropic SGS stress, and identify key modelling features required for accurate and robust predictions in complex separated flows – ultimately providing guidance for future SGS model development.

The remainder of the paper is organised as follows. Section 2 describes the numerical approach, flow configuration, and simulation set-up. In § 3, the sensitivity of mean flow separation to SGS models and mesh resolution is examined, followed by a numerical experiment in which anisotropic SGS stress is introduced in different regions of the computational domain to identify where SGS anisotropy is most critical. Section 4 explains the mechanisms underlying the sensitivity of separation prediction through budgets of the mean streamwise momentum and Reynolds stress transport equations. In § 5, the SGS stress properties are investigated and compared with *a priori* results from filtered DNS of a turbulent Couette–Poiseuille flow. Finally, § 6 summarises the key findings and provides insights for further improvement of WMLES.

2. Computational methodology

2.1. Numerical approach

Flow simulations are conducted using a finite-volume, unstructured-mesh LES code (You, Ham & Moin 2008), which has been validated in various turbulent flow configurations (Yang & Wang 2013; Zhou *et al.* 2020, 2024). The spatially filtered incompressible Navier–Stokes equations are solved with second-order accuracy using cell-based, low-dissipative and energy-conservative spatial discretisation and a fully implicit, fractional-step time-advancement method with the Crank–Nicolson scheme. The Poisson equation for pressure is solved using the bi-conjugate gradient stabilised method (van der Vorst 1992). The governing equations are

$$\frac{\partial \tilde{u}_i}{\partial x_i} = 0 \quad (2.1)$$

and

$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \tilde{p}}{\partial x_i} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j} \tau_{ij}^{sgs}, \quad (2.2)$$

where u_i is the instantaneous flow velocity, p is the instantaneous static pressure, ρ is fluid density, ν is fluid kinematic viscosity, and $(\cdot)$ denotes the grid-filtering operation. The SGS stress tensor is $\tau_{ij}^{sgs} = \widehat{u_i u_j} - \tilde{u}_i \tilde{u}_j$, whose deviatoric part is modelled using an SGS model for closure of the equations, and the isotropic component of the SGS stress is absorbed into pressure. In what follows, the tilde is omitted and u_i , p denote resolved-field quantities directly for brevity.

Two SGS models are investigated. The first is the classical Smagorinsky model (SM) (Smagorinsky 1963)

$$\tau_{ij}^{sgs} = \tau_{ij}^{SM} = -2\nu_t S_{ij} = -2(C_s \Delta)^2 |S| S_{ij}, \quad (2.3)$$

where ν_t is the eddy viscosity, S_{ij} is the strain-rate tensor, $|S| = (2S_{ij}S_{ij})^{1/2}$, Δ is the grid filter width, and $C_s = 0.16$ is calibrated for homogeneous isotropic turbulence. The second model is a modified SM (MSM), which augments the isotropic SM with an anisotropic SGS stress term

$$\tau_{ij}^{sgs} = \tau_{ij}^{SM} + \tau_{ij}^{ani}, \quad (2.4)$$

where $\tau_{ij}^{ani} = C_a \Delta^2 (S_{ik} R_{kj} - R_{ik} S_{kj})$, and R_{ij} is the rotation-rate tensor. This term is one of the six independent terms arising from the expansion of SGS stress in strain-rate and rotation-rate tensors (Lund & Novikov 1992; Gatski & Jongen 2000), and it is explicitly incorporated in several recently developed anisotropic SGS models that have shown promising results for flow over a Gaussian bump (Agrawal *et al.* 2022; Uzun & Malik 2025). Importantly, this term contributes no SGS kinetic energy dissipation, i.e. $\tau_{ij}^{ani} S_{ij} = 0$ (Lund & Novikov 1992; Silvis & Verstappen 2019; Inagaki & Kobayashi 2023), allowing the physical role of anisotropic SGS stress to be studied independently of energy transfer. To minimise differences in kinetic energy dissipation between the two models, $C_s = 0.16$ is retained in the MSM. For C_a , tests over the range $-1/30$ to $-1/6$ (consistent with values used in related anisotropic models Bardina 1983; Kosović 1997; Sarghini, Piomelli & Balaras 1999; Wang & Bergstrom 2005; Marstorp *et al.* 2009; Silvis & Verstappen 2019) showed no significant change in the predicted separation bubble; $C_a = -1/30$ is therefore adopted as the most conservative choice. No near-wall damping is applied to the eddy viscosity in either model, consistent with the coarse-mesh WMLES focus of this study.

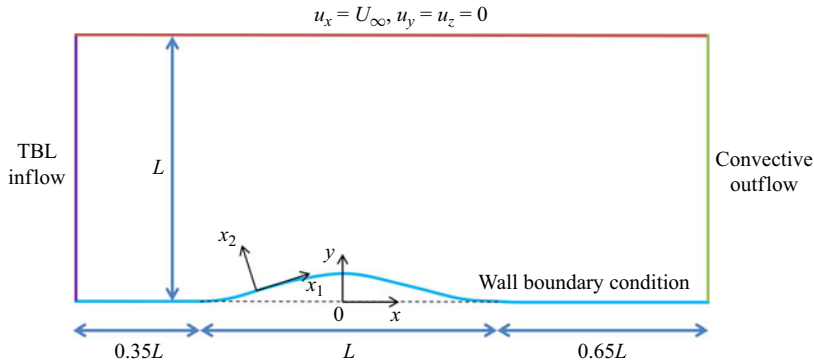


Figure 1. Simulation set-up for flow over a Gaussian-shaped bump.

2.2. Flow configuration and simulation set-up

The physical conditions follow the hybrid DNS–WRLES of Uzun & Malik (2022). The bump geometry is $y = f_b(x) = h \exp[-(x/x_0)^2]$, with maximum height $h = 0.085L$, $x_0 = 0.195L$, for L the bump width. The Reynolds number is $Re_L = U_\infty L/\nu = 2 \times 10^6$, where U_∞ is the free-stream velocity, matching the reference DNS (Uzun & Malik 2022). The flow configuration and boundary conditions are shown schematically in figure 1.

Simulations are conducted in a rectangular domain of length $2L$, height L , and spanwise depth $0.08L$, matching the DNS dimensions in the vertical and spanwise directions (Uzun & Malik 2022). A Cartesian coordinate system (x, y, z) with velocity components u_x, u_y, u_z serves as the global reference, with origin at the base of the bump peak, located $0.85L$ downstream from the inlet. A surface-fitted local coordinate system (x_1, x_2, x_3) is also used, with velocity components u_1, u_2, u_3 , where x_1 is tangential to the bump surface in the flow direction, x_2 is surface-normal, and x_3 is spanwise.

Boundary conditions consist of a TBL inflow at the inlet, a free-stream condition on the top boundary, a convective outflow at the exit, and periodic conditions in the spanwise direction. The TBL inflow is generated by a separate flat-plate LES using the rescale-and-recycle method of Lund, Wu & Squires (1998). We note that the inflow-generation method used in the study of Uzun & Malik (2022) is different from the current method. The friction Reynolds number is $Re_\tau \approx 620$, and the TBL thickness is $\delta_{in}/L = 0.0061$, which is approximately 10 % larger than in the DNS. The momentum thickness Reynolds number of the inflow is $Re_\theta \approx 1074$, compared to their slightly smaller value of approximately 1035. Further details are given in Zhou & Bae (2024a). To isolate SGS model effects from wall-model interactions, the no-slip condition is replaced by an idealised Neumann boundary condition prescribing the mean wall shear stress from DNS:

$$\left(\frac{\partial u_1}{\partial x_2} \right) \Big|_w = \frac{\tau_{w,1}^{DNS}}{\rho \nu}, \quad (2.5)$$

where $\tau_{w,1}^{DNS}$ is the mean wall shear stress taken directly from Uzun & Malik (2022), with a no-penetration condition enforced for u_2 . Since the spanwise mean wall shear stress is zero in the DNS, the corresponding boundary condition for $\partial u_3/\partial x_2$ at the wall is set to zero.

The computational mesh combines structured-mesh blocks near the bottom surface with unstructured-mesh blocks in the outer region, using isotropic cells to avoid resolution-induced anisotropy (Haering, Lee & Moser 2019). Mesh parameters are listed in table 1. Based on the TBL thickness at $x/L = -0.65$ from the DNS (Uzun & Malik 2022),

Mesh label	Δ_c/L	Cell number
Coarsest mesh	1.90×10^{-3}	$1050 \times 44 \times 42 \approx 1.94$ million
Coarse mesh	9.52×10^{-4}	$2100 \times 88 \times 84 \approx 15.5$ million
Medium mesh	4.76×10^{-4}	$4200 \times 176 \times 168 \approx 124$ million
Fine mesh	2.38×10^{-4}	$8400 \times 352 \times 336 \approx 993$ million

Table 1. Parameters of the computational meshes utilising isotropic cells.

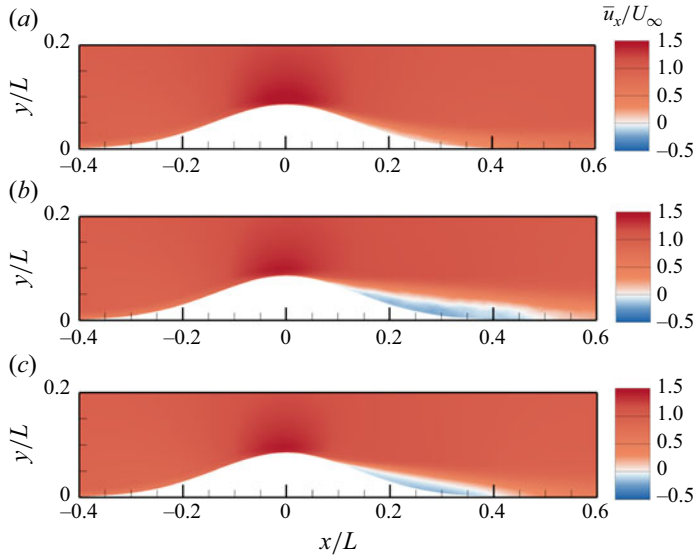


Figure 2. Isocontours of mean velocity $\bar{u}_x/U_\infty$ from the medium-mesh simulations with (a) the SM and (b) the MSM, and (c) from the reference DNS (Uzun & Malik 2022).

the TBL is resolved by approximately 5, 9, 18 and 36 cells for the coarsest, coarse, medium and fine meshes, respectively. The fine mesh resolution ranges from 10 to 30 wall units in attached-flow regions, comparable to standard WRLES in the streamwise and spanwise directions, but an order of magnitude coarser in the wall-normal direction near the wall.

A maximum Courant–Friedrichs–Lewy number 1.0 is used throughout. Simulations are run for two flow-through times ($4L/U_\infty$) to remove initial transients, followed by three flow-through times ($6L/U_\infty$) for statistics collection.

3. Sensitivity of mean flow separation prediction to SGS model

3.1. Separation prediction and grid convergence test

The flow field around the Gaussian-shaped bump, obtained using the medium mesh for the two SGS models, is shown in figure 2. Here, $\bar{(\cdot)}$ denotes both temporal averaging and spatial averaging along the homogeneous directions. With this definition, an instantaneous quantity φ can be decomposed as $\varphi = \bar{\varphi} + \varphi'$. Following the convention introduced in § 2.1, the tilde denoting grid filtering is omitted throughout, so that all quantities, including φ , its mean $\bar{\varphi}$ and the fluctuation φ' , refer to the resolved (filtered) field. For reference, the DNS results from Uzun & Malik (2022) are also included in the figure.

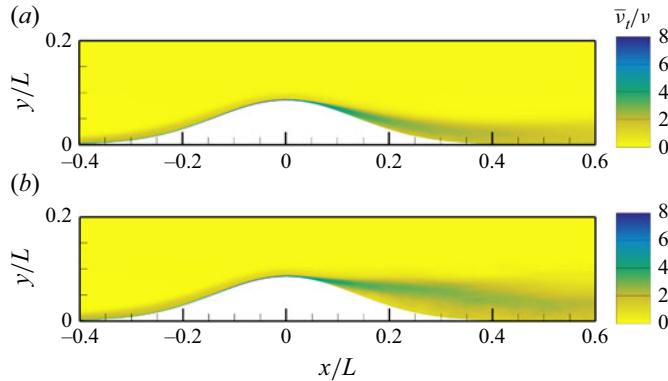


Figure 3. Isocontours of mean eddy viscosity $\bar{\nu}_t/\nu$ from the medium-mesh simulations with (a) the SM and (b) the MSM.

In the DNS flow field, the incoming TBL accelerates upstream of the bump peak, then decelerates downstream under the influence of an adverse pressure gradient (APG), leading to rapid thickening of the boundary layer on the leeward side. Farther downstream, a pronounced separation bubble forms. In contrast, for the present medium-mesh WMLES, the predicted separation behaviour is highly sensitive to the SGS model. The simulation using the SM does not predict any flow separation, whereas the simulation using the MSM exhibits a separation bubble that is larger than the one observed in the DNS.

The contours of the mean eddy viscosity of the SGS model, $\bar{\nu}_t/\nu$, from the medium-mesh simulations in an x - y plane are shown in figure 3. For the MSM, the eddy viscosity arises solely through the isotropic stress term, as defined in (2.4). The magnitude of the eddy viscosity within the TBL is of the same order of magnitude as the fluid viscosity. Upstream of the bump peak, the eddy viscosities produced by the two SGS models are similar in magnitude, indicating that the mean kinetic energy dissipation is similar for the two models in this region. However, farther downstream, noticeable differences emerge: although the MSM uses the same model coefficient C_s as the SM, the eddy viscosity distribution on the leeward side is significantly modified by the additional anisotropic stress term, indicating that the SGS dissipation of kinetic energy introduced by the MSM differs substantially from that of the SM in this region.

In figure 4, the distribution of the mean pressure coefficient $C_p = (\bar{p}_w - P_\infty)/((1/2)\rho U_\infty^2)$ on the bottom surface is compared with the DNS data (Uzun & Malik 2022). Here, p_w denotes the instantaneous static pressure at the wall, and the reference pressure P_∞ is taken near the top boundary at the inlet. The C_p distributions show a strong favourable pressure gradient (FPG) on the windward side near the bump peak. Downstream of the peak, the flow experiences a strong APG, followed by a milder APG over the majority of the leeward side. The two medium-mesh simulations agree reasonably well upstream of the bump peak and in the flat region downstream of the bump, but clear differences appear near the peak and along the leeward side. In particular, the MSM predicts a plateau in C_p on the leeward side corresponding to the presence of a separation bubble, in better agreement with the DNS.

A comparison of boundary layer profiles from the medium-mesh simulations with the DNS results of Uzun & Malik (2022) is shown in figure 5, where the mean streamwise velocity $\bar{u}_1$ is plotted at four streamwise stations in the computational domain. The results capture the flow acceleration on the windward side of the bump, followed by deceleration and boundary layer thickening or separation on the leeward side. Upstream of the bump

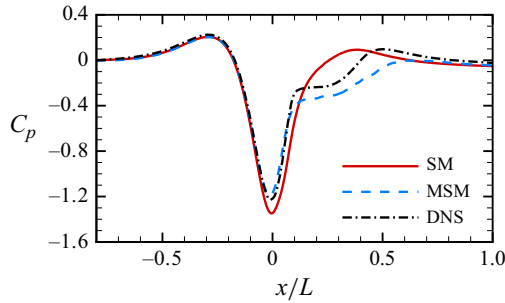


Figure 4. Mean pressure coefficient on the bottom surface from the medium-mesh simulations with the SM and MSM along with the reference DNS (Uzun & Malik 2022).

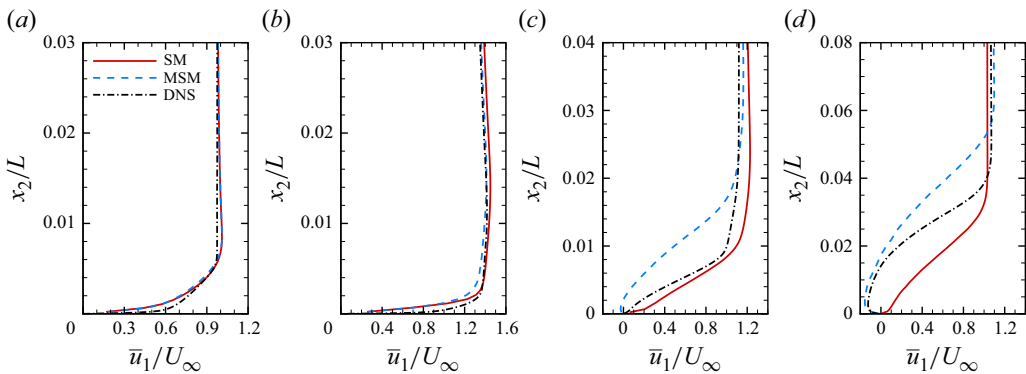


Figure 5. Profiles of mean streamwise velocity at (a) $x/L = -0.7$, (b) $x/L = 0$, (c) $x/L = 0.1$ and (d) $x/L = 0.2$ for the SM, the MSM, and the reference DNS (Uzun & Malik 2022).

peak, the two SGS model simulations agree reasonably well with each other and with the DNS. Downstream of the peak, however, clear differences emerge: the boundary layer thickens more rapidly in the MSM simulation, leading to the formation of a separation bubble, while the boundary layer in the SM simulation remains attached throughout the domain.

Since all simulations impose a wall shear stress matched to the local mean wall shear stress from the reference DNS (Uzun & Malik 2022), the length of the predicted separation bubble is estimated using the mean streamwise velocity at the first off-wall cell centre, as shown in figure 6. Upstream of the bump peak, the mean velocity at the first off-wall cell centre agrees well across all simulations. Downstream of the peak, the flow in the SM simulation approaches separation but remains attached, whereas the MSM simulation clearly exhibits a separation bubble. It should be noted that the wall shear stress is prescribed rather than derived from a wall-law constraint as in conventional wall modelling approaches. In addition, since the SGS model coefficients are not tuned for this configuration and no near-wall damping is applied, the inner-layer velocity profiles are not expected to match the DNS exactly even in the upstream region, despite the correct mean wall shear stress being imposed.

To quantitatively assess the effects of mesh resolution and the SGS model on the predicted mean separation bubble size, figure 7 shows the mean horizontal length (L_s/L) as a function of the characteristic mesh resolution (Δ_c/L). The DNS value of approximately $0.32L$ is included for reference (Uzun & Malik 2022). For the SM

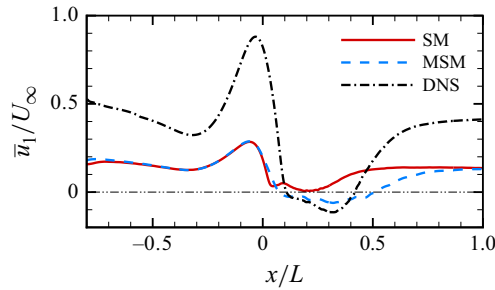


Figure 6. Mean streamwise velocity at the first off-wall cell centre from the medium-mesh simulations with the SM and MSM, and the reference DNS results (Uzun & Malik 2022) at the same wall-normal location. Here, $\bar{u}_1 = 0$ is indicated by the horizontal line.

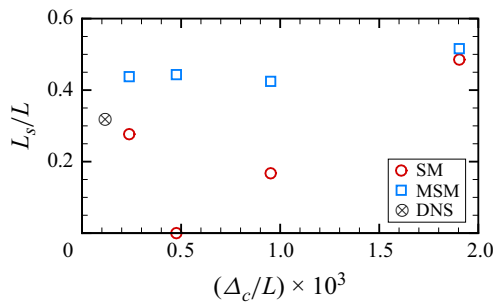


Figure 7. Mean separation bubble length on the leeward side of the bump from the simulations using the SM and MSM for different mesh resolutions and the reference DNS (Uzun & Malik 2022). Each symbol represents one simulation case.

simulations, convergence with mesh refinement is non-monotonic, producing a spurious reduction of the separation bubble upon refinement, a behaviour reported in previous studies with various isotropic SGS models (Whitmore *et al.* 2021; Agrawal *et al.* 2022; Zhou & Bae 2024a) and attributed to limitations in the SGS model. In contrast, the MSM predicts a larger separation bubble but yields consistent results across mesh resolutions, suggesting that including an anisotropic SGS stress term provides a beneficial effect. Similar consistency has been observed with the mixed model (Bardina 1983; Sarghini *et al.* 1999), as discussed in Appendix B and in Zhou & Bae (2024a). Since the current MSM has not been optimised and employs fixed coefficients, further improvement may be achieved through dynamic coefficients or other optimisation strategies. As the mesh is refined to fine-mesh resolution, results from all simulations converge towards the DNS, with the dependence on the SGS model significantly reduced at near-WRLES resolution.

The results highlight the sensitivity of the predicted mean velocity field in WMLES to both the SGS model and mesh resolution, consistent with observations from previous studies (Lozano-Durán & Bae 2019; Rezaeiravesh *et al.* 2019; Whitmore, Lozano-Durán & Moin 2020; Iyer & Malik 2022; Zhou & Bae 2024a). Two key questions follow. (i) Why does the mean velocity on the leeward side differ qualitatively between isotropic and anisotropic SGS models at medium resolution? (ii) Why do anisotropic models yield more consistent separation bubble predictions across mesh resolutions? These questions are addressed in the following subsections.

Group index	Virtual interface location x_0/L	SGS models in the upstream/downstream
SM-MSM	-0.3	SM/MSM
	-0.2	
	-0.1	
MSM-SM	0	MSM/SM
	0.05	

Table 2. List of parameters for the virtual interface set-up.

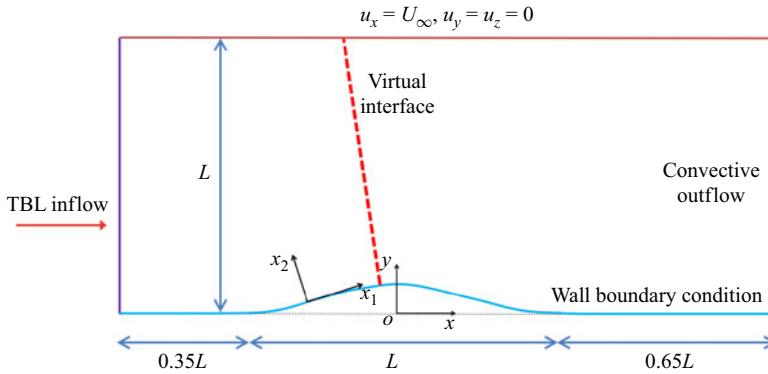


Figure 8. Virtual interface set-up dividing the domain into upstream and downstream regions using different SGS models for the flow over a Gaussian bump.

3.2. Identification of the critical region for the SGS anisotropy effect

To identify where the effects of anisotropy are most critical, a numerical experiment is designed in which the computational domain is divided into upstream and downstream sections, as illustrated in figure 8, using the medium mesh. A different SGS model is assigned to each section, with a logistic function providing a smooth transition at the virtual interface. All simulations use the same boundary conditions as before.

The SM and MSM are selectively assigned to the two domain sections. Five interface locations are considered near the bump peak, which prior studies have identified as important for downstream separation (Uzun & Malik 2022; Prakash *et al.* 2024; Xu & Bermejo-Moreno 2024). The interface is aligned with the local surface normal, and its x -coordinate at the bump surface is defined as x_0 . Details are listed in table 2. In group SM-MSM, the upstream section uses the SM, and the downstream section uses the MSM; in group MSM-SM, the assignment is reversed. The SGS stress is given by

$$\tau_{ij}^{sgs} = \tau_{ij}^{iso} + g \tau_{ij}^{ani} = \tau_{ij}^{SM} + g \tau_{ij}^{ani}. \quad (3.1)$$

Here, g is a logistic function used to achieve a smooth transition from one SGS model to another in the computational domain, and its form for the two groups of simulations is given by

$$g = \begin{cases} \frac{g_0}{1 + e^{k \times d \times \varphi}} & (\text{SM-MSM}), \\ \frac{g_0}{1 + e^{-k \times d \times \varphi}} & (\text{MSM-SM}), \end{cases} \quad (3.2)$$

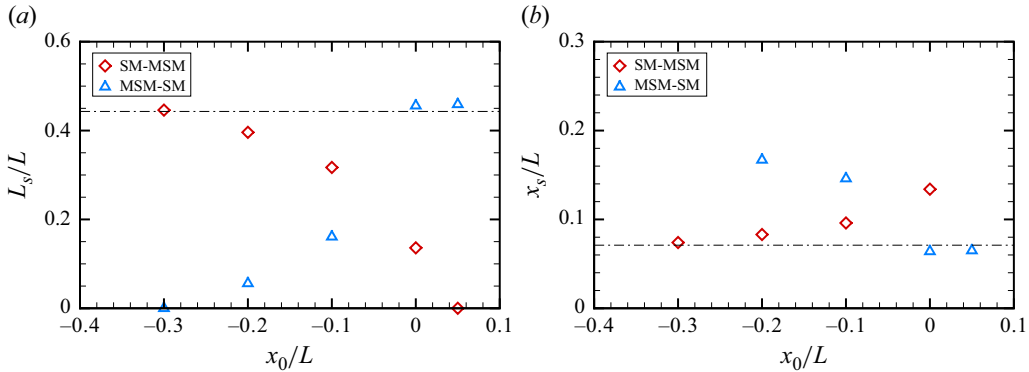


Figure 9. (a) Mean separation bubble length on the leeward side of the bump, and (b) the location of the mean separation point from medium-mesh simulations using SM-MSM and MSM-SM with different virtual interface locations. Horizontal lines indicate the results from the medium-mesh simulation with MSM.

where $g_0 = 1$, d is the spatial distance from a point to the interface at x_0 , and $k = 5000$, giving effective interface width $5 \times 10^{-3}L$ (ten cells in the medium mesh). Tests with interface widths ranging from $1 \times 10^{-3}L$ to $1 \times 10^{-2}L$ showed no significant influence on the resulting mean velocity fields. The parameters are $\varphi = 1$ upstream and $\varphi = -1$ downstream of the interface centreline. Statistics are collected over three flow-through times after discarding two for transient removal.

Figure 9 shows the predicted mean separation bubble size (L_s/L) and separation point location (x_s/L). When the interface is at the most upstream location ($x_0/L = -0.3$), predictions closely match those from the simulation using the downstream SGS model over the entire domain. As the interface shifts downstream, results increasingly reflect the upstream model. For example, in group SM-MSM with the interface at $x_0/L = -0.3$, the separation bubble matches that of the full-domain MSM simulation ($L_s \approx 0.44L$). As the interface moves downstream, the bubble shrinks progressively, disappearing when the interface reaches $x_0/L = 0.05$. An opposite trend is observed for group MSM-SM. The most pronounced variations occur as the interface moves from $x_0/L = -0.2$ to the bump peak, within the strong FPG region; beyond the peak, further downstream shifts have diminishing influence.

To examine how the TBL evolves with interface location within the critical FPG region ($x \in [-0.2, 0]$), velocity statistics at the bump peak ($x/L = 0$) are analysed. In addition to the mean streamwise velocity $\bar{u}_1$, the Reynolds shear stress $\overline{u'_1 u'_2}$ and wall-normal Reynolds normal stress $\overline{u'_2 u'_2}$ are examined, as these play important roles in shaping the near-wall mean flow, as discussed in §§ 4.1–4.3. Results are shown in figures 10 and 11. As the interface shifts downstream, results gradually approach those of the upstream model applied over the entire domain. The DNS profiles exhibit pronounced internal peaks in the Reynolds stresses at this location, which Uzun & Malik (2022) identify as crucial for the downstream TBL evolution and onset of separation.

From the comparisons among the simulations, it is found that applying the MSM within the critical FPG region improves the capture of these internal peaks. Since the resolved Reynolds stress is governed by a transport equation with mean convection, these stress distributions are carried downstream and directly influence the momentum balance near the separation point, ultimately driving the differences in leeward-side separation. This mechanism will be examined in detail in §§ 4.1 and 4.3. Overall, these results highlight that anisotropic SGS stress on the windward side is critical: the SGS model used in the

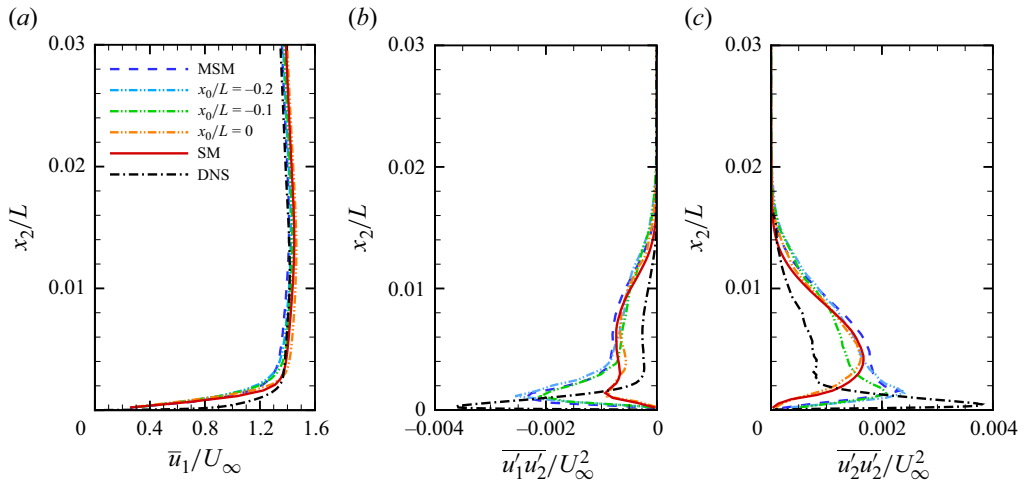


Figure 10. (a) Mean streamwise velocity $\bar{u}_1$, (b) Reynolds shear stress $\overline{u'_1 u'_2}$, and (c) Reynolds normal stress $\overline{u'_2 u'_2}$ profiles at the bump peak ($x/L = 0$) for SM-MSM with the virtual interface located at $x_0/L = -0.2, -0.1$ and 0 . The DNS (Uzun & Malik 2022), SM and MSM results are shown for reference.

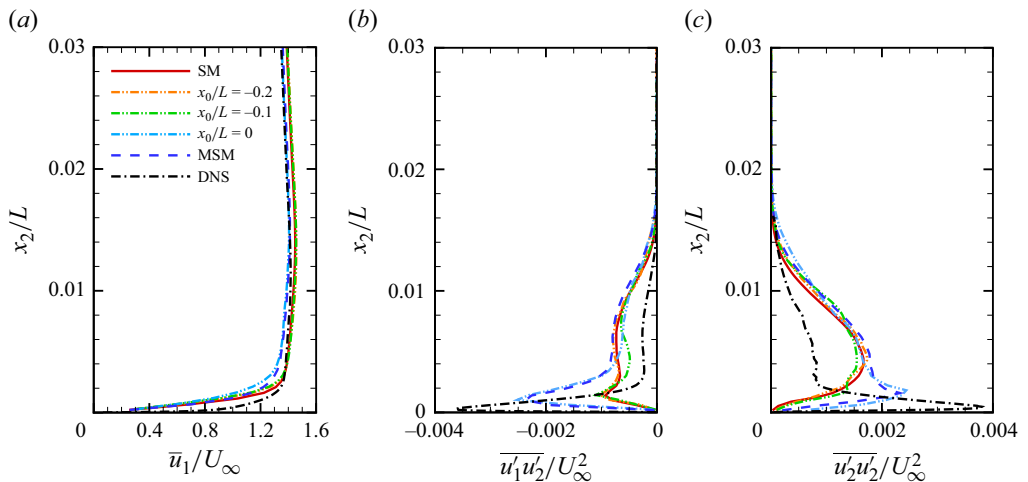


Figure 11. (a) Mean streamwise velocity $\bar{u}_1$, (b) Reynolds shear stress $\overline{u'_1 u'_2}$, and (c) Reynolds normal stress $\overline{u'_2 u'_2}$ profiles at the bump peak ($x/L = 0$) for MSM-SM with the virtual interface located at $x_0/L = -0.2, -0.1$ and 0 . The DNS (Uzun & Malik 2022), SM and MSM results are shown for reference.

strong FPG region governs the wall-normal Reynolds stress distributions, and through them, the downstream separation behaviour.

4. Budget analyses

In this section, we analyse the mean streamwise momentum and pressure equations slightly upstream of the mean separation point to isolate the effects of individual budget terms, demonstrating that the Reynolds stresses (particularly $\overline{u'_1 u'_2}$ and $\overline{u'_2 u'_2}$) have a significant impact on the mean flow field. We then analyse the Reynolds stress transport equations

to understand how their distributions within the TBL are influenced by anisotropic SGS stress. Only the medium-mesh simulations are considered in §§ 4.1–4.3, as this resolution shows the most pronounced differences in separation prediction between the two SGS models (see figure 7). The effect of mesh resolution is then examined in § 4.4.

4.1. Mean streamwise momentum equation

Considering the spanwise homogeneity of the statistically stationary flow, the streamwise mean momentum (Pope 2000; Abe 2019) is

$$\bar{u}_1 \frac{\partial \bar{u}_1}{\partial x_1} + \bar{u}_2 \frac{\partial \bar{u}_1}{\partial x_2} = P_g + V_{11} + V_{12} + T_{11} + T_{12} + R_{11} + R_{12}, \quad (4.1)$$

where the seven terms on the right-hand side are

$$P_g = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_1}, \quad V_{11} = \frac{\partial}{\partial x_1} (2\nu \bar{S}_{11}), \quad V_{12} = \frac{\partial}{\partial x_2} (2\nu \bar{S}_{12}), \quad (4.2)$$

$$T_{11} = \frac{\partial}{\partial x_1} (-\overline{\tau_{11}^{sgs}}), \quad T_{12} = \frac{\partial}{\partial x_2} (-\overline{\tau_{12}^{sgs}}), \quad (4.3)$$

$$R_{11} = \frac{\partial}{\partial x_1} (-\overline{u'_1 u'_1}), \quad R_{12} = \frac{\partial}{\partial x_2} (-\overline{u'_1 u'_2}), \quad (4.4)$$

corresponding to mean pressure gradient (P_g), viscous (V_{11} , V_{12}), SGS stress (T_{11} , T_{12}), and Reynolds stress (R_{11} , R_{12}) contributions, respectively. The magnitudes of these terms influence the distribution of mean momentum, and consequently the mean velocity field. Curvature effects are neglected as their influence upstream of the mean separation point ($x/L < 0.1$) is negligible, as shown in Appendix A.

Figure 12 shows the mean streamwise momentum budget at $x/L = 0.05$, approximately one boundary layer thickness ($0.02L$) upstream of the separation point in the MSM simulation. The same location is used for the SM case for consistency. In both simulations, the budget shows that mean convection $\bar{u}_k(\partial \bar{u}_1/\partial x_k)$, the mean pressure gradient P_g , and the Reynolds shear stress gradient R_{12} are the leading-order terms governing the near-wall momentum balance. All three terms differ quantitatively between the SM and MSM simulations. The convection term represents the transport of mean momentum by the established mean flow, and its quantitative difference between the two simulations reflects the differences in the upstream mean velocity field that have already been shaped by the SGS model. In contrast, P_g and R_{12} are forcing terms that directly act on the local momentum balance, and the upstream Reynolds stresses that contribute to both P_g and R_{12} are, as shown in § 3.2, the quantities through which the SGS model in the upstream FPG region influences the downstream flow field. The subsequent discussion therefore focuses on P_g and R_{12} as the terms that most directly connect the SGS model to the separation onset. The residual (total sum of all budget terms) is negligible relative to the leading-order terms throughout the boundary layer in both simulations, and is not shown for brevity.

In both cases, P_g is strongly negative across the boundary layer, but the MSM produces a noticeably larger magnitude, indicating stronger mean flow deceleration. The distributions of R_{12} also differ critically: while the peak magnitude is similar between the two simulations, the MSM exhibits a significantly broader region of negative R_{12} , implying that momentum is extracted over a thicker portion of the boundary layer, and redistributed towards the near-wall region. Consequently, less streamwise momentum remains available farther from the wall to resist the APG, making the near-wall flow more susceptible to separation. In contrast, the SM confines the momentum deficit to a thinner layer, helping the flow to remain attached.

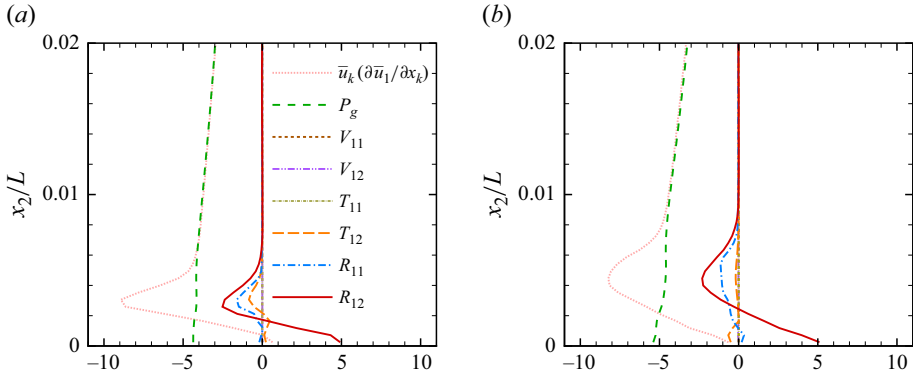


Figure 12. Mean streamwise momentum budget terms at $x/L = 0.05$ from medium-mesh simulations with (a) the SM and (b) the MSM. All terms are non-dimensionalised using U_∞ , L and ρ . The term $\bar{u}_k (\partial \bar{u}_1 / \partial x_k)$ denotes the net mean convection on the left-hand side of the mean momentum equation, and the other line notations correspond to (4.2)–(4.4).

4.2. Mean pressure equation

The Poisson equation for the mean pressure (Bradshaw & Koh 1981; Adrian 1982; Pope 2000) is

$$-\frac{1}{\rho} \nabla^2 \bar{p} = \frac{\partial \bar{u}_i}{\partial x_j} \frac{\partial \bar{u}_j}{\partial x_i} + \frac{\partial^2 \bar{u}'_i \bar{u}'_j}{\partial x_i \partial x_j} + \frac{\partial^2 \bar{\tau}_{ij}^{sgs}}{\partial x_i \partial x_j} \quad (4.5)$$

$$= U_{11} + U_{12} + U_{22} + W_{11} + W_{12} + W_{22} + G_{11} + G_{12} + G_{22},$$

where the nine terms on the right-hand side of the equation are

$$U_{11} = \left(\frac{\partial \bar{u}_1}{\partial x_1} \right)^2, \quad U_{12} = 2 \left(\frac{\partial \bar{u}_1}{\partial x_2} \frac{\partial \bar{u}_2}{\partial x_1} \right), \quad U_{22} = \left(\frac{\partial \bar{u}_2}{\partial x_2} \right)^2, \quad (4.6)$$

$$W_{11} = \frac{\partial^2 \bar{u}'_1 \bar{u}'_1}{\partial x_1^2}, \quad W_{12} = 2 \frac{\partial^2 \bar{u}'_1 \bar{u}'_2}{\partial x_1 \partial x_2}, \quad W_{22} = \frac{\partial^2 \bar{u}'_2 \bar{u}'_2}{\partial x_2^2}, \quad (4.7)$$

$$G_{11} = \frac{\partial^2 \bar{\tau}_{11}^{sgs}}{\partial x_1^2}, \quad G_{12} = 2 \frac{\partial^2 \bar{\tau}_{12}^{sgs}}{\partial x_1 \partial x_2}, \quad G_{22} = \frac{\partial^2 \bar{\tau}_{22}^{sgs}}{\partial x_2^2}, \quad (4.8)$$

representing contributions to the mean pressure from the mean velocity field (U_{ij}), Reynolds stress (W_{ij}) and mean SGS stress (G_{ij}), respectively.

Figure 13 shows profiles of all nine terms at $x/L = 0.05$. The results are qualitatively similar for both SGS models. Within the TBL, the dominant term is W_{22} , associated with the wall-normal Reynolds normal stress $\bar{u}'_2 \bar{u}'_2$, indicating that wall-normal turbulent fluctuations are the primary contributors to the local mean pressure distribution.

Taken together with the momentum budget analysis, these results highlight a consistent picture: in the medium-mesh simulations, the Reynolds stresses, not the mean SGS stresses, govern both the mean velocity and mean pressure fields immediately upstream of the separation point. Although the two SGS models produce different downstream separation behaviour, those differences arise primarily from how each model shapes the Reynolds stress distributions within the upstream FPG region. These stresses are then carried downstream by the mean flow, and influence the momentum balance near the

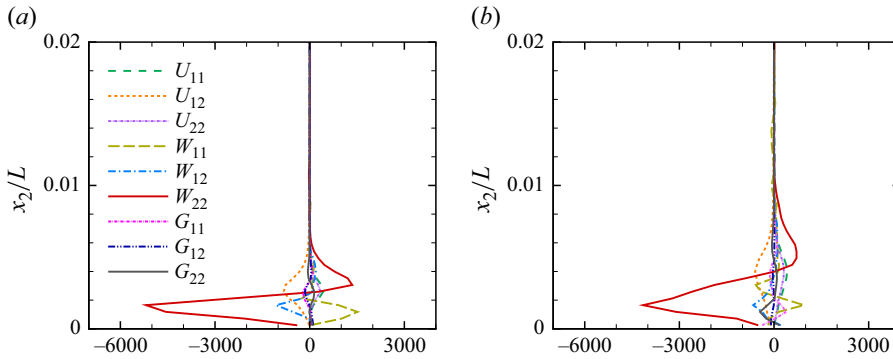


Figure 13. Mean pressure budget terms at $x/L = 0.05$ from medium-mesh simulations with (a) the SM and (b) the MSM. All terms are non-dimensionalised using U_∞ , L and ρ . The line notations correspond to (4.6)–(4.8).

separation point. The mean SGS stresses, by comparison, make only a minor contribution at this location.

4.3. Reynolds stress transport equation

Based on the mean momentum and pressure budget analyses, we examine the Reynolds shear stress $\overline{u'_1 u'_2}$ and Reynolds normal stress $\overline{u'_2 u'_2}$ at three streamwise locations within the FPG region upstream of the bump peak. Figure 14 shows wall-normal profiles from the two medium-mesh simulations along with DNS results (Uzun & Malik 2022).

The DNS data show that the magnitudes of both stresses gradually increase in the near-wall region, and exhibit distinct internal peaks within the FPG region. Both medium-mesh simulations deviate noticeably from the DNS, overpredicting Reynolds stresses in the outer layer. This is partly attributable to the use of constant, non-optimised model coefficients and the inability of the coarse mesh to resolve near-wall turbulent structures, which can lead to an imbalance in the resolved turbulence, a behaviour widely reported in WMLES (Bae *et al.* 2018; Lozano-Durán & Bae 2019; Rezaeiravesh *et al.* 2019). The most prominent distinction between the two simulations is that the MSM reproduces clear internal peaks of $\overline{u'_1 u'_2}$ and $\overline{u'_2 u'_2}$ at $x_2/L \approx 1.3 \times 10^{-3}$, slightly above the DNS location, while the SM does not capture these features. This upward shift is partly due to the coarse mesh resolution.

As shown in § 4.1, the wall-normal gradient of $\overline{u'_1 u'_2}$ shapes mean momentum transport, while that of $\overline{u'_2 u'_2}$ governs the mean pressure distribution. Accurately capturing the wall-normal variation of these stresses is therefore essential for predicting downstream separation. The improved prediction of Reynolds stress profiles in the MSM, particularly the internal stress peaks under FPG conditions, demonstrates the benefit of incorporating anisotropic SGS stress. As discussed by Uzun & Malik (2022), and consistent with the analysis in § 3.2, these internal peaks evolve downstream and strongly influence the mean flow and separation onset. Taken together, the results indicate that the improved separation prediction in the medium-mesh simulation with MSM is linked to its ability to better reproduce Reynolds stress distributions, particularly the internal peak in the FPG region.

To understand how the SGS model influences these Reynolds stress distributions, we analyse the Reynolds stress transport equation (Meneveau 1994; Abe 2019; Inagaki & Kobayashi 2023)

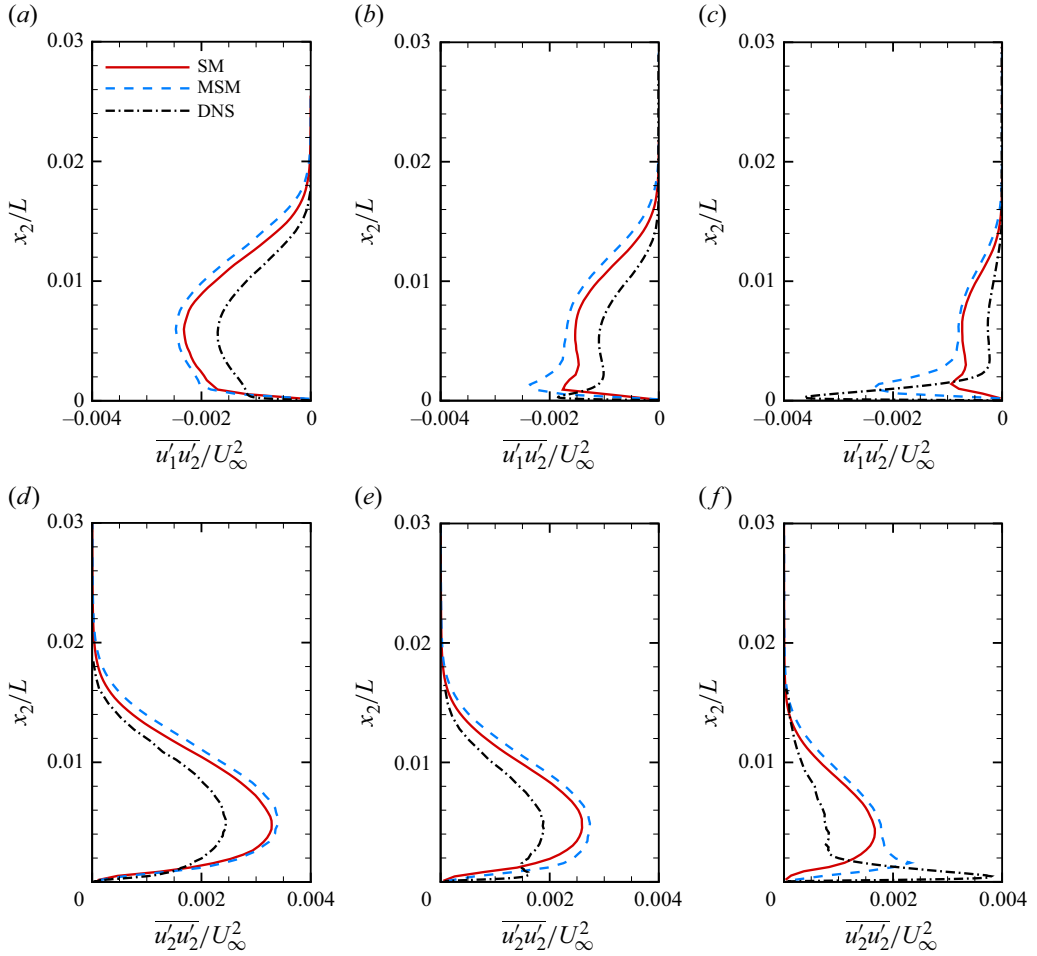


Figure 14. (a–c) Reynolds shear stress $\overline{u'_1 u'_2}$ and (d–f) Reynolds normal stress $\overline{u'_2 u'_2}$ profiles at (a,d) $x/L = -0.2$, (b,e) $x/L = -0.1$, and (c,f) $x/L = 0$ for the SM, the MSM and reference DNS (Uzun & Malik 2022).

$$\frac{\partial \overline{u'_i u'_j}}{\partial t} + \overline{u_k} \frac{\partial \overline{u'_i u'_j}}{\partial x_k} = P_{ij} - \varepsilon_{ij} + \phi_{ij} + \xi_{ij} + \frac{\partial}{\partial x_k} (\zeta_{ijk} + D_{ijk} + T_{ijk} + J_{ijk}), \quad (4.9)$$

where

$$P_{ij} = -\overline{u'_i u'_k} \frac{\partial \overline{u_j}}{\partial x_k} - \overline{u'_j u'_k} \frac{\partial \overline{u_i}}{\partial x_k}, \quad (4.10)$$

$$\varepsilon_{ij} = 2\nu \overline{\left(S'_{ik} \frac{\partial u'_j}{\partial x_k} + S'_{jk} \frac{\partial u'_i}{\partial x_k} \right)} = 2\nu \overline{\left(\frac{\partial u'_i}{\partial x_k} \frac{\partial u'_j}{\partial x_k} \right)}, \quad (4.11)$$

$$\phi_{ij} = \frac{p'}{\rho} \overline{\left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right)} = 2 \frac{p'}{\rho} \overline{S'_{ij}}, \quad (4.12)$$

$$\xi_{ij} = \overline{(\tau_{ik}^{sgs})'} \frac{\partial u_j'}{\partial x_k} + \overline{(\tau_{jk}^{sgs})'} \frac{\partial u_i'}{\partial x_k}, \quad (4.13)$$

$$\frac{\partial}{\partial x_k} \zeta_{ijk} = -\frac{\partial}{\partial x_k} \left[\overline{(\tau_{ik}^{sgs})' u_j'} + \overline{(\tau_{jk}^{sgs})' u_i'} \right], \quad (4.14)$$

$$\frac{\partial}{\partial x_k} D_{ijk} = \nu \frac{\partial^2 \overline{u_i' u_j'}}{\partial x_k^2} = 2\nu \frac{\partial}{\partial x_k} \left(\overline{S_{ik}' u_j'} + \overline{S_{jk}' u_i'} \right), \quad (4.15)$$

$$\frac{\partial}{\partial x_k} T_{ijk} = -\frac{\partial}{\partial x_k} \overline{u_i' u_j' u_k'}, \quad (4.16)$$

$$\frac{\partial}{\partial x_k} J_{ijk} = -\frac{\partial}{\partial x_k} \left(\overline{p' u_i'} \delta_{jk} + \overline{p' u_j'} \delta_{ik} \right), \quad (4.17)$$

corresponding to production, viscous dissipation, pressure strain, SGS dissipation, SGS diffusion, viscous diffusion, turbulent diffusion and pressure diffusion, respectively.

The SGS-related terms in (4.13) and (4.14) directly represent the contributions from the SGS model. Although they are generally smaller in magnitude than leading-order terms such as production and pressure strain (see Appendix C), they remain physically meaningful. The SGS dissipation represents local transfer of resolved-scale energy to unresolved scales, while SGS diffusion represents spatial redistribution. Crucially, differences in these terms between the two models modify the evolution of the resolved Reynolds stresses, which in turn drives changes in production, pressure strain and other terms through the coupled governing equations. The differences observed in the remaining budget terms between the SM and MSM can therefore be interpreted as a response of the resolved flow to different SGS forcing. The following analysis focuses on these SGS terms.

Figure 15 shows the SGS dissipation and diffusion from the medium-mesh simulations at the three streamwise locations. The SM and MSM produce qualitatively different behaviours for both $\overline{u_1' u_2'}$ and $\overline{u_2' u_2'}$. In the SM, SGS dissipation remains positive for $\overline{u_1' u_2'}$ and negative for $\overline{u_2' u_2'}$, consistently acting as a sink of resolved energy. In contrast, the MSM yields negative SGS dissipation for $\overline{u_1' u_2'}$ in the inner layer, and positive values for $\overline{u_2' u_2'}$ very near the wall, indicating net backscatter, a transfer of energy from unresolved to resolved scales that locally enhances the Reynolds stresses. This backscatter is physically significant: on a coarse mesh, the cut-off lies in the energy-containing range where unresolved motions carry substantial turbulent energy and momentum fluxes, and the near-wall cycle involves essential small-scale dynamics that become especially important under pressure gradients. A coarse-mesh LES omitting these near-wall scales loses an important pathway by which energy is both removed from and returned to the larger scales.

The SGS diffusion results reveal further important differences. For $\overline{u_1' u_2'}$, the SM produces negative diffusion near the wall that becomes positive farther away, implying a redistribution of energy from the outer part of the inner layer towards the near-wall region. The MSM yields negative diffusion throughout, continuously moving $\overline{u_1' u_2'}$ towards the wall. For $\overline{u_2' u_2'}$, the SM gives positive near-wall diffusion decreasing with distance, redistributing energy from the outer region towards the wall. The MSM shows negative diffusion very close to the wall, positive values near the internal peak, and negative values farther out, moving $\overline{u_2' u_2'}$ into the internal-peak region from both sides. These SGS diffusion behaviours, combined with the differences in SGS dissipation, help to explain

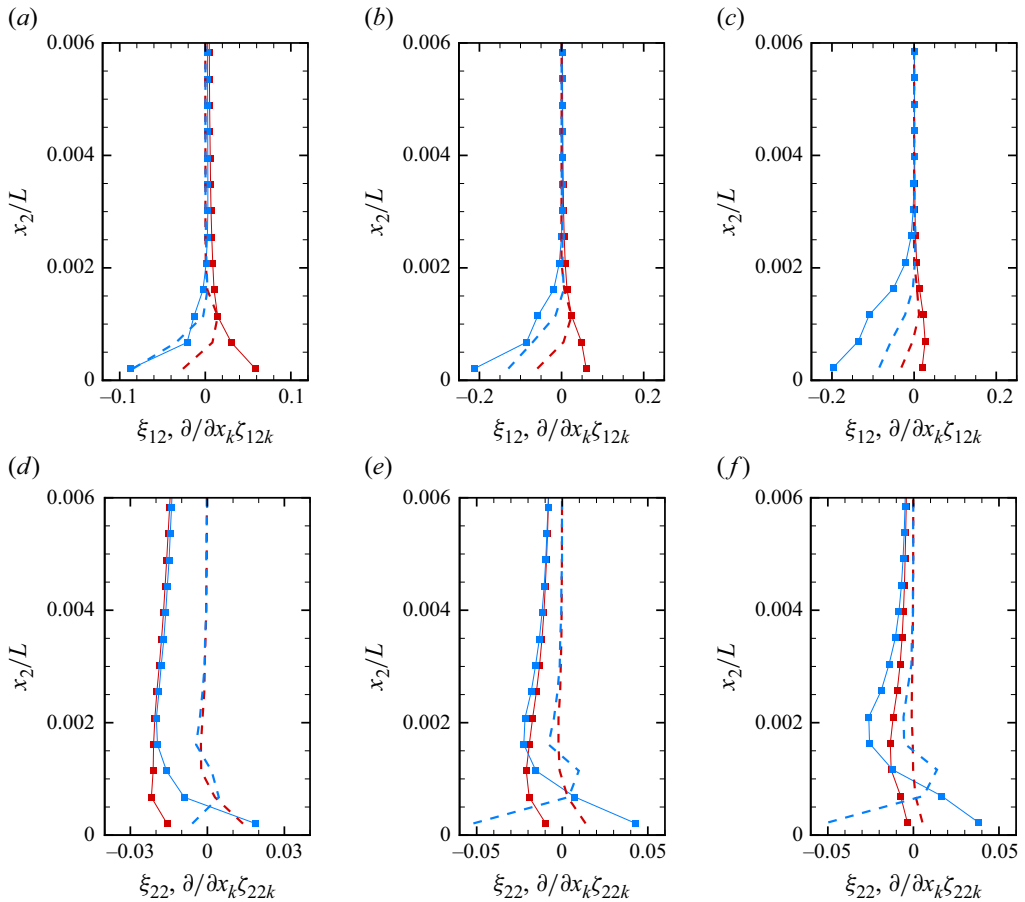


Figure 15. (a–c) The SGS dissipation ξ_{ij} (solid squares) and diffusion $\partial\zeta_{ijk}/\partial x_k$ (dashed lines) for the Reynolds shear stress $\overline{u'_1 u'_2}$, and (d–f) the wall-normal Reynolds normal stress $\overline{u'_2 u'_2}$ at (a,d) $x/L = -0.2$, (b,e) $x/L = -0.1$, and (c,f) $x/L = 0$ for SM (red) and MSM (blue) simulations on the medium mesh. All terms are non-dimensionalised using U_∞ , L and ρ .

why the MSM produces the internal Reynolds stress peaks in figure 14 whereas the SM does not.

To clarify the roles of the isotropic and anisotropic components, figure 16 shows their individual contributions to SGS dissipation and diffusion in the MSM. For both $\overline{u'_1 u'_2}$ and $\overline{u'_2 u'_2}$, the isotropic component behaves similarly to the SM, consistently acting as a dissipative sink with larger magnitude. In contrast, the anisotropic component generates significant local production in the near-wall inner layer, confirming it as the primary source of backscatter in the MSM. For the SGS diffusion of $\overline{u'_1 u'_2}$, the isotropic part mirrors the SM pattern (negative near the wall, positive farther out), while the anisotropic part shows the opposite, with negative values near the internal peak at $x_2/L \approx 1.3 \times 10^{-3}$. For $\overline{u'_2 u'_2}$, the isotropic part redistributes energy from the outer region towards the wall, while the anisotropic part redistributes energy into the internal-peak location from neighbouring wall-normal positions. Taken together, the anisotropic SGS stress in the MSM is responsible for the redistribution of energy towards the wall-normal locations of the internal Reynolds stress peaks.

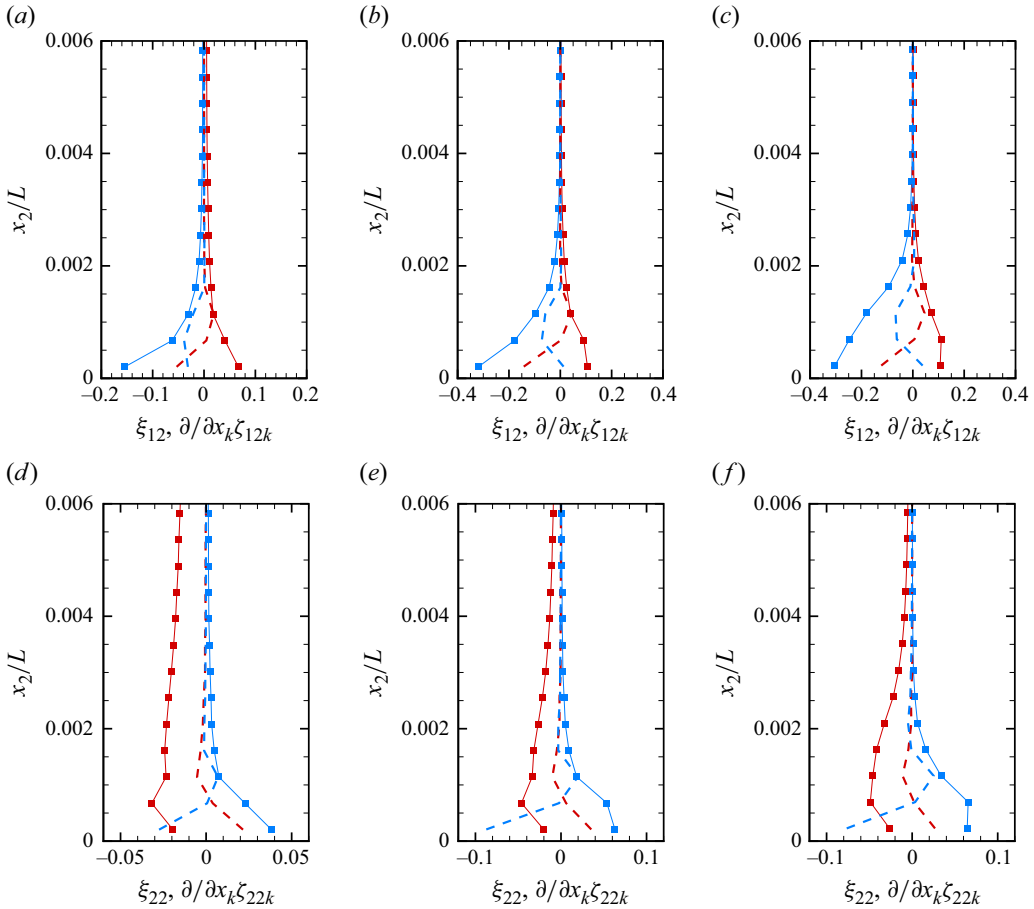


Figure 16. (a–c) The SGS dissipation ξ_{ij} (solid squares) and diffusion $\partial\zeta_{ijk}/\partial x_k$ (dashed lines) from the isotropic (red) and anisotropic (blue) stress components of the MSM for the Reynolds shear stress $\overline{u'_1 u'_2}$, and (d–f) the wall-normal Reynolds normal stress $\overline{u'_2 u'_2}$ at (a,d) $x/L = -0.2$, (b,e) $x/L = -0.1$, and (c,f) $x/L = 0$. All terms are non-dimensionalised using U_∞ , $\bar{L}$ and ρ .

The analyses in this subsection demonstrate that SGS stress fluctuations strongly influence Reynolds stress distributions within the critical FPG region. The isotropic SGS stress acts primarily as a dissipative sink for $\overline{u'_1 u'_2}$ and $\overline{u'_2 u'_2}$, while the anisotropic SGS stress provides significant backscatter and wall-normal redistribution of energy, facilitating the formation of internal Reynolds stress peaks in the near-wall region of the TBL. This mechanism drives the downstream Reynolds stress evolution, and in turn, alters the mean flow on the leeward side, particularly the onset of separation. Accurate WMLES predictions therefore require properly representing these complex near-wall SGS dynamics, including SGS stress fluctuations.

4.4. Influence of mesh resolution

We now address the second question from § 3: why do anisotropic SGS models yield more consistent separation bubble predictions across mesh resolutions than isotropic models? To this end, we examine mean streamwise momentum budgets at a location immediately

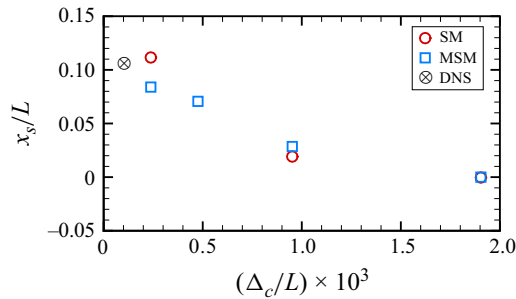


Figure 17. Mean separation point from simulations using the SM and MSM for different mesh resolutions, and from the reference DNS (Uzun & Malik 2022). No separation is detected in the medium-mesh simulation ($\Delta_c/L \approx 4.76 \times 10^{-4}$) with the SM.

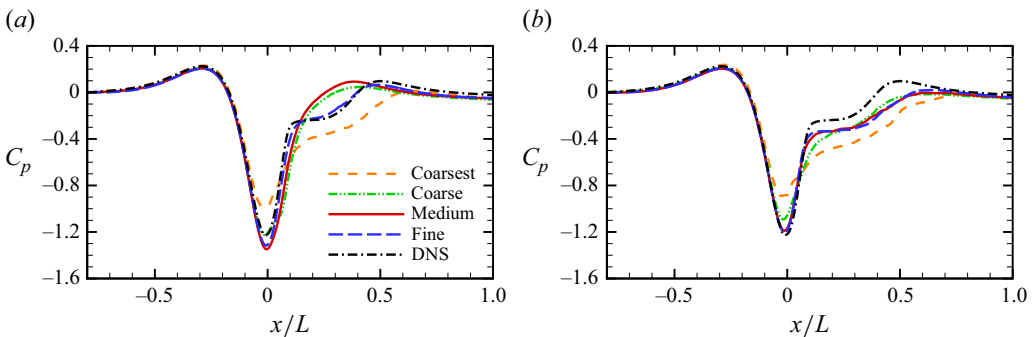


Figure 18. Mean pressure coefficient on the bottom surface from the (a) SM and (b) MSM simulations with different computational meshes, and from the reference DNS (Uzun & Malik 2022).

upstream of the mean separation point across all mesh resolutions, following the approach of § 4.1.

The predicted mean separation points are shown in figure 17. On the coarsest mesh, both models capture separation with similar bubble sizes and separation locations near the bump peak. As the mesh is refined, the separation point shifts downstream, and differences between the two models grow, with the SM losing the separation bubble entirely on the medium mesh. On the fine mesh, both models again predict separation, and their predictions approach the DNS reference (Uzun & Malik 2022).

Figure 18 shows the distributions of the mean pressure coefficient C_p on the bottom surface for all mesh resolutions and both SGS models. Upstream of the bump peak, results agree well across resolutions and models. On the leeward side, the SM simulations are sensitive to mesh resolution: the medium mesh produces no pressure plateau (consistent with the absence of separation), while the coarsest, coarse and fine meshes all produce some plateau or inflection. In contrast, the MSM consistently produces a separation-indicative pressure plateau across all resolutions. With the fine mesh, both models approach the DNS. These C_p distributions directly determine the streamwise pressure gradient and thus the P_g term in the mean momentum equation.

The momentum budget analysis is performed at $0.02L$ upstream of each simulation's mean separation point along the x direction, approximately one boundary layer thickness upstream of separation, with results shown in figure 19. For the medium-mesh simulation with the SM, where no separation bubble forms on the leeward side of the bump,

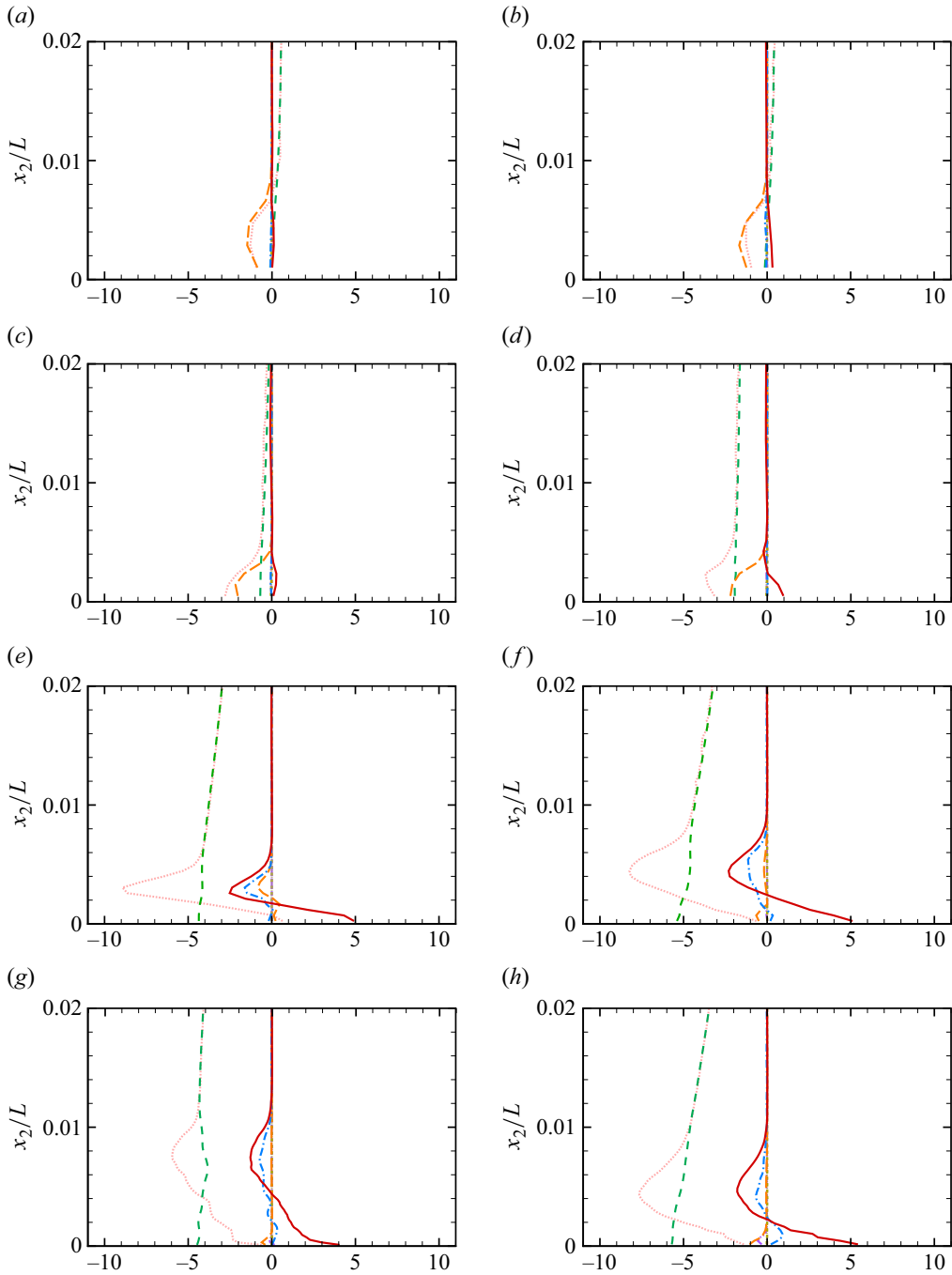


Figure 19. Mean streamwise momentum budget terms at $0.02L$ upstream of the corresponding mean separation point along the x direction from cases with (a,c,e,g) SM and (b,d,f,h) MSM for the (a,b) coarsest mesh, (c,d) coarse mesh, (e,f) medium mesh and (g,h) fine mesh. All terms are non-dimensionalised using U_∞ , L and ρ . The line styles correspond to those in figure 12.

the same location as in the medium-mesh MSM simulation is used for consistency. On the coarsest mesh, both models behave similarly, with the mean SGS shear stress gradient T_{12} dominating the lower half of the TBL. On the coarse mesh, the separation point shifts downstream, and the APG strengthens. While T_{12} remains important, R_{12} begins to play a more notable role in the MSM simulation. On the medium mesh, T_{12} decreases substantially in both cases, and the qualitative difference in separation behaviour emerges as discussed in § 4.3. On the fine mesh, R_{12} and P_g become the dominant contributions in both models. Notably, the negative region of R_{12} in the SM case extends over a wider wall-normal range than on the medium mesh, extracting momentum over a thicker layer, and redistributing it towards the wall. Such redistribution reduces the streamwise momentum available downstream to resist the APG, ultimately enabling separation. The contribution of R_{11} , associated with $\overline{u'_1 u'_1}$, also becomes important at this resolution.

The mean convection term itself exhibits a resolution-dependent character that is consistent with the broader picture above. On the coarsest and coarse meshes, where the SGS shear stress gradient T_{12} dominates the budget, the convection term is modest in magnitude throughout the boundary layer. As the mesh is refined to medium and fine resolutions, the convection term develops substantially richer structure. Specifically, it becomes large in the outer layer, where it grows to balance the pressure gradient, and it develops pronounced structure in the near-wall region as well. This shift parallels the broader transition from an SGS-dominated near-wall momentum balance at coarse resolution to a balance dominated by mean convection, pressure gradient, and Reynolds stress at finer resolutions. As discussed in § 4.1, the convection term is a kinematic consequence of the established mean flow rather than a direct forcing of the local momentum balance, and the present discussion therefore focuses on the forcing terms P_g and R_{12} that connect the SGS model to the separation onset mechanism. The residual is small relative to the leading-order terms at all four mesh resolutions, confirming the numerical accuracy of the budgets, and is not shown for brevity.

These results reveal how the relative importance of mean SGS stress and Reynolds stress shifts with mesh resolution. At coarse resolutions, many flow structures, including the internal Reynolds stress peaks in the FPG region, remain unresolved, and the mean SGS shear stress $\overline{\tau_{12}^{sgs}}$ dominates the momentum balance. Since the anisotropic term's contribution to $\overline{\tau_{12}^{sgs}}$ in the near-wall region is limited (as shown in § 5), both models behave similarly. At medium resolution, more scales are resolved, Reynolds stresses play a more significant role, and the anisotropic SGS term improves flow prediction through its effect on SGS stress fluctuations, as discussed in § 4.3. At fine resolution, the SGS model accounts only for the smallest motions, and the influence of both models on mean velocity and Reynolds stress predictions weakens. These findings highlight that robust SGS models for WMLES must accurately represent both mean SGS stress and SGS stress fluctuations across varying mesh resolutions.

5. Properties of SGS stress

In this section, we examine the properties of the SGS stress and the underlying mechanism of the anisotropic SGS stress. The investigation focuses on the critical FPG region upstream of the bump peak, where the anisotropic SGS stress has a strong effect and significantly influences the downstream mean flow separation.

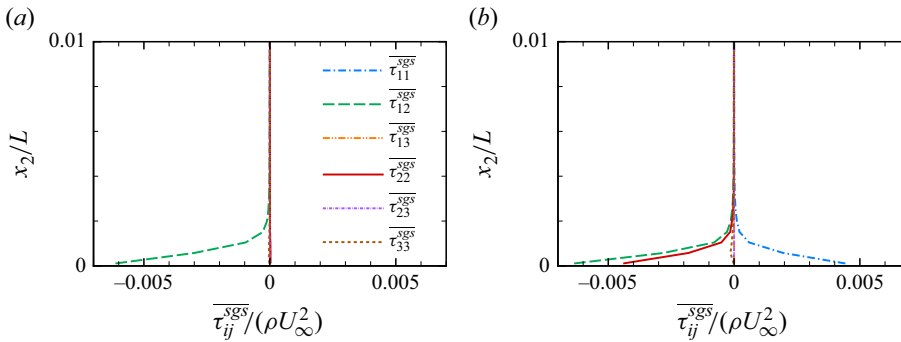


Figure 20. Mean SGS stress tensor components $\overline{\tau_{ij}^{sgs}}$ at $x/L = -0.1$ for medium-mesh simulations with (a) the SM and (b) the MSM.

5.1. Mean SGS stress

We first examine the mean SGS stress at $x/L = -0.1$, near the centre of the critical FPG region where the TBL remains attached in all simulations. Figure 20 shows the wall-normal distributions of the six independent components of the mean SGS stress from the medium-mesh simulations with the SM and MSM.

In both simulations, the mean SGS stress is significant only very near the wall, with the dominant component being the shear stress $\overline{\tau_{12}^{sgs}}$, as expected from the large wall-normal gradient of the streamwise velocity. In the SM, the remaining components are negligible relative to this dominant shear stress. In contrast, the MSM produces two additional components of appreciable magnitude, $\overline{\tau_{11}^{sgs}}$ and $\overline{\tau_{22}^{sgs}}$, which modify the principal directions of the mean SGS stress tensor, and alter the associated momentum transfer. This qualitative behaviour persists throughout the FPG region. Since $\overline{\tau_{11}^{sgs}}$, $\overline{\tau_{22}^{sgs}}$ and $\overline{\tau_{12}^{sgs}}$ are the three main components and the others are negligible, only these are presented in subsequent figures.

The behaviour of the mean SGS stress at $x/L = -0.1$ is consistent across mesh resolutions. As the mesh is refined, the magnitude of the dominant SGS stress components decreases, and the wall-normal extent of significant SGS shear stress narrows. For the SM, $\overline{\tau_{12}^{sgs}}$ remains dominant at all resolutions, though at fine resolution the normal components $\overline{\tau_{11}^{sgs}}$ and $\overline{\tau_{22}^{sgs}}$ become non-negligible near the wall. This indicates that the anisotropic dynamics of near-wall turbulence begin to be resolved, with a shift in the principal directions of the mean SGS stress and strain-rate tensors. The signs of these normal components are consistent with those from the MSM, suggesting that the principal direction of the mean SGS stress in the fine-mesh SM case begins to align with that of the MSM. For the MSM, the contributions of all three main components remain significant near the wall and qualitatively unchanged across resolutions.

Figure 21 shows the isotropic and anisotropic contributions to the mean SGS stress at $x/L = -0.1$ for the medium-mesh MSM simulation. The mean isotropic stress behaves almost identically to the SM (see figure 20), indicating that the anisotropic term does not significantly modify the isotropic part. A similar trend is observed in the eddy-viscosity distributions within the FPG region (figure 3), and since SGS dissipation in the MSM arises exclusively from the isotropic term, the SGS kinetic energy dissipation is comparable between the SM and the MSM at this location. In contrast, the mean anisotropic stress is dominated by the normal components, with a negligible shear component. Comparison with figure 20 confirms that the normal stresses in the MSM

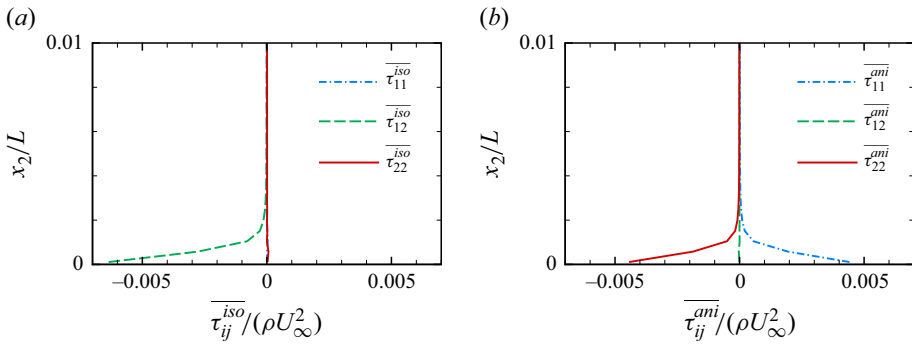


Figure 21. (a) Mean isotropic SGS stress tensor components ($\overline{\tau_{11}^{iso}}$, $\overline{\tau_{12}^{iso}}$ and $\overline{\tau_{22}^{iso}}$) and (b) mean anisotropic SGS stress tensor components ($\overline{\tau_{11}^{ani}}$, $\overline{\tau_{12}^{ani}}$ and $\overline{\tau_{22}^{ani}}$) at $x/L = -0.1$ for medium-mesh simulations with the MSM.

originate almost entirely from the anisotropic term, which is therefore the primary source of the difference in principal directions of the mean SGS stress tensor between the two models.

The differences in mean SGS stresses between the SM and MSM thus arise from the normal components τ_{11}^{sgs} and τ_{22}^{sgs} , which originate from the anisotropic term. However, as shown in the momentum budget analyses of §§ 4.1 and 4.4, these normal stresses contribute little to mean streamwise momentum transport. The mean shear stress τ_{12}^{sgs} , which is most relevant to momentum transport and separation onset at coarse resolutions, is predicted similarly by both models in the critical FPG region. This explains why the coarsest- and coarse-mesh simulations yield similar separation behaviour for both SGS models.

5.2. The SGS stress fluctuations

Figure 22 shows wall-normal distributions of the root mean square (r.m.s.) values of SGS stress fluctuations at $x/L = -0.1$ for the medium-mesh simulations. As with the mean SGS stresses, fluctuations are largest near the wall, with τ_{12}^{sgs} exhibiting the strongest fluctuations in both simulations. In the MSM, however, the normal components τ_{11}^{sgs} and τ_{22}^{sgs} also display substantial fluctuations, unlike in the SM. Since fluctuations of the normal stresses contribute directly to SGS dissipation and diffusion in the Reynolds stress transport equation, these enhanced fluctuations explain the distinct SGS dissipation and diffusion behaviour observed for the MSM in § 4.3. This qualitative behaviour remains consistent throughout the FPG region.

Across mesh resolutions, the qualitative behaviour of SGS stress fluctuations within each model remains similar upon refinement. In contrast to the mean SGS stresses, which decrease rapidly with mesh refinement, the fluctuations exhibit only a mild reduction, indicating their increasing relative importance on finer grids. The MSM consistently yields larger fluctuations than the SM across all resolutions at this location.

Figure 23 shows the isotropic and anisotropic contributions to the SGS stress fluctuations at $x/L = -0.1$ for the medium-mesh simulation with the MSM. The isotropic fluctuations behave similarly to those in the SM, confirming that the anisotropic term does not substantially alter the isotropic component. The anisotropic fluctuations are dominated by the normal components τ_{11}^{ani} and τ_{22}^{ani} , consistent with the mean anisotropic stress

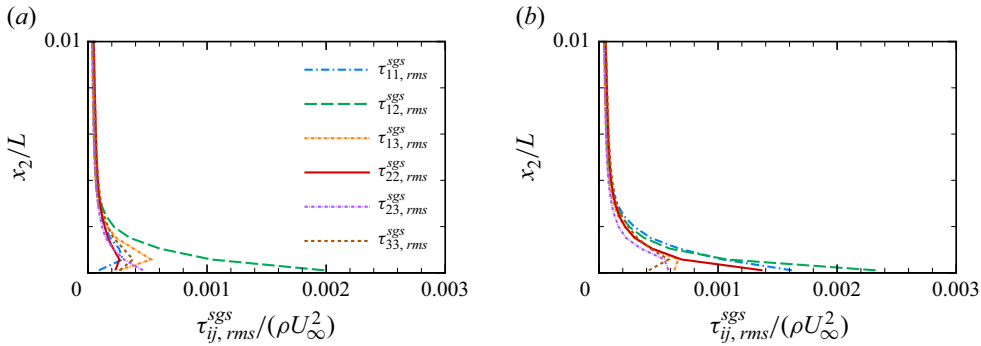


Figure 22. The SGS stress tensor r.m.s. components τ_{ij}^{sgs} at $x/L = -0.1$ for medium-mesh simulations with (a) the SM and (b) the MSM.

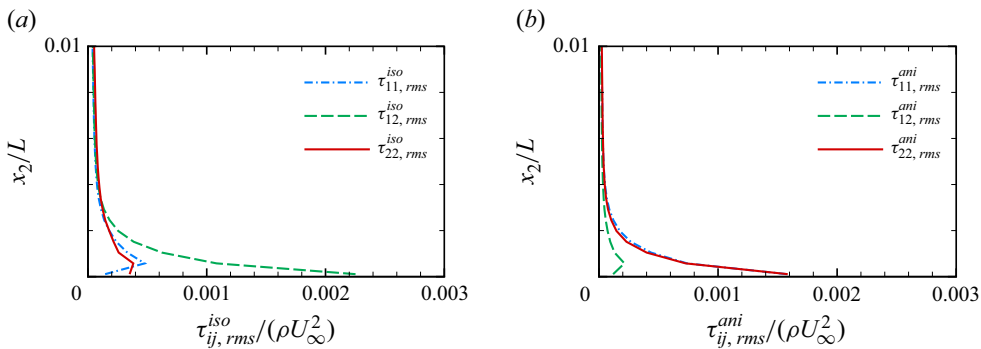


Figure 23. (a) Isotropic SGS stress tensor r.m.s. components (τ_{11}^{iso} , τ_{12}^{iso} and τ_{22}^{iso}) and (b) anisotropic SGS stress tensor r.m.s. components (τ_{11}^{ani} , τ_{12}^{ani} and τ_{22}^{ani}) at $x/L = -0.1$ for medium-mesh simulations with the MSM.

behaviour, and comparison with [figure 22](#) confirms these as the primary source of the enhanced normal stress fluctuations in the MSM.

These observations confirm that the differences in SGS dissipation and diffusion of Reynolds stresses discussed in § 4.3 stem from the anisotropic stress term in the MSM, not from modifications to the isotropic component. This suggests that optimising the coefficient and formulation of the anisotropic stress term offers an effective pathway for controlling SGS stress fluctuations and improving Reynolds stress and mean velocity predictions.

5.3. *A priori* analysis of filtered DNS

To further validate the *a posteriori* findings and gain additional insight into SGS stress characteristics in wall-bounded turbulence under FPG, we perform an *a priori* analysis using Gaussian-filtered DNS of turbulent Couette–Poiseuille flow at $Re_H = U_c H/\nu = 2500$, where H is the channel half-height, and U_c is the wall motion speed. Details of the DNS and filtering procedure are provided in [Appendices D](#) and [E](#).

The analysis focuses on the lower half of the channel, where the flow near the bottom wall experiences an FPG and behaves qualitatively similarly to a TBL with FPG. Gaussian filtering is applied only in the streamwise (x) and spanwise (z) directions, with standard

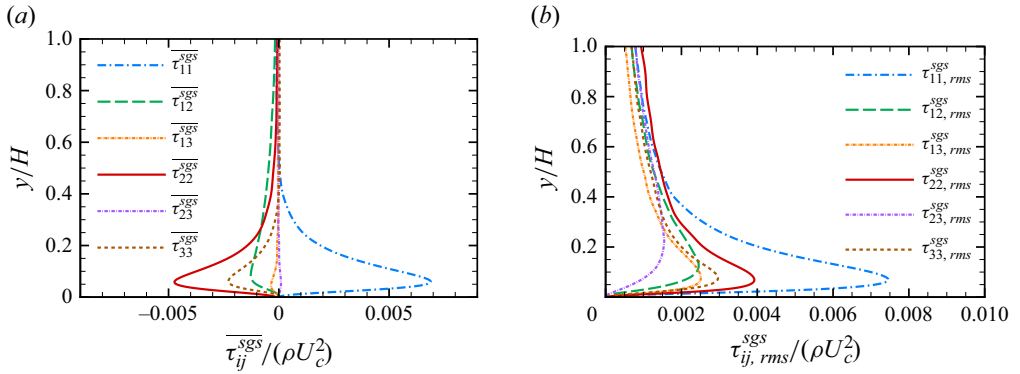


Figure 24. (a) Mean SGS stress tensor components $\overline{\tau_{ij}^{sgs}}$ and (b) SGS stress tensor r.m.s. components $\tau_{ij,rms}^{sgs}$ obtained from the Gaussian filtering of the velocity field from the DNS of turbulent Couette–Poiseuille flow with the standard deviations of the Gaussian kernel $\sigma_x/\Delta_x = \sigma_z/\Delta_z = 2$.

deviations $\sigma_x/\Delta_x = \sigma_z/\Delta_z = 1, 2$ and 4 , where Δ_x and Δ_z are the uniform DNS grid spacings. The SGS stress is computed as $\tau_{ij}^{sgs} = \widehat{u_i u_j} - \widehat{u_i} \widehat{u_j}$, where $\widehat{(\cdot)}$ denotes Gaussian filtering, and only the deviatoric part is considered.

Figure 24(a) shows the wall-normal distributions of mean SGS stress components for $\sigma_x/\Delta_x = \sigma_z/\Delta_z = 2$. The magnitudes are largest near the wall, peaking at $y/H \approx 0.06$ (approximately 20 wall units). The normal components τ_{11}^{sgs} and τ_{22}^{sgs} are significantly larger in magnitude than the other components, indicating strong SGS anisotropy near the wall. Despite the large mean shear in this region, the small-scale cross-correlation between streamwise and wall-normal velocity fluctuations is weaker than the variance of each component, keeping the mean shear stress τ_{12}^{sgs} comparatively small.

Compared with the mean SGS stress in the TBL within the FPG region of the WMLES, the filtered DNS results are more consistent with the MSM predictions. In both cases, the anisotropic stress introduces dominant normal stress components near the wall with consistent signs for τ_{11}^{sgs} and τ_{22}^{sgs} . The peaks are not fully captured in the WMLES due to the coarse wall-normal resolution.

It is also worth noting that the qualitative behaviour of the mean SGS stresses remains similar across different standard deviations of the Gaussian filter, or equivalently, different effective filter widths. For brevity, results for other filter widths (e.g. $\sigma_x/\Delta_x = \sigma_z/\Delta_z = 1$ or 4) are not shown. Examination of these cases indicates that increasing the filter standard deviation leads to larger mean SGS stress magnitudes, consistent with the fact that wider filters remove more turbulent scales and therefore attribute a greater portion of the momentum transfer to the unresolved motions.

Figure 24(b) shows the r.m.s. values of SGS stress fluctuations from the filtered DNS. The fluctuation intensities peak near the wall at approximately the same location as the mean SGS stresses, reflecting the highly intermittent energy transfer between resolved and subgrid scales driven by the bursting and ejection–sweep cycles of near-wall turbulence. The particularly large fluctuations in τ_{11}^{sgs} and τ_{22}^{sgs} indicate substantial temporal variability in the normal stress components, likely associated with rapid distortion of streaks and vortical structures. These strong normal SGS stress fluctuations are captured by the MSM in the WMLES but are nearly absent in the SM results, reinforcing the role of the anisotropic stress term in reproducing correct near-wall SGS dynamics. The qualitative behaviour is similar across filter widths, with fluctuation intensities increasing with filter width.

From the filtered DNS analysis and its comparison with the WMLES results, it is evident that in wall-bounded turbulence under FPG, SGS dynamics is strongly anisotropic near the wall, with the normal SGS stress components playing a major role. Classical eddy-viscosity models cannot represent these effects adequately. In contrast, the anisotropic SGS model reproduces the near-wall anisotropy much more realistically, consistent with the filtered DNS characteristics. These findings suggest that improving or optimising the anisotropic stress term and its coefficient in SGS models offers a promising pathway for enhancing model performance in simulations of complex turbulent flows.

6. Conclusions

This study performed a comprehensive *a posteriori* analysis of the effect of anisotropic SGS stress on WMLES of separated turbulent flow over a spanwise-uniform Gaussian bump. An idealised wall boundary condition prescribing the local mean wall shear stress from DNS data (Uzun & Malik 2022) was used to isolate the impact of the SGS model. Two models were compared: the classical Smagorinsky model (SM) and a modified Smagorinsky model (MSM) that includes an additional anisotropic stress term.

The main findings are summarised as follows. First, the predicted flow separation on the leeward side of the bump depends strongly on the SGS model. The isotropic SM exhibits non-monotonic convergence of the mean separation bubble length with mesh refinement, whereas the anisotropic MSM provides consistent predictions across resolutions. Second, the influence of anisotropic SGS stress is found to be most critical upstream of the bump peak, within the region of strong FPG. Changes to the SGS model in this region substantially alter the downstream separation, revealing a pronounced history effect in determining separation onset. Third, inclusion of anisotropic SGS stress improves the prediction of Reynolds shear and normal stress distributions in the FPG region. These modifications propagate downstream and influence the onset and size of the separation bubble. Analysis of the Reynolds stress budget shows that anisotropic SGS stress fluctuations enable both dissipation and backscatter, facilitating the bidirectional energy transfer that isotropic models fail to represent. The dependence of flow-separation prediction on mesh resolution is also clarified. On coarse meshes, the mean SGS shear stress dominates the streamwise momentum balance upstream of the separation point, and both models behave similarly. As the resolution increases, Reynolds stresses become more influential, and the anisotropic MSM better captures their distribution and yields more consistent flow predictions. At fine resolution, model differences diminish as more turbulent scales are resolved. The key physical distinction between the SM and the MSM arises from the normal SGS stress components τ_{11}^{sgs} and τ_{22}^{sgs} , which in the MSM significantly contribute to SGS dissipation and diffusion of Reynolds stresses, particularly under FPG. An *a priori* analysis based on filtered DNS of Couette–Poiseuille flow further confirms that near-wall turbulence under FPG is highly anisotropic and dominated by these normal stress components, which are not captured by isotropic eddy-viscosity models.

Taken together, these findings explain why isotropic and anisotropic SGS models yield qualitatively different predictions of separation behaviour, and why the anisotropic model achieves more consistent convergence across mesh resolutions. The results emphasise that accurate WMLES predictions require proper representation of both mean and fluctuating SGS stresses, especially their anisotropy in the near-wall region.

Beyond elucidating the role of anisotropic SGS stress, this study highlights directions for improving WMLES of complex wall-bounded turbulence. Since the unresolved motions in WMLES carry substantial energy and momentum fluxes, the SGS model must account for anisotropic stress dynamics near the wall and under pressure gradients. Developing

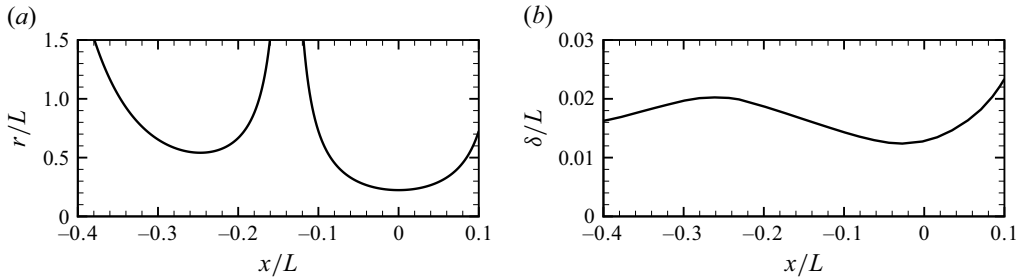


Figure 25. (a) Curvature radius of the Gaussian bump surface and (b) local boundary layer thickness for the medium-mesh simulation with the SM in $x/L \in [-0.4, 0.1]$.

more advanced anisotropic SGS models, potentially through optimised extensions of eddy-viscosity formulations (Marstorp *et al.* 2009; Silvis & Verstappen 2019; Agrawal *et al.* 2022; Uzun & Malik 2025), is therefore a promising path forward. Although this work employed an idealised wall model based on the exact mean wall shear stress to isolate SGS effects, simulations using other idealised wall-model formulations (Zhou & Bae 2024a), as well as an equilibrium wall model (Wang & Moin 2002), also show improved prediction of flow separation when combined with the anisotropic SGS models considered in the present study. Moreover, to further enhance the overall performance of WMLES, the coupling between realistic wall models and SGS models remains a critical challenge that needs to be addressed. In this context, unified SGS/wall modelling frameworks (Ling *et al.* 2022; Arranz *et al.* 2023, 2024; Zhou *et al.* 2025) represent a promising direction to improve the robustness and predictive accuracy of WMLES for complex turbulent flows.

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Declaration of interests. The authors report no conflict of interest.

Appendix A. Investigation of the curvature effect on the region upstream of separation

Figure 25 shows the local curvature radius (r) of the present Gaussian bump surface and the local boundary layer thickness (δ) from the medium-mesh simulation with the SM in the region $x/L \in [-0.4, 0.1]$. The curvature radius is substantially larger than the local boundary layer thickness throughout, with the ratio exceeding 10 even at the bump peak where curvature is largest. These observations indicate that curvature effects in this region are negligible, consistent with findings of previous studies (Prakash *et al.* 2024; Spalart, Jansen & Coleman 2024).

Appendix B. Simulations using the mixed model

An additional anisotropic SGS model evaluated in the present study is the mixed model (MM) (Bardina 1983; Sarghini *et al.* 1999). It combines the SM (Smagorinsky 1963) with a scale-similarity term computed using explicit filtering (Meneveau & Katz 2000), which is given by

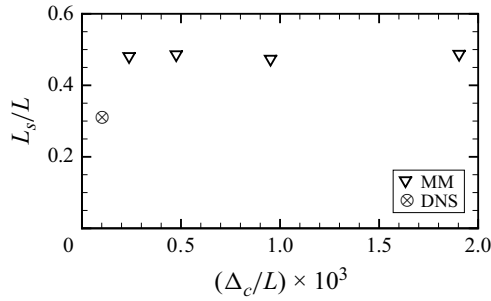


Figure 26. Mean separation bubble length on the leeward side of the bump from the simulations using the MM for different mesh resolutions and the reference DNS (Uzun & Malik 2022). Each symbol represents one simulation case.

$$\tau_{ij}^{ani} = \widehat{u_i u_j} - \hat{u}_i \hat{u}_j, \quad (\text{B1})$$

where $\widehat{(\cdot)}$ denotes an explicit filtering operation, chosen here as Gaussian filtering. With the use of Gaussian filtering, the anisotropic term can be approximated (Clark *et al.* 1979) by

$$\tau_{ij}^{ani} = \widehat{u_i u_j} - \hat{u}_i \hat{u}_j \approx \frac{\Delta^2}{12} \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k} = \frac{\Delta^2}{12} [S_{ik} S_{kj} - R_{ik} R_{kj} - (S_{ik} R_{kj} - R_{ik} S_{kj})]. \quad (\text{B2})$$

In addition to introducing anisotropic SGS stress, this term can produce kinetic energy dissipation. In particular, it can also produce negative dissipation, which allows for local backscatter of kinetic energy. The isotropic SGS stress term in the MM is given by the SM with coefficient $C_s = 0.127$ (Bhushan & Warsi 2005; Bhushan, Warsi & Walters 2006), chosen to ensure that the total kinetic energy dissipation for homogeneous isotropic turbulence from both the isotropic and anisotropic stress terms matches that from the SM and MSM used in the present study. The MM simulations are conducted using the same computational meshes and boundary conditions as described in § 2.

Figure 26 shows the mean separation bubble length on the leeward side of the bump as a function of characteristic mesh resolution. The MM simulations consistently overpredict the separation bubble length, but the predictions remain nearly insensitive to mesh refinement. This trend mirrors the behaviour observed for the MSM simulations (see figure 7), further underscoring the robustness of anisotropic SGS models for WMLES.

Additional analyses of the mean streamwise momentum, mean pressure and Reynolds stress transport equations using data from multiple mesh resolutions show that the MM exhibits behaviour qualitatively similar to the MSM, particularly in its representation of anisotropic stress effects. Thus the conclusions drawn in the main text remain applicable to the MM simulations.

Appendix C. Budget terms for the transport equation of Reynolds stresses $\overline{u'_1 u'_2}$ and $\overline{u'_2 u'_2}$

Figure 27 shows the wall-normal distributions of all budget terms in the transport equations for the Reynolds shear stress $\overline{u'_1 u'_2}$ and the wall-normal Reynolds normal stress $\overline{u'_2 u'_2}$ at $x/L = -0.1$ from the medium-mesh simulations with the SM and MSM, within the critical FPG region identified in § 3.2. In addition to the source, sink and transport

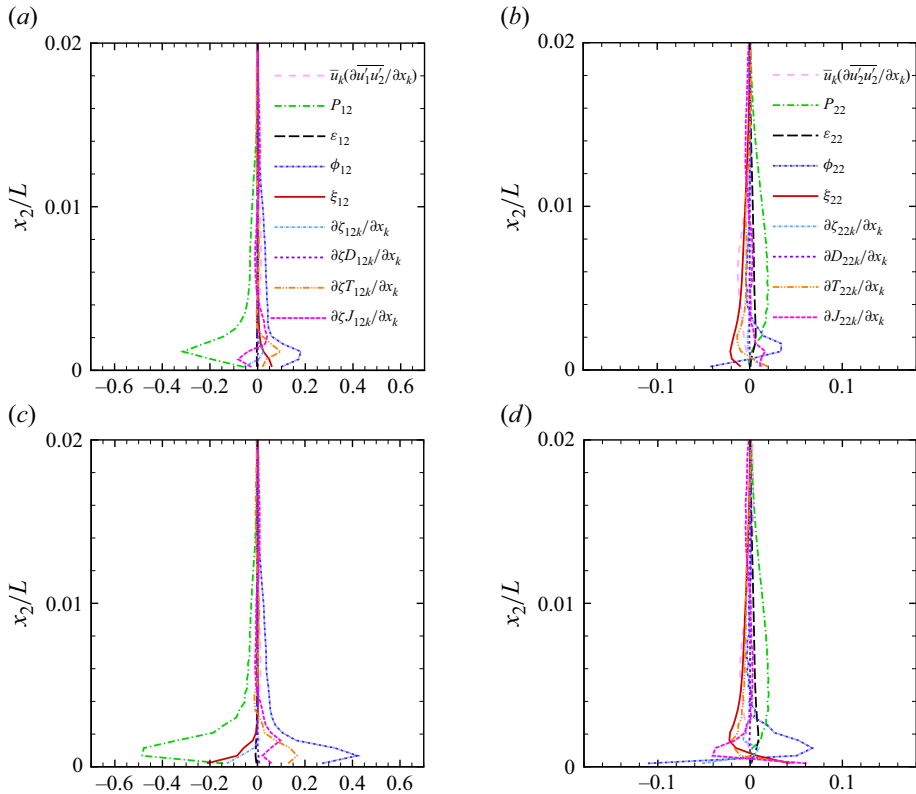


Figure 27. Budget terms for (a,c) the Reynolds shear stress $\overline{u'_1 u'_2}$ and (b,d) the Reynolds normal stress $\overline{u'_2 u'_2}$ transport equations at $x/L = -0.1$ from medium-mesh simulations with (a,b) the SM and (c,d) the MSM. The line notations correspond to the terms in (4.9), excluding the unsteady term that vanishes for the statistically stationary flow, and all terms are non-dimensionalised using U_∞ , L and ρ .

terms on the right-hand side of (4.9), the convection term on the left-hand side is also shown.

For $\overline{u'_1 u'_2}$, the dominant terms are production P_{12} and pressure strain ϕ_{12} , with the MSM exhibiting larger magnitudes of both in the near-wall region. Turbulent and pressure diffusion contribute mainly in the region very close to the wall, while viscous diffusion and dissipation remain small throughout. The SGS dissipation ξ_{12} and diffusion $\partial \zeta_{12k} / \partial x_k$ are not the largest terms in absolute magnitude, but show clear qualitative differences between the two models, as discussed in § 4.3.

For $\overline{u'_2 u'_2}$, production P_{22} is small near the wall, and the dominant source is pressure strain ϕ_{22} , which redistributes turbulent kinetic energy into the wall-normal stress. The MSM produces more pronounced ϕ_{22} , consistent with the internal peak in the $\overline{u'_2 u'_2}$ profiles within the FPG region (figure 14). As with the shear stress budget, the SGS dissipation and diffusion terms show qualitative differences between the two models.

The convection term is small throughout the boundary layer for both Reynolds stress components, indicating that the budget is dominated locally by the source, sink and transport terms on the right-hand side rather than by streamwise mean flow transport. The residual reflecting the numerical and statistical closure error is small relative to the leading-order terms, and smaller than the SGS dissipation and SGS diffusion terms throughout the boundary layer, thus it is not shown for brevity.

Overall, the budget comparison shows that production and pressure strain are the leading-order terms governing the Reynolds stress evolution, while the SGS-related terms are smaller in magnitude. However, the SGS dissipation and SGS diffusion terms provide the pathway through which the SGS model enters the Reynolds stress transport. Their different behaviours between the SM and MSM, particularly the backscatter and wall-normal redistribution introduced by the anisotropic stress term, modify the overall budget balance. Through the coupled nature of the governing equations, these changes lead to corresponding adjustments in the other budget terms. The enhanced production and pressure strain observed in the MSM simulation can therefore be interpreted as a response of the resolved flow to the different SGS forcing. Although the SGS dissipation represents the transfer of energy from resolved to unresolved scales, and the SGS diffusion corresponds to its spatial redistribution, their importance here lies less in their absolute magnitude than in their influence on the overall balance. In this sense, differences between SGS models do not act in isolation but propagate through the coupled system, ultimately affecting the distribution of Reynolds stresses.

Appendix D. The DNS of plane Couette–Poiseuille flow

The DNS of turbulent plane Couette–Poiseuille flow is conducted at $Re_H = 2500$, where H denotes the channel half-height. In the simulation, the incompressible Navier–Stokes equations are solved using a staggered finite-difference scheme that is second-order accurate in space and advanced in time with an explicit third-order Runge–Kutta method. The flow solver has been validated in previous studies of turbulent channel flows (Bae *et al.* 2018, 2019). In the computational domain, periodic boundary conditions are imposed in the streamwise (x) and spanwise (z) directions. The top and bottom walls move parallel to each other in opposite directions along the x axis. A Dirichlet boundary condition with constant velocity $u_x = U_c = 1$ is applied at the top wall ($y/H = 2$), while a constant velocity $u_x = -U_c = -1$ is imposed at the bottom wall ($y/H = 0$). A constant streamwise pressure gradient, corresponding to $(H/(\rho U_c^2)) \, dp/dx = -0.003$, is applied to drive the flow, indicating that the mean pressure decreases in the positive x direction. The computational domain extends over $L_x/H = 6\pi$, $L_y/H = 2$ and $L_z/H = 3\pi$ in the streamwise, wall-normal and spanwise directions, respectively. Uniform grids with 512 points are used in both the streamwise and spanwise directions. In the wall-normal direction, 256 non-uniformly spaced points are distributed according to a hyperbolic tangent stretching, yielding $\min(\Delta_y)/H = 3.5 \times 10^{-4}$ and $\max(\Delta_y)/H = 2.2 \times 10^{-2}$. The simulation is first advanced for 100 flow-through times ($6\pi H/U_c$) to eliminate initial transients. After the flow reaches a statistically stationary state, 1500 temporal snapshots are collected for the present analysis. Moreover, the statistical quantities in the present study are obtained by performing temporal and spatial averaging along the homogeneous streamwise and spanwise directions of the corresponding instantaneous fields.

Figure 28 shows the instantaneous streamwise velocity relative to the bottom wall, $u_r = u_x + U_c$, from the DNS. Due to the motion of the parallel walls and the imposed mean pressure gradient, the wall-bounded turbulence in the upper half of the channel experiences an APG and behaves similarly to an APG TBL containing many large-scale flow structures. In contrast, the turbulence near the bottom wall is subjected to an FPG and is qualitatively similar to an FPG TBL. Figure 29 presents the profile of inner-scaled mean streamwise velocity relative to the bottom wall, defined as $\overline{u_r^+} = (\overline{u_x + U_c})/u_{\tau,b}$. Here, the superscript ‘+’ denotes inner-scaled quantity by wall units, and $u_{\tau,b}$ is the bottom-wall friction velocity.

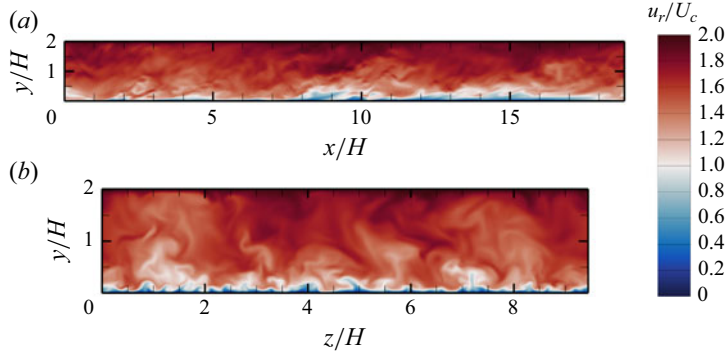


Figure 28. Isocontours of the instantaneous streamwise velocity relative to the bottom wall u_r/U_c in (a) an x - y plane and (b) a z - y plane from the DNS of turbulent Couette–Poiseuille flow.

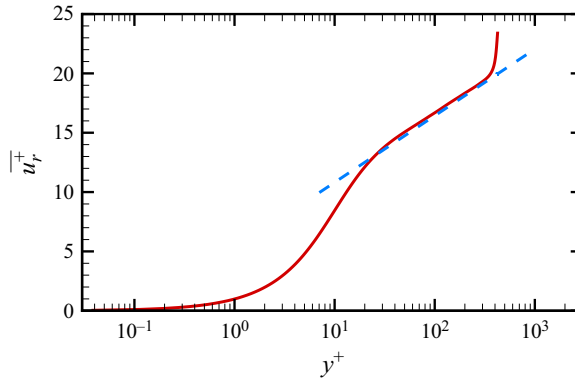


Figure 29. Mean streamwise velocity relative to the bottom wall from the DNS of turbulent Couette–Poiseuille flow. The dashed line represents the classical log law of the wall $\overline{u_r^+} = (1/0.41) \ln(y^+) + 5.2$.

Appendix E. Filtering of the DNS velocity field

To obtain the filtered velocity field from the DNS of the turbulent Couette–Poiseuille flow, a Gaussian filter is applied to the instantaneous velocity field $\mathbf{u} = (u_x, u_y, u_z)$. Since the DNS grid is non-uniform in the wall-normal (y) direction, the filtering operation is performed only in the streamwise (x) and spanwise (z) directions to avoid commutation errors. The filtered velocity field $\widehat{\mathbf{u}} = (\widehat{u}_x, \widehat{u}_y, \widehat{u}_z)$ is obtained through a two-dimensional convolution in the x and z directions:

$$\widehat{u}_i(x, y, z) = \iint G(x - r_x, z - r_z) u_i(r_x, y, r_z) dr_x dr_z, \quad (\text{E1})$$

where the Gaussian kernel is defined as

$$G(r_x, r_z) = \frac{1}{2\pi \sigma_x \sigma_z} \exp \left[-\frac{1}{2} \left(\frac{r_x^2}{\sigma_x^2} + \frac{r_z^2}{\sigma_z^2} \right) \right]. \quad (\text{E2})$$

Here, r_x and r_z denote spatial separations in the streamwise and spanwise directions, and σ_x and σ_z are the corresponding standard deviations of the Gaussian kernel. Since the DNS grid is uniform in both directions, the filtering is implemented as a discrete convolution using symmetric one-dimensional Gaussian kernels applied successively in x and z .

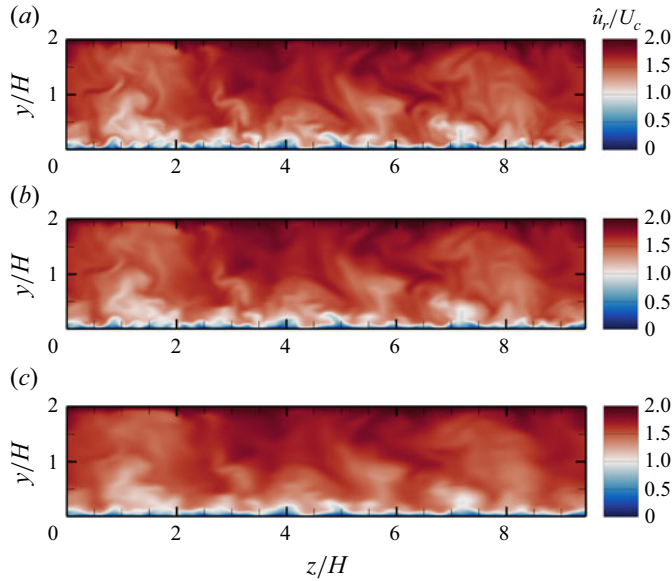


Figure 30. Isocontours of the filtered streamwise velocity $\hat{u}_r/U_c$ with $\sigma_x/\Delta_x = \sigma_z/\Delta_z$ values (a) 1, (b) 2, (c) 4.

The effective filter width $\Delta_{f,i}$ in each direction $i \in \{x, z\}$ is defined by matching the second moment of the Gaussian filter with that of a top-hat filter, giving

$$\Delta_{f,i} = 2\sqrt{3} \sigma_i. \quad (\text{E3})$$

In this study, the standard deviations are set as multiples of the uniform DNS grid spacings such that $\sigma_x/\Delta_x = \sigma_z/\Delta_z = 1, 2$ and 4, corresponding to moderate to coarse filter widths that remove small-scale motions while retaining large-scale flow structures. The resulting effective filter widths $\Delta_{f,x}$ and $\Delta_{f,z}$ are approximately 3.464, 6.928 and 13.856 times the grid spacings Δ_x and Δ_z , respectively.

Figure 30 shows the filtered instantaneous streamwise velocity relative to the bottom wall $\hat{u}_r/U_c$ in an x – y plane for these three filter widths. As the standard deviations increase, progressively finer structures are removed, demonstrating how the Gaussian filter systematically isolates the larger-scale motions.

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