Modal triadic interaction informed restricted nonlinear models

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1 Introduction

The restricted nonlinear (RNL) model is a low order flow representation obtained by decomposing the flow field into a large-scale streamwise mean ($k_x = 0$ in a Fourier representation) and small-scale perturbations about that mean ($k_x \neq 0$). Order reduction is achieved by neglecting nonlinear interactions between perturbations that do not contribute to the mean. This model reproduces low order statistics, energetics and structural features of turbulent flows at low to moderate Reynolds numbers (Re) with the perturbation dynamics supported by as few as one non-zero streamwise mode [2, 3]. However as Re increases, additional nonlinearity is required to capture the critical scale interactions. The required additional model fidelity can achieved by augmenting the RNL model with additional large-scale modes ($K_x \neq 0$) formed by nonlinear interactions between small-scale models (perturbations $k_x^{(j)}$) that interact in a consistent manner such that $k_x^{(2)} = K_x^{(1)} + k_x^{(1)}$. Order reduction is maintained by neglecting nonlinear interactions that do not contribute to the large scale (i.e., the set $\{k_x = 0, K_x \neq 0\}$ are neglected) [6]. Results from a simulation of this augmented RNL (ARNL) model at $\mathrm{Re}_\tau = 180$ with only three non-zero modes is plotted alongside DNS data [5] in figure 1(a). While these and prior studies up to $Re_\tau = 2000$ illustrate the promise of the ARNL paradigm [6], a systematic means of selecting the right set of nonlinear scale interactions to support the ARNL dynamics has yet to be developed.

Recently, there has been a number of studies that attempt to employ modal decompositions to identify key nonlinear interactions in wall-bounded turbulence [1, 4, 7]. For example, the suppression of the principal resolvent forcing mode, which has contributions from limited nonlinear interactions, has been shown to inhibit turbulence within a minimal flow unit [1] (figure 1(b)). This method allows the identification of key nonlinear-interactions in the self-sustaining process of near-wall turbulence.

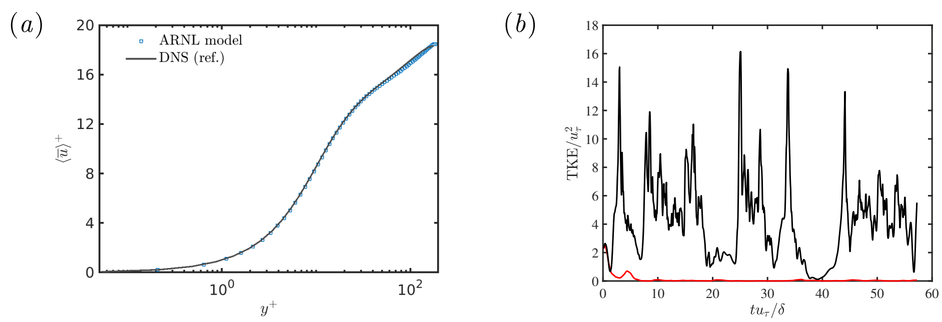

Figure 1: Left: Time- and plane-averaged streamwise velocity as a function of wall normal location for the ARNL model alongside DNS data from Lee & Moser [5]. Right: Temporal evolution of turbulent kinetic energy (TKE) at $y^+ = 15$ for the undamped minimal channel (black) and the damped minimal channel (red).

In the proposed work, we seek to exploit resolvent methods to identify the nonlinear interactions that need to be accounted for to obtain the highest fidelity RNL/ARNL models. In particular, we will compute key nonlinear interactions occurring among resolvent response modes. Then we will include these nonlinear interactions in ARNL/RNL simulations to understand the dynamically significant modal interactions required to reproduce key features of the turbulent dynamics. The outcome of this study will inform a wide range of coherent structure based reduced-order models and analysis techniques, e.g. closure models for the nonlinearity in linearized Navier-Stokes equations etc.

2 Work Program

The goal of this project is to perform modal analysis of nonlinear interactions in the Navier–Stokes equations and implement optimal modes within RNL simulations of a minimal flow unit to better understand

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turbulence dynamics. The work program will be divided into two sections: (1) preliminary work prior to the summer program and (2) anticipated work during the summer program.

(1) Preliminary Work

Prior to the program, code infrastructure and compatibility between simulations/data will be established. A full-scale DNS of a minimal flow unit will be conducted to capture relevant turbulent dynamics of the flow. The DNS, along with RNL/ARNL, will be run using RNLGO, a code modified from `lesgo.me.jhu.edu`. All simulations will be performed in a full channel at $Re_\tau = 180$.

In addition, the full set of resolvent modes and their nonlinear interactions for this flow will be computed. To identify the nonlinear interactions that contribute most strongly to the system dynamics, we will calculate the resolvent mode energy transferred through triadic interactions:

$$\alpha(k_x, k_z, m_y; k'_x, k'_z, m'_y, k''_x, k''_z, m''_y) = -\left\langle \widehat{\psi}_{i,k_x,k_z,m_y}(y), \frac{\partial}{\partial x_j}\left(\widehat{\phi}_{i,k'_x,k'_z,m'_y}(y)\widehat{\phi}_{j,k''_x,k''_z,m''_y}(y)\right)\right\rangle, \quad (1)$$

where α is the amount of resolvent mode energy transferred nonlinearly to (k_x, k_z, m_y) via (k'_x, k'_z, m'_y) and (k''_x, k''_z, m''_y), where $k_x = k'_x + k''_x$ and $k_z = k'_z + k''_z$ are the streamwise and spanwise wavenumbers, and m_y indicates the resolvent mode index in order of decreasing gain. Here, ϕ and ψ are resolvent response and forcing modes, respectively. To help with the tractability of this method, we will only keep the $m_y \leq N_y$ modes in each k_x and k_z pair and focus predominantly on the $\omega = 0$ modes. With this approach, we hope to identify the resolvent mode triplets with the most contribution to the dynamically significant modes.

(2) Anticipated Work

Week 1-2: We will implement the optimal mode results from the resolvent analysis into the ARNL/RNL simulation framework. These results will be compared to the original DNS simulated prior to the program, and DNS from literature to ensure turbulent statistics are in good agreement. The first presentation will include work done prior to attending the program such as code development, mathematical derivations of the nonlinear interaction coefficient, resolvent/POD analysis, and preliminary simulation results.

Week 3-5: More ARNL/RNL simulations will be run to parameterize the model's response to different triadic interactions. A full comprehensive study of varying ARNL/RNL model inputs will be done and in depth analysis of the results will be completed. We will look at mean streamwise velocity and TKE profiles, energy spectra, TKE budget analysis, and model energy budgets. The final presentation will showcase detailed results and conclusions made for the duration of the program.

Connections to Other Projects: The work will have significant impact in the field because we will be able to identify important nonlinear interactions in turbulent channel flows that can be exploited to develop higher fidelity low order modeling and analysis tools. The resolvent analysis methodology to be developed can be extended to more applied settings, such as turbulent boundary layers, to better understand important nonlinear interactions responsible for development of coherent structures that have been associated with skin friction drag. This work helps with the RNL model development because it provides insight for what modes and scales are directly correlated with turbulent flow dynamics.

3 Computational and Financial Requirements

Computational Requirements: The project requires computational resources to run DNS and RNL/ARNL simulations of a minimal flow unit at $Re_\tau = 180$. We request 2 TB of storage for time averaged and instantaneous output data and a monitor for AR to use during the program would be appreciated.

Financial Requirements: Room-and-board near UPM for the duration of the program, funding for flights to Madrid and back, and a limited per diem for AR would be appreciated. We request partial support including funding for flights, and a week of accommodations near UPM for BV and DG.

4 Brief Description of Applicants

AR and EL are Ph.D. students at JHU and Caltech, respectively; BV and HJB are assistant professors at Polytechnique Montréal and Caltech; DG is a professor at JHU. AR will attend the full duration of the program; BV and DG will each attend 1-2 weeks of the program.

References

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