

Abstract: We observe that a theorem of Morel implies that the class of linear orders that are multiplicatively absorbing on the left is closed under lexicographic products.

Given two linear orders X and Y , we write XY for the lexicographically ordered cartesian product of X and Y . A linear order X is *left-absorbing* if there is an order A with at least two points such that $AX \cong X$. We write \mathbf{L} and \mathbf{nL} for the classes of left-absorbing and non-left-absorbing linear orders, respectively.

Morel showed that a linear order X is cancellable on the right in lexicographic products if and only if X is non-left-absorbing.

Theorem (Morel): Suppose X is a linear order. Then there exist non-isomorphic linear orders $A \not\cong B$ such that $AX \cong BX$ if and only if X is left-absorbing.

It follows from Morel's theorem that the class \mathbf{nL} is closed under lexicographic products.

Corollary Suppose B and X are non-left-absorbing linear orders. Then BX is also non-left-absorbing.

Proof. Suppose there is an order A with at least two points such that $ABX \cong BX$. If $AB \cong B$ then B is left-absorbing, contrary to hypothesis. Hence $AB \not\cong B$. Then since $ABX \cong BX$, by Morel's theorem we have that X is left-absorbing, contrary to hypothesis again. Hence there is no such A . \square