Approximating sol'ns numerically

- Sps we have a diff eq of the form
$$y' = F(x,y)$$
we can't solve (not separable, e.g.) for y.

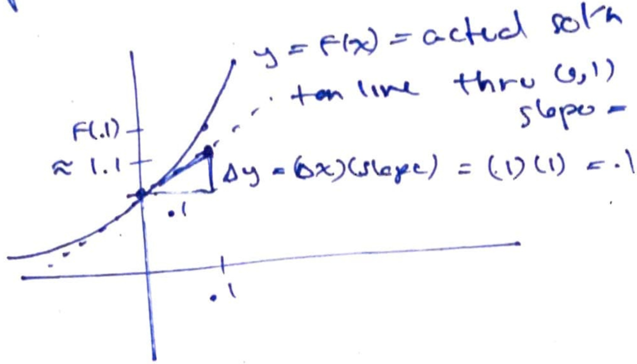
- can try to approximate a particular sol'n $y = f(x)$ thru a given point (x_0, y_0) numerically

idea: use eq'n (which gives info about f') to approx f.

ex: consider eq'n
$$y' = x+y$$
we will approx sol'n $y = f(x)$ thru $(0,1)$ (i.e. satisfying $y(0) = 1$)
$= (x_0, y_0)$

eq'n tells us: tan line to $y = f(x)$ there has slope $y' = x+y = 0+1 = 1$

- thus tan line gives a rough approx'n to f near (0,1)



If we want to approx. f(.1), can follow tan line.

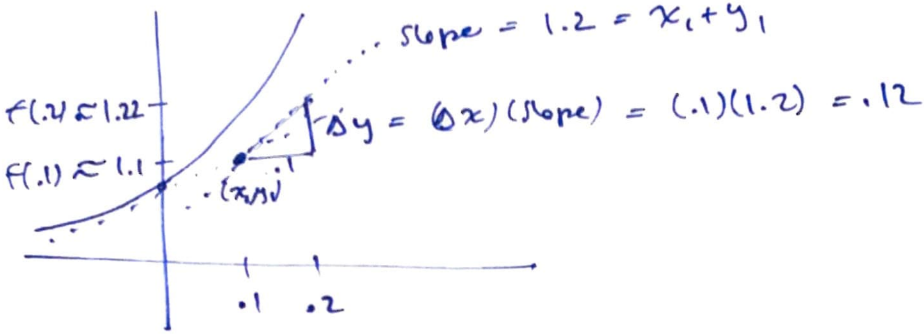
$$f(.1) \approx y_0 + \Delta y$$
$$= 1 + (\text{slope})\,\Delta x$$
$$= 1 + 1(.1) = 1.1 \quad \text{call this } y_1$$

- what if we want to approx $f(.2)$?
- as we get further from $x_0 = 0$ original tan line becomes a worse approx'n for f.
- instead, let's use diff'eq to get a new slope @ $(x_1, y_1) = (.1, 1.1)$

then we get:

$$y' = x+y = x_1 + y_1 = .1 + 1.1$$
$$= 1.2$$

- to approx $f(.2)$ we follow tan line w/ this slope beginning @ $(x_1, y_1) = (.1, 1.1)$



we get: $f(.2) \approx y_1 + \Delta y$

$$= 1.1 + (\text{slope})(\Delta x)$$
$$= 1.1 + (1.2)(.1)$$
$$= 1.22$$

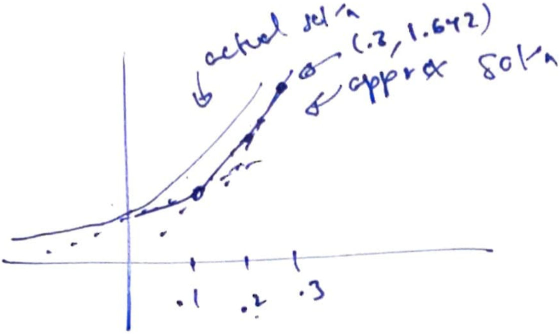
can repeat to approx $f(.3)$

$f(.3) \approx y_2 + \Delta y$ $\quad y' = x_2 + y_2 = .2 + 1.22 = 1.42$

$= 1.22 + (\text{slope})(\Delta x)$

$= 1.22 + (1.42)(.1)$

$= 1.642$



In general: to get approximate values $y_n \approx f(x_n)$ for the sol'n $y = f(x)$ to the eq'n $y' = F(x,y)$ thru (x_0, y_0) we have

$$y_n = y_{n-1} + (\Delta x)(F(x_{n-1}, y_{n-1}))$$

step size

new y

old y

slope from (x_{n-1}, y_{n-1})

ex: (9.3.21) using step size .5, approximate y values y_1, y_2, y_3 for sol'n of eq'n $y' = y - 2x$ satisfying $y(1) = 0$

sol'n: know: $y(1) = 0 \quad y' = y - 2x$

want to approx: $y(1.5), y(2), y(2.5)$

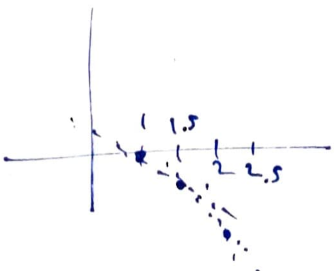
$\quad\quad\quad\quad\quad\quad\quad\approx \quad\quad \approx \quad\quad \approx$

$\quad\quad\quad\quad\quad\quad\quad y_1, \quad y_2, \quad y_3$

$$
\begin{aligned}
y(1.5) \approx y_1 &= y_0 + .5(y'(1,0)) \\
&= 0 + .5(0 - 2\cdot 1) \\
&= 0 - 1 = -1
\end{aligned}
$$

$$
\begin{aligned}
y(2) \approx y_2 &= y_1 + .5(y'(1.5,-1)) \\
&= -1 + .5(-1 - 2(1.5)) \\
&= -3
\end{aligned}
$$

$$
\begin{aligned}
y(2.5) \approx y_3 &= y_2 + .5(y'(2,-3)) \\
&= -3 + .5(-3 - 2(2)) \\
&= -6.5
\end{aligned}
$$



11.1 Sequences

- a **sequence** is an infinite list of numbers, indexed by positive integers:

$$a_1, a_2, a_3, \dots$$

ex's:

1) $\overset{a_1}{2}, \overset{a_2}{4}, \overset{a_3}{6}, 8, \dots$
2) $1, 1/2, 1/3, 1/4, \dots$
3) $1, -1, 1, -1, \dots$
4) $1, 11, 21, 1211, 111221, 312211, \dots$

are sequences

- Sometimes the nth term a_n of a sequence can be computed by a formula.

e.g. for sequences above we have:

1) $a_n = 2n$
2) $a_n = 1/n$
3) $a_n = (-1)^{n+1}$
4) no nice fmla for a_n (what is the pattern?)