9.2 Direction Fields

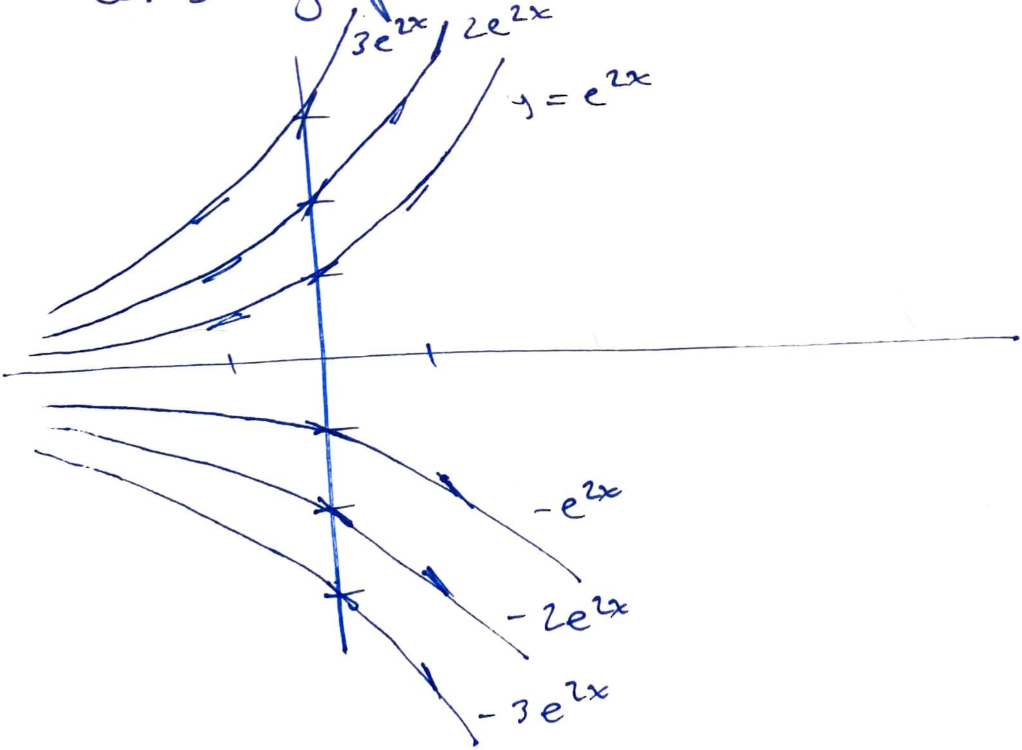
- we've seen: how to solve diff 'eq'ns of form $y' = f(x)g(y)$
- "separable eq'ns"
- e.g. we showed that:

$$y' = 2y$$

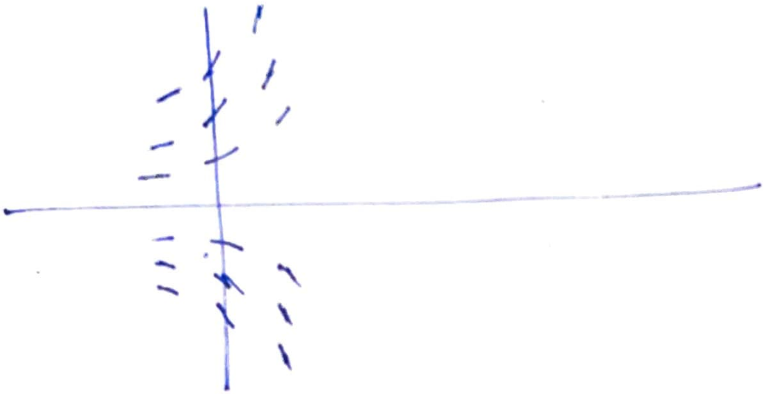
has family of sol'ns:

$$y = Ae^{2x}$$

- let's graph sol'ns for $A = -3, -2, -1, 1, 2, 3$



If we draw some mini tan lines at various points along curves and then erase curves, retain a "slope field" for our family:



↳ observe: can determine slopes of these tan lines from original eq'n

↳ can use this idea to sketch sol'ns to diff'l eq'ns we can't solve explicitly

Ex: consider eq'n

$$y' = x + y$$

- not a separable eq'n: instead of solving for the family of sol'ns we sketch their graphs using a slope field.
- the point: if $y = f(x)$ is sol'n passing thru (x_0, y_0), eq'n tells us tan line to f there has slope $x_0 + y_0$.

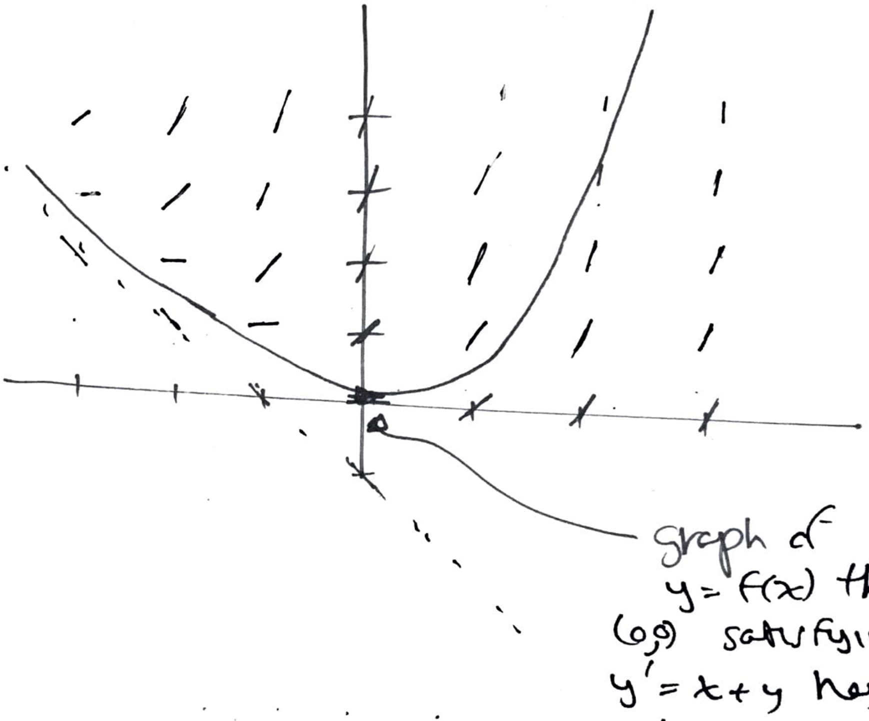
			$x + y = y'$
- so, e.g. sol'n through	$(0,1)$	has slope	$0+1=1$
" "	$(0,2)$	" "	$0+2=2$
" "	$(1,1)$	" "	$1+1=2$

Table of slopes:

$\boxed{y}$ \ $\boxed{x}$	-3	-2	-1	0	1	2
1	-2	-1	0	1	2	3
2	-1	0	1	2	3	4
3	0	1	2	3	4	5
4	1	2	3	4	5	6

y'

Slope field:



- can get rough sketch for a particular sol'n $y = f(x)$ going thru (e.g.) $(0,c)$ by "following field lines"

Note: actually, family of sol'ns given by: $y = ce^{x} - x - 1$

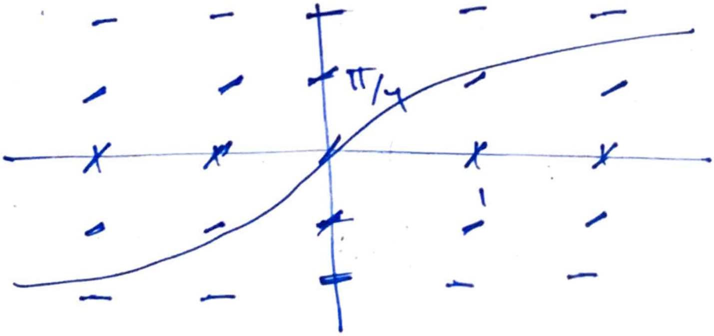
ex: a) sketch slope field for eq'n
$y' = \cos^2(y)$ for $-\pi/2 \le y \le \pi/2$
$-2 \le x \le 2$

b) solve eq'n ~~explicitly~~ for sol'n thru $(0,0)$.

Sol'n: a)

$\boxed{y}$ \ $\boxed{x}$	-2	-1	0	1	2
$-\pi/2$	0	0	0	0	0
$-\pi/4$	1/2	1/2	1/2	1/2	1/2
0	1	1	1	1	1
$\pi/4$	1/2	1/2	1/2	1/2	1/2
$\pi/2$	0	0	0	0	0

notice: y' depends only on y



b) eq'n is separable:

$$\frac{dy}{dx} = \cos^2 y$$

$$\Rightarrow \frac{1}{\cos^2 y}\, dy = dx$$

$$\Rightarrow \sec^2 y\, dy = dx$$

$$\Rightarrow \int \sec^2 y\, dy = \int dx$$

$$\Rightarrow \tan(y) = x + C$$

If $x = y = 0$ then $\tan(0) = 0 + C$

$$\Rightarrow C = 0$$

So sol'n thru $(0,0)$ is given by $\tan(y) = x$

$$\Rightarrow y = \tan^{-1}(x) \checkmark$$