repeated quad. factors handled similarly to linear case, e.g.:

to find p.f.d. for

$$\frac{x^2-1}{x(x^2+1)^2(x^2+4)}$$

we set

$$= \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2} + \frac{Fx+G}{x^2+4}$$

things get ugly quick...

# 7.5 More integral practice

**#5:** $\int \frac{t}{t^4+2}\,dt$

$u = t^2$

$du = 2t\,dt$

$\frac{1}{2}du = t\,dt$

$= \frac{1}{2}\int \frac{1}{u^2+2}\,du$

$u = \sqrt{2}\tan\theta$

$du = \sqrt{2}\sec^2\theta\,d\theta$

$= \frac{1}{2}\int \frac{1}{2\tan^2\theta+2}\sqrt{2}\sec^2\theta\,d\theta$

$= \frac{1}{2}\int \frac{\sqrt{2}}{2}\frac{\sec^2\theta}{\sec^2\theta}\,d\theta$

$= \frac{\sqrt{2}}{4}\theta = \frac{\sqrt{2}}{4}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right)$

$= \frac{\sqrt{2}}{4}\tan^{-1}\left(\frac{t^2}{\sqrt{2}}\right)+C$

#8 $\int t \sin t \cos t \, dt$

$= \frac{1}{2} \int t \sin(2t) \, dt$

$u = t \qquad du = dt$

$v = -\frac{1}{2}\cos 2t \qquad dv = \sin 2t$

$= \frac{1}{2}\left(-\frac{1}{2} t \cos 2t + \frac{1}{2}\int \cos 2t \, dt\right)$

$= \frac{1}{2}\left(-\frac{1}{2} t \cos 2t + \frac{1}{2}\cdot\frac{1}{2}\sin 2t\right)$

$= -\frac{1}{4} t \cos 2t + \frac{1}{8}\sin 2t + C$

#12 $\int \frac{2x-3}{x^3+3x} \, dx$

$\frac{2x-3}{x^3+3x} = \frac{2x-3}{x(x^2+3)}$

$= \frac{A}{x} + \frac{Bx+C}{x^2+3}$

⋮ ✓

#14: $\int \ln(1+x^2)\,dx$

spicy!

$x = \tan\theta \qquad dx = \sec^2\theta\, d\theta$

$= \int \ln(1+\tan^2\theta)\sec^2\theta\, d\theta$

$= \int \ln(\sec^2\theta)\sec^2\theta\, d\theta$

$u = \ln(\sec^2\theta) \qquad dv = \sec^2\theta\, d\theta$

$du = \frac{1}{\sec^2\theta} 2\sec\theta\sec\theta\tan\theta \qquad v = \tan\theta$

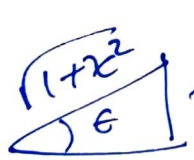
$= 2\tan\theta$

IBP $= \tan\theta\ln(\sec^2\theta) - 2\int \tan^2\theta\, d\theta$

$-2\int \sec^2\theta - 1\, d\theta$

$-2(\tan\theta - \theta)$

$= \tan\theta\ln(\sec^2\theta) - 2\tan\theta + 2\theta$




$\Rightarrow \sec\theta = \sqrt{1+x^2}$

so $= x\ln(1+x^2)$

$-2x + 2\tan^{-1}(x)$

$+C$ ✓

#19 $\int e^{x+e^x}\,dx$

$= \int e^x e^{e^x}\,dx$

$u = e^x$

$du = e^x\,dx$

$= \int e^u\,du$

$= e^u = e^{e^x} + C$ ✓

#23 $\int_0^1 (1+\sqrt{x})^8\,dx$

$u = 1+\sqrt{x} \Rightarrow \sqrt{x} = u-1$

$du = \frac{1}{2\sqrt{x}}\,dx$

$\Rightarrow 2\sqrt{x}\,du = dx$

$\Rightarrow 2(u-1)\,du = dx$

$\rightarrow 2\int_1^2 (u-1)u^8\,du$

$2\int_1^2 u^9 - u^8 = 2\left(\frac{1}{10}u^{10} - \frac{1}{9}u^9\right)\Big|_1^2$

$= \ldots$ ✓