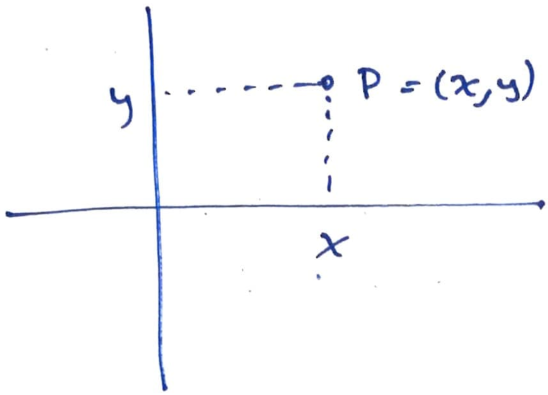
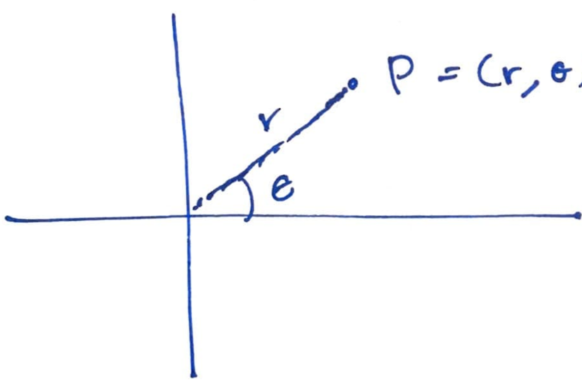
10.3 Polar coordinates

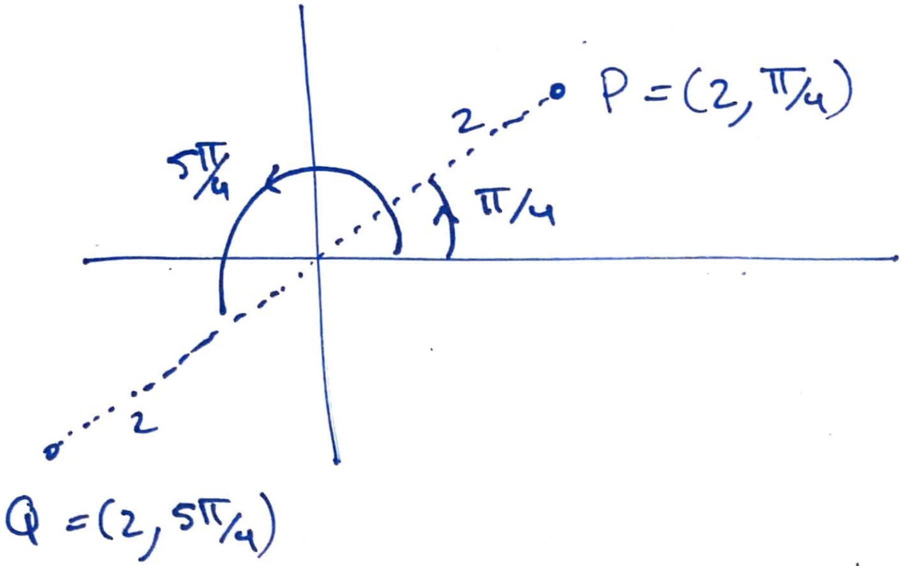
- a point P in the plane is uniquely determined by its rectangular coordinates (x,y)



- can also specify P by its polar coordinates (r, θ) where: r = distance to origin
  θ = angle made w/ x-axis

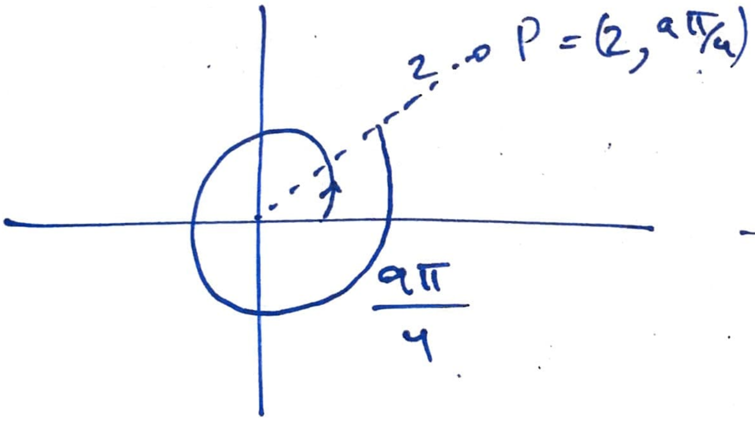


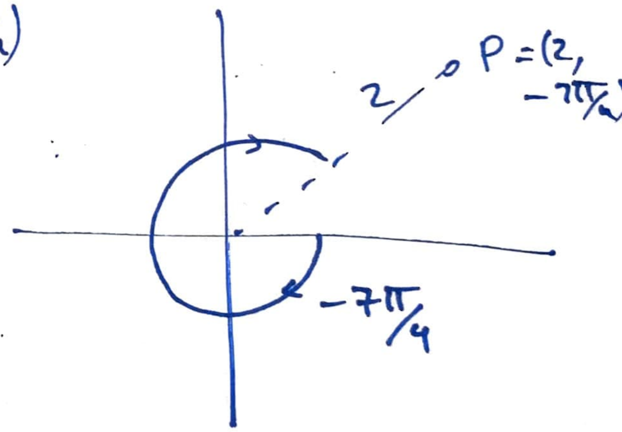
ex: $P = (2, \pi/4)$
$Q = (2, 5\pi/4)$ are shown below:



<u>Note</u>: we allow $\theta > 2\pi$ and $\theta < 0$

e.g: P also has coords $(2, \frac{9\pi}{4})$ and $(2, -\frac{7\pi}{4})$

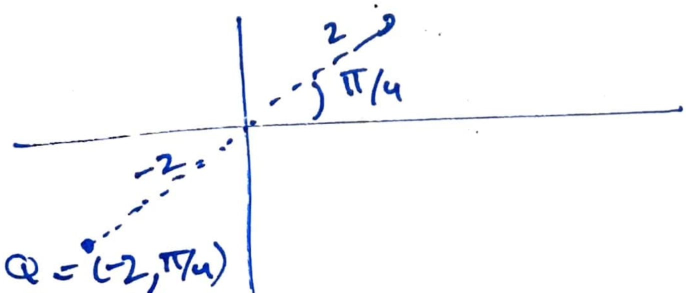




So: polar coords are <u>not</u> unique

<u>Note</u>: also allow $r < 0$.

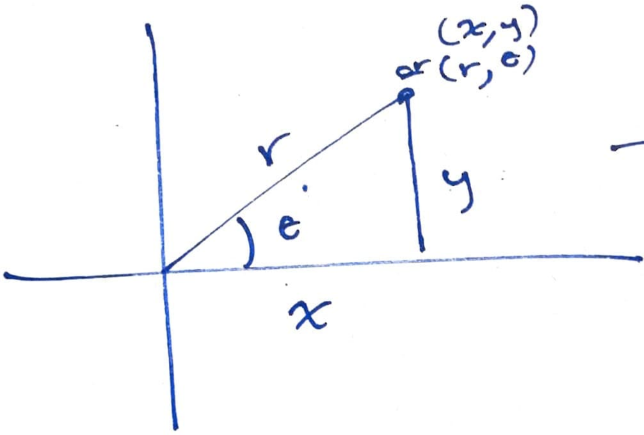
e.g.: Q also has coords $(-2, \pi/4)$



Translating:

from polar to rect: use $x = r\cos\theta$
$y = r\sin\theta$

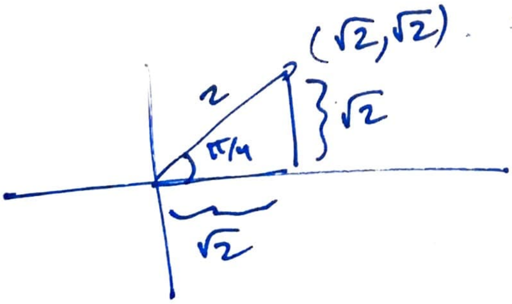
from rect. to polar: use $x^2 + y^2 = r^2$
$\tan\theta = \frac{y}{x}$



$\longrightarrow$ $x = r\cos\theta$
$y = r\sin\theta$
$x^2 + y^2 = r^2$
$\tan\theta = \frac{y}{x}$

ex: if P has polar coords $(2, \pi/4)$
then P has rectangular coords

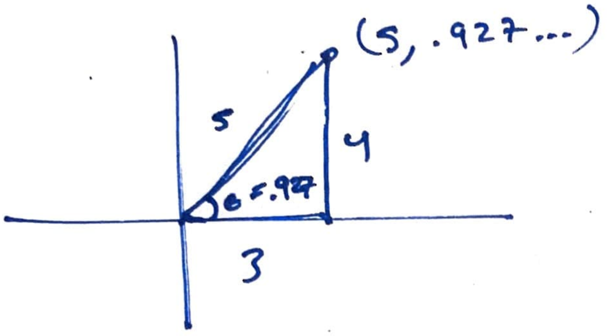
$x = 2\cos(\pi/4) = 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$
$y = 2\sin(\pi/4) = 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$



If P has rectangular coords (3,4) then P has polar coords given by:

$$r^2 = 3^2 + 4^2 = 25 \Rightarrow r = 5$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = .927\ldots$$

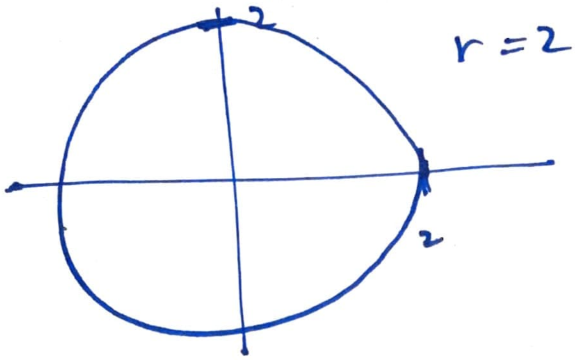


Polar curves

- can also specify curves w/ polar eq'ns.
- usually we consider equations of the form $r = f(\theta)$, i.e. specifying r as a function of θ.
- graph of such an eq'n is all polar (r, θ) where $r = f(\theta)$

ex: graph the polar curve $r = 2$.

soln: consists of all points (r, θ) where $r = 2$



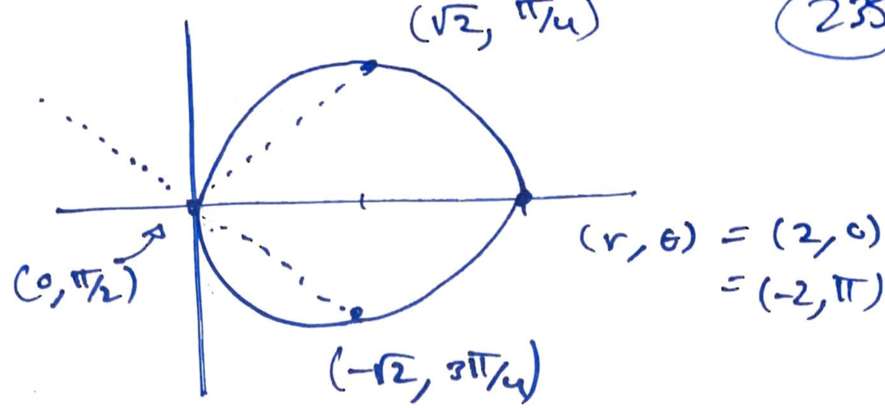
to graph more complicated curves $r = f(\theta)$, various approaches:

- plot points (not usually effective by itself)
- translate to rectangular coords (doesn't always work)
- use calculus to find points w/ horiz. or vert. tan lines (see second ex below)
- use brain

ex: graph $r = 2\cos\theta$

sol'n: can plot some points to start.

θ	$r = 2\cos\theta$
0	2
$\pi/4$	$\sqrt{2}$
$\pi/2$	0
$3\pi/4$	$-\sqrt{2}$
π	-2

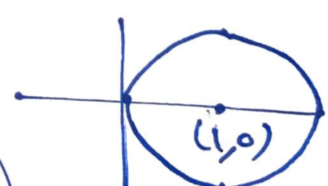


- only gives a rough sense of curve.
- in this case we can translate to rectangular coords, but requires some creativit

use: $x = r\cos\theta$ $\quad x^2 + y^2 = r^2$
$y = r\sin\theta$

to get $r = 2\cos\theta$ into x, y

$$\Rightarrow \cos\theta = \frac{x}{r}$$
$$\Rightarrow 2\cos\theta = 2\frac{x}{r} = r$$
$$\Rightarrow r^2 = 2x$$
$$\Rightarrow x^2 + y^2 = 2x$$
$$\Rightarrow x^2 - 2x + y^2 = 0$$
(complete square) $$\Rightarrow x^2 - 2x + 1 + y^2 = 1$$
$$\Rightarrow (x-1)^2 + y^2 = 1$$



circle of radius 1 centered @ $(1,0)$

<u>using calculus</u>: to graph a curve $r = f(\theta)$ finding $\frac{dy}{dx}$ at various points gives more reliable info than plotting points randomly and trying to interpolate

↳ so: we need a fmla for $\frac{dy}{dx}$

using $x = r\cos\theta = f(\theta)\cos(\theta)$
$y = r\sin\theta = f(\theta)\sin(\theta)$

<u>idea</u>: can view a polar curve $r = f(\theta)$ as a parametric curve (using θ as a parameter instead of t)

- we have: $\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta$

$$\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$$

- from our previous fmla we know:

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$