Ma 116a Homework #7

Due Tuesday, November 21 at 1:00pm

1) Consider the following graph on the set of natural numbers. Let m, n be two distinct natural numbers, with $m < n$. Then join m, n by an edge iff when n is written as a sum of distinct powers of 2, 2^m occurs in this sum, i.e., in the binary expansion of n there is a 1 in the mth place (starting from 0 and reading from right to left). For example, if $n = 2^2 + 2^3 + 2^5 = 101100$, then $3, n$ are connected by an edge but $4, n$ are not.

   Show that this graph is isomorphic to the random graph.

2) Let $\mathcal{R} = \langle R, E \rangle$ be the random graph. Show that if we split the vertices R into two sets R_1, R_2, then the substructure (induced subgraph) determined by at least one of the them is isomorphic to the random graph. (Thus when the random graph is divided into two subgraphs at least one of them is isomorphic to the random graph.)

3) Exercise 4.5.1 in Marker.

   Note 1: As usual, "dense linear order" for Marker means "dense linear order without endpoints."

   Note 2: You may assume $\mathcal{M}$ is *countable* in part (a). (Can you see how to lift the countable case to the general one?)

   Note 3: There are two typos in the problem statement for part (a): it should say "Show that $\mathrm{tp}^{\mathcal{M}}(\bar{b}/A) = \mathrm{tp}^{\mathcal{M}}(\bar{c}/A)$ if and only if..." and also in the third line it should read "$b_i > a \Leftrightarrow c_i > a$" instead of "$b_i > a \Leftrightarrow c_i < a$."

   Note 4: In part (c), it should really say that 1 and 2 realize the same *complete* type over A, or, much more simply, that $\mathrm{tp}^{\mathcal{M}}(1/A) = \mathrm{tp}^{\mathcal{M}}(2/A)$.