Ma 116a Homework #5

Due Tuesday, November 7th at 1:00pm

All numbered exercises are from Marker.

1) Suppose that G_1, G_2 are elementarily equivalent groups. Then show that for any group H the groups $G_1 \times H, G_2 \times H$ are elementarily equivalent. Conclude that if G_1, G_2 are elementarily equivalent and H_1, H_2 are elementarily equivalent, then $G_1 \times H_1, G_2 \times H_2$ are also elementarily equivalent. (*Hint*: View groups as structures in a relational language by replacing multiplication by its graph. For example, see Exercise 1.4.17 in Marker.)

   Note: This is a special case of a much more general theorem of Feferman–Vaught to the effect that elementary equivalence of arbitrary structures is preserved under products.

2) Exercise 2.5.28 from Marker. [**Correction**: In part c), T_4 should be obtained by adding to T_3 the displayed sentence *plus* the set of sentences:

$$P(c_i),$$

   for $i = 0, 1, 2, \ldots$. Also in T_3 it is meant that we consider the theory of dense linear orders without endpoints.]

   Note: This exercise shows that a complete theory can have exactly n countable infinite models (up to isomorphism), when $n = 1, 3, 4, 5, \ldots$. Vaught has shown that it cannot have exactly 2 such models.