Ma 116a Homework #3
Due Tuesday, October 24th at 1:00pm

All numbered exercises are from Marker.

1) 2.5.3

2) 2.5.4

3) 2.5.7 (the axioms for Peano arithmetic can be found on page 18)

4) Let $\mathcal{L}$ be a first order language and let $\mathcal{C}$ be a class of structures in $\mathcal{L}$. Denote by $\mathcal{C}'$ the class of structures that are not in $\mathcal{C}$. Show that if *both* $\mathcal{C}$ and $\mathcal{C}'$ are elementary, then there is a single sentence σ such that $\mathcal{C} = \{\mathcal{M} : \mathcal{M} \models \sigma\}$.

5) Consider the language of rings. Let σ be a sentence in this language that is true in every field of characteristic 0. Show that there is a number p such that σ is true in every field of characteristic at least p. Conclude that the theory of fields of characteristic 0 is *not* finitely axiomatizable.

An $\mathcal{L}$-theory T is *finitely axiomatizable* if there is a finite $\mathcal{L}$-theory T' such that for every sentence σ we have $T \models \sigma$ if and only if $T' \models \sigma$. (Observe that if T is finitely axiomatizable, then actually it can be axiomatized by a single sentence.)