Polarization and Campaign Spending in Elections*

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Abstract

We develop a Downsian model of electoral competition in which candidates with policy and office motivations use platforms and campaign spending to gain the median voter's support. The unique equilibrium involves randomizing over platforms and spending, and exhibits the following properties – (i) ex-ante uncertainty about the winner, (ii) platform divergence, (iii) inefficiency in spending and outcomes, (iv) polarization, and (v) voter extremism. We show that platform polarization and spending move in tandem, since spending is used by candidates to gain support for extreme platforms. Factors that contribute to both include the candidates' desire for extreme policies and their capability at translating spending into support for them. We also show that strong incumbents parlay an advantage into more extreme platforms, consistent with the classic "marginality hypothesis," but contrasting with a large theoretical literature in which candidates with an (exogenous or endogenous) valence advantage tend to moderate their platforms.

Keywords: elections, valence, polarization, campaign spending, extremism

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Two of the most prominent features of American elections are the growing polarization in federal and state legislatures (Hall (2019); McCarty et al. (2019)) and the ever-increasing amounts of campaign spending, leading several scholars to speculate that these phenomena are linked (e.g., McCarty, Poole and Rosenthal (2006); Meirowitz (2008)). In this paper we develop a simple model that organizes a collection of empirical regularities about US elections, offers a plausible rationale for this link, and generates new hypotheses about the potential causes of both phenomena.

To do so we build on the classic Downsian model of electoral competition. In the Downsian model, two candidates simultaneously offer policy platforms to an electorate with known preferences. The well-known prediction is the "median voter theorem" – the inescapable fact that winning must be the "proximate goal" needed to achieve all other goals inexorably pushes both candidates to adopt policy platforms that reflect the ideal point of the median voter. This holds *even if* the candidates are wholly ideologically motivated. To this baseline model we simply add the ability to make costly campaign expenditures. Specifically, in our model each candidate simultaneously chooses how much to spend alongside her choice of policy platform. Spending improves the voters' evaluation of her non-policy attributes or *valence* (e.g. Ashworth and Bueno de Mesquita (2009); Meirowitz (2008); Serra (2010); Wiseman (2006)), in keeping with previous works studying the effects of campaign spending.

The central strategic force in our model is that ideologically-motivated candidates try to exploit campaign spending to gain electoral support for more extreme platforms. We show that this force produces five striking equilibrium effects. The first is *platform divergence* – both candidates always position strictly away from the median voter. The second is *electoral uncertainty* – both the platforms of candidates, and the behavior of voters, are ex-ante unpredictable. The third is *inefficiency* – candidates always waste money trying to win, and there is ex-ante uncertainty over the winning platform that harms both voters and candidates. The fourth is *platform polarization* – candidates are more likely to take extreme positions far from the median, and outcomes are more likely to be far from the median as well. The fifth is *voter extremism* – the median voter always paradoxically selects the candidate with the most extreme platform due to that candidate's significantly higher spending. This last finding, while surprising, is consistent with recent empirical evidence suggesting that voters' evaluations of Congressional candidates exhibit a *positive association* between candidates' perceived ideological extremism and "valence" attributes like integrity and competence (Simas (2020)).

We next examine how the candidates' platform polarization and spending are affected by the candidates' motivations, the voters' motivations, and the cost of raising funds and influencing voters with them. Across all comparative statics we find that polarization and spending *covary positively*, suggesting a simple but plausible rationale for the empirical correlation between them; polarization simply reflects candidates' motivation to exploit campaign spending to gain voter support for more extreme platforms. Two key factors in the model, in turn, jointly determine this motivation.

The first is the candidates' *desire* to influence policy outcomes; this is determined by both the *extremism* of their ideal policies and the *intensity* of their ideological preferences. Our model thus reproduces an intuitive hypothesis generated by the classic Calvert (1985) and Wittman (1983) without resorting to its assumption that candidates are uncertain about the electorate's preferences – strategic candidates will adopt more ideologically-extreme platforms when they have more extreme underlying policy preferences (Fiorina (1999)). In other words, campaign spending competition *dampens the centripetal force* of the electoral imperative, thereby creating a link between candidates' true policy preferences and their platforms that is absent in the classic Downsian model.

The second is the candidates' *capability* at translating spending into support for more-extreme platforms; as candidates become more capable, both increased spending and polarization result. This capability in turn derives from three parameters in the model. The first is the *marginal cost* of a dollar of campaign spending. The second is the *marginal impact* of a dollar of spending on voter behavior. The third is the *intensity* of the voters' policy preferences, which affects how much spending is needed to "buy" support for more extreme platforms. In terms of testable implications, a variety of real word factors plausibly affect candidates' capability at translating spending into votes through one or more of these channels. The most obvious is the technology of electoral campaigns; recent years have seen dramatic improvements in the ability of candidates to target donors (effectively reducing the marginal cost of fundraising and thereby a dollar of spending) (Hassell and Monson (2014)), and to target individual voters through online advertising and social media (effectively increasing the marginal impact of a dollar of spending) (Nickerson and Rogers (2014)). A less obvious example is the legal environment of campaign finance; restrictions on "soft money" fundraising by political parties have made it easier for donors to channel funds to individual candidates relative to party organizations, potentially raising the marginal return to candidate effort on fundraising (Raja and Schaffner (2015)).

We next examine the welfare of voters and candidates. Results about voter welfare hinge crucially on the interpretation of campaign spending's effect on voter behavior. If the effect is interpreted as simply biasing voters' decisions away from their "true" preferences (consistent with a literature showing that spending effects are transient (Gerber et al. (2011); Iaryczower, Lopez-Moctezuma and Meirowitz (2021))), then spending competition harms voters. Moreover, factors that increase polarization also reduce voter welfare, in line with the conventional wisdom about the harmful effects of polarization. Conversely, if the effect is interpreted as actually benefitting voters (e.g., if candidates spend to improve "character valence" attributes like competence and expertise (Stone and Simas (2010))), then spending competition actually *helps* voters. Moreover, factors that increase polarization also increase voter welfare (despite polarized outcomes) due to the higher candidate valence that also results (see also Ashworth and Bueno de Mesquita (2009)), and constraints on fundraising and spending would harm voters despite observably reducing polarization. In contrast, we show that the candidates are unambiguously harmed by their ability to compete through campaign spending, in line with frequent and growing complaints by members of Congress about the excessive time spent fundraising.[1] Moreover, they become increasingly worse off as either their interest in policy, or capability at influencing voters via spending, go up. For them, "dialing for dollars" is thus a wasteful race to the bottom that both would prefer to avoid, but neither can commit to.

We last extend our model to reexamine the influential "marginality hypothesis," which states that "marginal (i.e. electorally weak) incumbents will tend to moderate more than nonmarginal incumbents" because a nonmarginal (i.e. strong) incumbent will "parlay the advantage into a policy position that is closer to her ideal point" (Groseclose (2001)). Consistent with this hypothesis, we show that an incumbent whose marginal cost of campaign spending is lower (i.e., who has a fundraising advantage) will both win more often, and take more ideologically-extreme platforms closer to her ideal. While intuitive and consistent with recent empirical evidence (Iaryczower, Lopez-Moctezuma and Meirowitz (2021)), this result contrasts sharply with an existing literature of both exogenous and endogenous valence models in which strong office-motivated incumbents tend to moderate more than weak ones (e.g. Aragones and Palfrey (2002); Casas, Balart and Troumpounis (2020)).

[1] https://www.vox.com/polyarchy/2016/1/8/10736402/congress-fundraising-miserable

Divergence, Models, and Mechanisms

The literature on the causes and consequences of polarization in US legislatures is too vast to review here (see McCarty (2019)). However, two generally accepted findings are worthy of note. First, whether it be through sorting or opinion change, the *underlying preferences* of political elites have become increasingly polarized across the two major parties. Second, a considerable share of polarization in the *observed behavior* of US legislators can be attributed to candidate platform divergence – that is, to differences in how Democrats and Republicans represent the same constituency. Any comprehensive explanation for rising polarization must therefore address this platform divergence.

It remains frequently asserted that divergence is a puzzle in light of the median voter theorem, as the centripetal force of the median voter is strong enough to overwhelm even the most ideologically motivated candidates in the classic Downsian model (Duggan (2017)). However, the theoretical literature has long offered a simple and plausible explanation for platform divergence – in the Calvert (1985) and Wittman (1983) model, *candidate uncertainty* about the electorate's preferences weakens the centripetal force of the median voter enough to make ideologically-motivated candidates diverge. This explanation is attractively parsimonious, predicting that greater divergence in candidate platforms merely reflects greater divergence in elites' underlying preferences. However, this extreme parsimony is also something of a weakness, as it simply regresses the empirical puzzle back one level to explaining the underlying causes of elite preference divergence. The only other predictor in the theory is the candidates' *uncertainty* about the electorate's preferences, which is difficult to measure (although recent work has made progress doing so at the state level (McCarty et al. (2019))).

Divergence and Exogenous Valence More recently, scholars have considered whether a candidate possessing non-policy or "valence" advantages like charisma or competence could fruitfully explain divergence. These analyses frequently attributed valence advantages to incumbency, and considered the "marginality hypothesis," which states that "marginal (i.e. electorally weak) incumbents will tend to moderate their platforms more than nonmarginal incumbents" (Groseclose (2001)). This hypothesis is premised on the intuitive idea that a policy-motivated candidate with a valence advantage will want to "parlay the advantage into a policy position that is closer to her ideal point" (Groseclose (2001)). Formal analysis, however, uncovered another important force for divergence that runs counter to this logic; strong candidates might want to "chase" their weak competitor to overwhelm them with their valence advantage (pulling them closer to the political center), while weak candidate would therefore seek to "evade" the strong by hiding in the extremes of the policy space (where the median voter might nevertheless serendipitously appear (Aragones and Palfrey (2002); Aragonès and Xefteris (2012)))).

As it turns out, whether the marginality hypothesis holds in any given model depends crucially on its assumptions about the knowledge and preferences of candidates. In particular, candidate uncertainty about the electorate's preferences is a crucial driver of "chase and evade" incentives; without it, the strong will still chase the weak, but the weak cannot evade the strong. Thus, in exogenous valence models lacking uncertainty, the stronger candidate positions himself to ensure victory; this yields the marginality hypothesis if candidates have policy motives (Peress (2010)) and inconclusive predictions otherwise (Ansolabehere and Snyder (2000)). Conversely, in models where candidates are *both* uncertain about the electorate and purely office-motivated, there are *only* chase-and-evade incentives; equilibria are only in mixed strategies, and the marginality hypothesis fails (e.g. Aragones and Palfrey (2002); Aragonès and Xefteris (2012)). Finally, in Groseclose (2001) both forces are present. He restricts attention to equilibria in which the candidates' policy motives weaken the strong candidate's incentive to chase *enough* to yield pure strategy equilibria, and shows that in such equilibria chase-and-evade incentives may remain sufficiently strong to contradict the marginality hypothesis.[2]

Divergence and Endogenous Valence Most recently scholars have studied candidates who can endogenously increase their valence through costly effort (e.g. Ashworth and Bueno de Mesquita (2009); Casas, Balart and Troumpounis (2020); Serra (2010); Wiseman (2006); Zakharov (2009)). In some models this effort is interpreted as wasteful campaign spending, whereas in others it is interpreted as a personal investment in productive attributes like policy expertise. While most such models predict platform divergence, they differ starkly in the mechanisms that drive it, again due to differing assumptions about the candidates' preferences and information.

One class of models considers office-motivated candidate who are uncertain about the electorate's preferences (e.g. Ashworth and Bueno de Mesquita (2009) and Zakharov (2009)). The underlying mechanism driving divergence in these models is closely connected to chase-and-evade incentives; can-

[2] A key assumption in Groseclose (2001) is that the median voter has quadratic utility; this makes valence loom larger in the voter's calculus the closer are the candidates, which strengthens chase-and-evade incentives (see pp. 870-74).

didates strategically "evade each other" so that valence will be less important to the voters, reducing the intensity of spending competition and benefitting both candidates.[3] To explain increasing divergence such models point to changes in campaign technology (Casas, Balart and Troumpounis (2020)); as candidates become better at translating spending into votes, spending competition becomes more intense, which leads them to further evade each other to tamp it down. While theoretically interesting, there are several reasons to doubt this mechanism as a primary driver of divergence in US legislative elections. First, if candidates are diverging to try and avoid wasteful spending competition, they are not doing a very good job of it; the cost of running a Congressional campaign has nearly doubled since 1980 (Hall (2019), p. 58), and if anything extreme positions encourage donations from the individuals who have come to increasingly dominate fundraising (Ansolabehere, de Figueiredo and Snyder (2003); Barber (2016)). Second, this explanation assumes away policy goals, which both anecdotal and empirical evidence suggest is a central motivation of US legislators (Gagliarducci, Paserman and Patacchini (2019); Iaryczower, Lopez-Moctezuma and Meirowitz (2021); Fenno (1973)). This is problematic to the extent that many theoretical predictions depend on whether office-seeking is truly legislators' only goal, or their "proximate" goal needed to achieve all other goals (including policy).[4]

Another class of models (including ours) considers policy-motivated candidates who know the electorate's preferences (e.g. Serra (2010), Wiseman (2006)). In these models, the logic that drives platform divergence is the same as that which underlies the marginality hypotheses; candidates seek to parlay a (now endogenous) valence advantage into support for a more ideologically-appealing platform. Our model has two key differences with previous works. First, we study candidates with a mixture of policy and office motivations, allowing us to explore their differences. Second, our model's sequence is different; candidates choose both platforms and spending simultaneously, while in Serra (2010) candidates first choose spending levels (simultaneously) and *then* platforms (simultaneously), and in Wiseman (2006) candidates move in a prdetermined order. Our model turns out to be much more

[3]This effect also requires a particular type of uncertainty about the electorate's preferences, as pointed out by Ashworth and Bueno de Mesquita (2009). Specifically, the "stochastic partisanship" model considered in related work by Herrera, Levine and Martinelli (2008) (in which the candidates are uncertain about the median's bias rather than her ideal point) cannot generate this effect because there is no way for a weak candidate to "evade" a strong one.

[4]This class of models also exhibits tractability issues. Ashworth and Bueno de Mesquita (2009) features very complex and discontinuous strategies well as equilibrium multiplicity and lacks general comparative statics results. Zakharov (2009) does not solve for Nash equilibria. Casas, Balart and Troumpounis (2020) requires voters to have non-standard "lexicographic" preferences in which they are randomized between caring only about policy and only about spending.

tractable (despite the apparent difficulty of working with mixed strategies), and exhibits equilibrium properties that better fit real-world elections.[5] For example, in both Serra (2010) and Wiseman (2006) voters and candidates know ex-ante where candidates will position and who will win, whereas the equilibrium of our model reflects the uncertainty of real-world elections. In addition, in Serra (2010) the losing candidate always converges to the median, and polarization depends on the *asymmetry* in candidates' underlying extremism – as candidate preferences become similarly extreme, polarization vanishes. Our model, in contrast, does not rely on asymmetric preferences to generate divergence, and captures the empirical regularity that candidates from opposing parties appear to "leapfrog" over the district median voter, often in an approximately symmetric fashion (Bafumi and Herron (2010)).

The Model

Two candidates $i \in \{-1, 1\}$ simultaneously choose ideological platforms $\gamma_i \in \mathbb{R}$ and costly spending levels $q_i \in [0, \infty) = \mathbb{R}^+$. The median voter (V) then votes for her preferred candidate. Platforms are commitments to spatial policies that will be implemented in office. Spending is a reduced-form representation for costly actions that make voting for a given candidate more appealing. The median ideal point is normalized to 0, and her utility for candidate i is $\mu q_i - \lambda_v \gamma_i^2$. She thus places a weight of λ_v on policy outcomes, and a candidate spending q_i generates a *valence* return of μq_i; μ thus reflects the *sensitivity* of the voter's valence perception to a dollar of campaign spending. For now we are agnostic about the interpretation of valence. Interpreted literally, valence increases welfare. However, it could instead proxy for how spending biases voters without increasing welfare (e.g. uninformative campaigning). We further note that the model analysis and equilibrium would be unchanged if candidates could allocate their spending freely between increasing their own valence, i.e. "positive campaigning," and reducing their opponent's valence, i.e., "negative campaigning" (see Appendix D).

Candidates may have both policy and office motivations. Letting w denote the election winner and γ_w the winning platform, candidate i's final utility is $\mathbf{1}_{i=w}\theta - \lambda_c (x_i - \gamma_w)^2$, where x_i is candidate i's ideal ideology. As in the canonical model by Calvert (1985), candidates have policy goals, but must first achieve the "proximate" goal of election to achieve those goals. θ represents office-holding

[5] Specifically, model has a unique, continuous, easily expressed equilibrium, with analytical comparative statics across the full parameter space. In contrast, Serra (2010) only explores the part of the parameter space with pure strategy equilibria, and Wiseman (2006) derives equilibria and performs comparative statics computationally.

benefits such as ego rents or salary. λ_c is the candidates' weight on ideology. Candidates prefer to avoid spending, which costs aq_i and enters additively into their utility. We consider candidates who are equidistant from the median ($|x_i| = |x_{-i}| = x$) on opposite sides, so candidate i's ideal is $x_i = ix$.

Discussion of Assumptions

The model is a variant of an all-pay contest (Siegel (2009)) and is related to the policy development model of Hirsch and Shotts (2015). In both models players' strategies have a costly up-front component alongside a "spatial" component, and they care directly (at least in part) about the strategy of the winning player. Here a candidate's strategy is a two-dimensional "bid" (γ_i, q_i) consisting of a policy platform and spending level in a "contest" to win the median's support. The median voter's utility $s_i = \mu q_i - \lambda_v \gamma_i^2$ for candidate i is the "score" of candidate i's "bid," meaning it is the quantity that determines who wins. A candidate can increase her score in two ways. First she can increase spending, which is costly up front, but has no effect on the benefits from winning. Second she can position closer to the median, which is "free" up front, but makes winning less valuable for both players.

Despite several similarities, our model has a number of key differences that complicate the equilibrium analysis and yield different equilibrium properties. First, in the policy-development model the "decisionmaker" (analogous to the median voter) has an "outside policy option" that she may select if she finds all of the endogenously-developed proposals unappealing. Here the median voter has no alternative to the two candidates, and it is only the disciplining effect of competition that bounds the harm she might suffer from divergence. Second, in the policy-development model the "developers" (analogous to the candidates) are purely policy-motivated; here the candidates may have a mixture of policy and office motivations. Third, in the policy-development model the developers put costly effort into "policy quality" that benefits all players (including their competitors); here spending only influences the electorate. In addition to yielding different predictions about equilibrium platforms, this property also allows our model to be easily extended to permit both "positive" and "negative" campaigning without perturbing the analysis and main results (excluding voter welfare).[6]

[6]There are also two other minor distinctions. First, the present model permits variation in voters' and candidates' utility weights on ideology and spending; this does not affect equilibrium analysis but is relevant for the empirical interpretation of comparative statics results. Second, we conduct voter welfare analysis both including and excluding the "valence term" from the voter's utility calculation, to allow for differing interpretations of candidate valence.

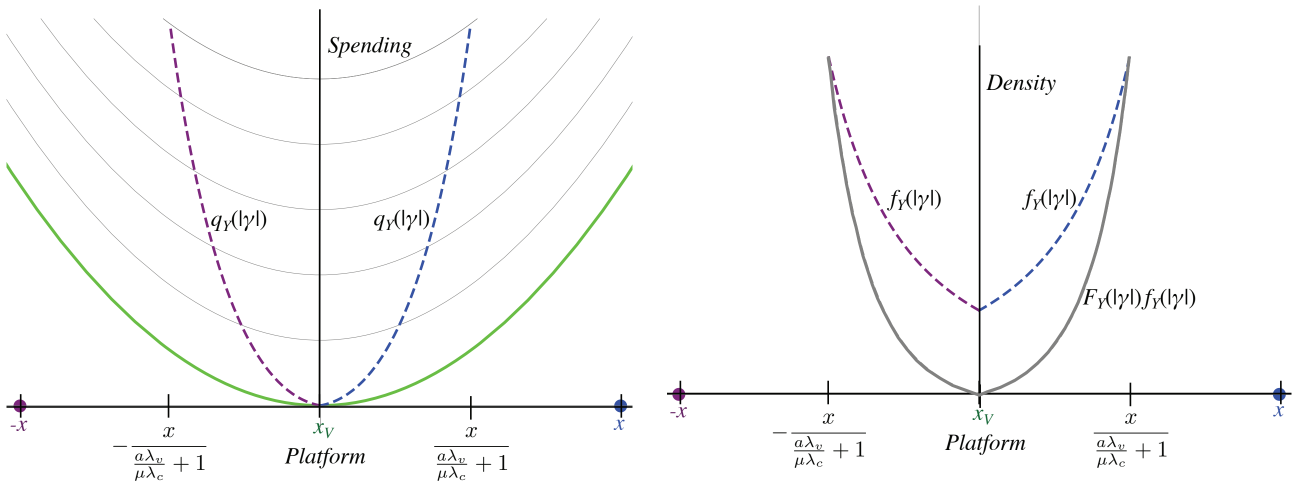

Figure 1: *Equilibrium Strategies and Outcomes*

Equilibrium

Our model exhibits a simple and unique equilibrium that is in symmetric mixed strategies; candidates randomize over both platforms and spending. Candidates must randomize for similar reasons as in a classic all-pay contest; a candidate who is spending something but losing for sure has an incentive to instead spend nothing, a candidate who is winning by a strictly positive margin has a strict incentive to spend less, and candidates who are tied both have an incentive to spend a little bit more and win.[7] All analysis and proofs are located in the Online Appendix.

Proposition 1. *The probability that a candidate positions closer to the median voter than distance y is $F_Y(y) = \min\left\{\left(\frac{a\lambda_v}{\mu\lambda_c}\right)\left(\frac{x}{x-y}\right), 1\right\}$. When positioning at distance y, candidate i selects platform $\gamma_i(y) = iy$ and spends $q_Y(y) = \frac{1}{\mu}\left(s_Y(y) + \lambda_v y^2\right)$, where $s_Y(y) = \int_0^y \lambda_v \frac{x}{(x-\hat{y})^2}\left(\frac{\theta}{\lambda_c} + 4x\hat{y}\right) d\hat{y}$.*

Strategies are depicted in Figure 1. The dashed curves in the left panel depict the pairs of platforms (on the x-axis) and spending levels (on the y-axis) that the left and right candidate mix over. The median's indifference curves, i.e., the pairs of platforms and spending levels that she is equally willing to vote for, are the solid curves. The dashed curves in the right panel depict the density (PDF) over the left and right candidates' platforms, and the solid curve depicts the density of the final policy outcome (i.e., the platform of the winning candidate).[8]

[7] Proving this is more involved than in the all pay contest because payoffs from winning and losing are endogenous.
[8] See Appendix D for details on calculating the distribution over the final policy outcome.

Properties of Equilibrium

Uncertainty Although the game is of complete and perfect information, in the unique equilibrium both candidates and voters are uncertain about where candidates will position, how much they will spend, and who will win. This unpredictability is a unique feature of the model that arises from the candidates' need to remain electorally competitive in the presence of campaign spending.

Divergence The unique equilibrium exhibits divergence – both candidates position away from the median with probability 1 ($F_Y(0) = 0$). The reason is similar to the classic Calvert model, but that model requires that candidates be uncertain about the median's preferences; this causes them to gamble on sometimes winning with a divergent platform. In our model both the median's preferences and the effect of spending are known. However, the *strategic* uncertainty produced by campaign spending, and the resulting uncertainty in an opponent's competitiveness, serves a similar role – platforms away from the median's ideal may still win the election, and are therefore sometimes adopted.

Inefficiency The unique equilibrium is inefficient in both spending and policy outcomes. Spending is inefficient in two senses. First, it is pure waste if assumed to only affect the voters "behaviorally." Second, the losing candidate always wastes money trying to win. Equilibrium policy outcomes are efficient "on average" because they are centered on the median. However, they are also uncertain ex-ante, harming both candidates and voters due to risk aversion. The root of these inefficiencies is the nature of campaign spending itself. Spending allows the candidates to gain voter support for policies away from the median. However, unlike a promised transfer in exchange for the voters' support, a candidate must pay for campaign spending *before* she knows whether it will yield an electoral victory.

Polarization The unique equilibrium exhibits polarization in candidate positions – each candidate is *more likely* to take extreme positions closer to her own ideal point than moderate ones closer to the median ($f_Y(y) > 0$ in the support), and places vanishing weight on platforms near the median. The reason is that candidates are risk averse over policy, so the marginal benefit of "buying" support for a more extreme platform with spending is largest for platforms near the median.

Voter Extremism In equilibrium candidates spend more, and have higher valence, when taking more extreme platforms ($q'_Y(y) > 0$). As previously discussed, this differs starkly from models of both exogenous and endogenous valence, in which candidates are both office-motivated and uncertain about the electorate's preferences (see Stone and Simas (2010) p. 373 for a review). More surprising is that candidates spend *so much more* when taking extreme platforms that the median actually evaluates their overall candidacy more favorably. (In the left panel of Figure 1 the candidates' spending functions are steeper than the median's indifference curves). The median thus appears to have a preference for extremism! Consequently, policy outcomes (i.e., the platform of the winning candidate) are even more polarized than the policy platforms (see the right panel of Figure 1). The reason for this counterintuitive effect is as follows. To win, candidates trade of spending against making concessions. When a candidate aims to be more competitive, ideological concessions become a costlier way to gain support because her platform is more likely to be implemented. Reversing the statement, a platform that makes fewer ideological concessions (i.e. that is more extreme) must also be more likely to win.

Comparative Statics

We next analyze how changes in the model parameters affect various outcomes. Because candidates' equilibrium strategies involve uncertainty, this requires analyzing *first-order stochastic* changes in the distribution of these outcomes. Recall that a candidate's spending is q_i while her valence is μq_i; we analyze the median voter's equilibrium welfare both including valence μq_i (i.e., interpreting valence literally as a productive attribute) and excluding it (i.e., interpreting valence "behaviorally" as a bias).

Proposition 2. *The six model parameters (in columns) first order stochastically increase (+), decrease (-), or have no effect on (0), the six equilibrium outcome quantities (in rows).*[9]

	x	λ_c	θ	a	λ_v	μ
Platform Extremism	$+$	$+$	0	$-$	$-$	$+$
Spending	$+$	$+$	$+$	$-$	$-^*$	$+$
Valence	$+$	$+$	$+$	$-$	$-^*$	$+^*$
Candidate Welfare	$-$	$-$	0	$+$	$+$	$-$
Voter Welfare (Behavioral)	$-$	$-$	0	$+$	$+$	$-$
Voter Welfare	$+$	$+$	$+$	$-$	$-$	$+$

Platforms, Spending, and Valence The model predicts a positive association between platform polarization and spending. The reason is simple: polarization reflects the candidates' motivation to

[9]Starred results further require $\frac{a}{\mu} \geq \frac{\lambda_c}{\lambda_v}$. This states that the marginal cost generating valence – that is, of influencing voters via spending – is sufficiently high, and specifically exceeds the candidates' (relative) valuation of ideology. When this condition fails, the indicated parameters do not cause a first-order stochastic change in the indicated outcomes.

spend to gain support for more extreme platforms. To interpret how the model parameters influence this willingness (and thus both polarization and spending), we divide them into three categories.

The first is the candidates' *desire for extreme policies*, as determined by their ideological strength λ_v and extremism x. Unsurprisingly, increasing the candidates' desire for extreme policies magnifies their willingness to spend to gain support for them, resulting in more extreme platforms and greater spending and valence. The second is the candidates' *capability* at translating spending into support for extreme policies, which is jointly determined by *(i)* the candidates' marginal cost a of spending, *(ii)* the effectiveness μ of a dollar of spending on valence, and *(iii)* the median's ideological strength λ_v, which affects how much spending is needed to "compensate" her for extreme platforms. Increasing capability through any of these channels also generates more extreme platforms and polarization. The third is the candidates' nonideological officeholding benefits θ. Higher officeholding benefits increase spending and valence, but surprisingly, have *no effect* on candidate platforms despite the greater benefit to winning. The reason is that the level of officeholding benefits does not influence the calculus of using spending to gain support for more extreme platforms, which is what drives divergence.

Welfare Candidates spend to both pull policy toward their ideal when they win, and to prevent losing to something ideologically unappealing. On average, however, they do not move policy outcomes away from the median, and waste money and generate policy uncertainty trying to do so. They are thus clearly harmed by spending competition. Interestingly, the degree of equilibrium platform polarization indicates by how much. Thus, they become *worse off* when their own desire for extreme policies (higher x or λ_c), or capability at translating spending into support (lower a, higher μ, or lower λ_v), go up. (They are unaffected by changes in officeholding benefits θ, because they compete all these away through campaign spending.) Thus, for the candidates campaign spending competition is a wasteful race to the bottom that both would prefer to avoid, but neither can commit to.

Whether the voter benefits from or is harmed by spending competition depends on whether her "revealed preference" for candidates who spend more is a behavioral bias or a genuine reflection of greater welfare, confirming a conjecture in Serra (2010). With the former assumption the median is harmed, and the degree of platform polarization indicates by how much. With the latter assumption the median benefits, and the level of *spending* indicates by how much. Thus, she becomes better off when the candidates become more extreme (higher x) or better able to translate spending into valence (higher μ) even though platforms further polarize. She would also be harmed by constraints or bans on fundraising and spending (see also Ashworth and Bueno de Mesquita (2009)).

Incumbency Advantages and the Marginality Hypothesis

We conclude by extending the model to reexamine the marginality hypothesis, i.e., that "marginal (i.e. electorally weak) incumbents will tend to moderate their platforms more than nonmarginal incumbents" (Groseclose (2001)). In the exogenous valence literature a strong incumbent is modeled as one endowed with greater valence. Here we model a strong incumbent as one with a *greater ability at spending to generate valence* (as in Meirowitz (2008)), an arguably more appropriate formalization given the well-documented fundraising advantage of US incumbents (Fouirnaies and Hall (2014)). The marginality hypothesis is of particular interest because in exogenous valence models it helps to differentiate office-motivated candidates from policy-motivated ones; both types of models predict platform divergence, but the marginality hypothesis fails in the former due to "chase and evade" incentives. Similarly, our endogenous valence model with policy-motivated candidates yields several common predictions with endogenous valence models with office-motivated candidates despite very different underlying forces, including platform divergence and a correlation between the effectiveness of campaign spending and polarization (Ashworth and Bueno de Mesquita (2009); Casas, Balart and Troumpounis (2020)). Can the marginality hypothesis again differentiate these classes of models?

Although a complete analysis of an asymmetric version of our model is beyond the scope of this paper, we can preliminarily answer this question in the affirmative. Specifically, in the endogenous valence literature with uncertain office-motivated candidates, the marginality hypothesis fails; a technologically-advantaged candidate uses her advantage to spend more and (to some extent) chase her opponent (Casas, Balart and Troumpounis (2020)). In contrast, in our model the marginality hypothesis holds; a technologically-advantaged incumbent with ideological motives exploits that advantage to both win more often, and to do so with a more ideologically-extreme platform. To show this result analytically we consider a simple extension of our model in which an *incumbent* (I) with a mixture of policy and office motives faces a *challenger* (C) who is purely office-motivated.

Proposition 3. *Suppose that an incumbent* (I) *and challenger* (C) *have different abilities and weights on ideology; the incumbent is cost-advantaged* ($0 < a_I \leq a_C$) *while the challenger is purely office-*

motivated ($0 = \lambda_C < \lambda_I$). As the incumbent becomes stronger (lower a_I), both her probability of winning and the ideological extremism of her platforms (first-order stochastically) increase.

Thus, when an incumbent's technological advantage increases (in the sense of a lower marginal cost of spending), she spends more, wins more, and does so with more extreme platforms.

Conclusion

In this paper we develop a simple Downsian model of electoral competition with campaign spending, show that it exhibits properties consistent with empirical regularities of US legislative elections, and use the model to develop further insight into the causes and consequences of polarization. We conclude by discussing several issues we believe to be worthy of greater consideration given our results.

First, our model yields surprising predictions about the endogenous relationship between a candidate's platform extremism, spending, valence, and electoral prospects; they will all be *positively correlated*. However, validating these predictions with empirical data is a challenging exercise. While a growing empirical literature examines the relationship between extremism and votes (e.g. Hall (2015); Shor and Rogowski (2018); Tausanovitch and Warshaw (2018)), in general these works do not distinguish between candidates' *preferences* and their *platforms*, nor control for spending or other costly campaign activities. This presents substantial difficulty because strategic candidates' true preferences are likely correlated with other characteristics that would substantially impact their platforms, costly campaign activities, and overall electoral prospects (for example, fundraising ability), rendering cross-candidate comparisons suspect. Indeed, this difficulty may explain why the literature has yielded such conflicting results, with some works finding the absence of an electoral penalty to extremism (Tausanovitch and Warshaw (2018)) and others finding a strong one (Hall (2015)). Recent work by Iaryczower, Lopez-Moctezuma and Meirowitz (2021) comes closest to addressing these issues by examining within-candidate variation in roll-call votes over time (as a proxy for changing platforms) and explicitly incorporating campaign expenditures, but in a dynamic setting very different from that studied in our model. Nevertheless, they do find support for two central premises of our model; that candidates position strategically to trade off personal ideology and electoral success, and use campaign spending to "buy" support for more ideologically-extreme platforms. Finally, recent work by Simas (2020) finds a positive association between candidate extremism and voters' valence evaluations, although they propose a very different mechanism (in which voters draw favorable inferences from extreme platforms) than in our model.

Second, a perennial topic of interest in the literature on polarization is how to reform electoral and governmental institutions to reduce it. For example, Hall (2019) plausibly argues that polarization has risen in part due to a devaluation of public service, and proposes making officeholding more valuable to draw in less ideologically-motivated candidates. However, our model illustrates the weakness of antidotes that fail to account for the complex and sometimes surprising interaction between candidates, parties, donors, and voters. In this particular case, our model suggests that candidates would simply compete away any new benefits of officeholding through campaign spending, leaving unchanged both the value of moderation to extreme candidates and the value of candidacy to moderate ones.

Finally and most importantly, a now-large empirical literature that documents candidate divergence implicitly or explicitly assumes that it is prima facie evidence of a failure of representation (Ansolabehere, Snyder and Stewart (2001), McCarty, Poole and Rosenthal (2006), Fowler and Hall (2016), Burden (2004)). Our analysis, however, reaffirms the difficulty of assessing whether and by how much voters are actually harmed by this rising polarization once all of its causes and consequences are taken into consideration. In our model and valence models generally, voters may be responsive to spending and other costly campaign activities because they are actually getting something of value, like greater governance skill or policy expertise (Stone and Simas (2010)). Indeed, when this productive interpretation of valence is adopted, rising polarization is actually associated with *greater* voter welfare in our model. Thus, to the extent that the strategic considerations captured by our model have indeed contributed to rising polarization, future work that unpacks the "black box" of valence to understand why and when voters respond favorably to costly campaign activities is needed to gain a fuller understanding its consequences.

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Polarization and Campaign Spending in Elections

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Online Appendix

This Appendix has five parts. Appendix A informally derives necessary equilibrium conditions. Appendix B uses these to derive the unique equilibrium; this subsumes the characterization in Proposition 1. Appendix C proves comparative statics in Propositions 2 and 3. Appendix D rigorously proves the necessary equilibrium conditions stated in Appendix A as well as two other propositions used in Appendix B, and explains two footnotes in the main text. Appendix E analyzes the asymmetric variant of the model in which an "incumbent" with mixed motives competes against a purely office-motivated "challenger."

A Informal Derivation of Necessary Conditions

The game is a multidimensional contest in which a candidate's "proposal" (γ, q) is her combination of platform and spending, and the "scoring rule" applied to proposals is just the median voter's utility $\mu q - \lambda_v \gamma^2$, since she decides which candidate to elect. We henceforth refer to candidate strategies as proposals. We also sometimes refer to the median voter as the decisionmaker (DM). To facilitate the analysis we reparameterize proposals (γ, q) to be expressed in terms of (s, γ), where $s = \mu q - \lambda_v \gamma^2$ is the DM's utility for a proposal or its *score*. The implied spending on a proposal (s, γ) is then $q = \frac{1}{\mu}\left(s + \lambda_v \gamma^2\right)$ which costs $\alpha\left(s + \lambda_v \gamma^2\right)$ where $\alpha = \frac{a}{\mu}$ is the cost to the candidate of generating one unit of valence. Note that the DM's ideal point with no spending has exactly 0 score, and is the most competitive "free" proposal to make.

In the reparameterized game, a candidate's pure strategy (s_i, γ_i) is a two-dimensional element of $\mathbb{B} \equiv \{(s, y) \in R^2 \mid s + \gamma^2 \geq 0\}$. A mixed strategy σ_i is a probability measure over the Borel subsets of $\mathbb{B}$. Let $F_i(s)$ denote the CDF over scores induced by i's mixed strategy σ_i. For technical convenience we restrict attention to strategies generating score CDFs that can be written as the sum of an absolutely continuous and a discrete distribution.

In this section we informally derive necessary conditions for equilibrium by assuming that the strategies satisfy the following intuitive and simplifying conditions. First, each candidate uses an absolutely continuous and atomless score CDF $F_i(s_i)$ with support over a common interval of scores $[0, \bar{s}]$. Second, each candidate positions at a unique platform $\gamma_i^S(s_i)$ at each score in the support. Finally, at most one candidate has an atom at score 0 ($F_k(0) > 0$ for at most one $k \in \{L, R\}$). In Appendix D we discard these assumptions and rigorously prove the stated necessary conditions.

Optimal Platforms

With our assumptions, a candidate's utility from making proposal (s_i, γ_i) is

$$-\alpha\left(s_i + \lambda_v \gamma_i^2\right) + F_{-i}(s_i) \cdot \left(\theta - \lambda_c (ix - \gamma_i)^2\right) + \int_{s_{-i} > s_i} -\lambda_c \left(ix - \gamma_{-i}^S(s_{-i})\right)^2 d\sigma_{-i}. \quad \text{(A.1)}$$

The first term is the up-front cost of spending. The second term is the probability i's proposal wins the election times her utility for it, which includes both policy losses $-\lambda_c (ix - \gamma_i)^2$ and office-holding benefits θ. The third term is i's utility should she lose, which requires integrating over all her opponent's proposals at potentially different platforms with higher scores than s_i. Taking the derivative with respect to γ_i and setting equal to 0 yields that i's optimal platform $\gamma_i^S(s_i)$ at each score must be equal to

$$\gamma_i^S(s_i) = i\left(\frac{F_{-i}(s_i)}{\Lambda\alpha + F_{-i}(s_i)}\right)x$$

where $\Lambda = \frac{\lambda_v}{\lambda_c}$. Thus, optimal platform at a score s_i becomes more extreme the further are the candidates from the DM, and more moderate as the cost α of generating a unit of valence or the DM's relative ideological strength $\Lambda = \frac{\lambda_v}{\lambda_c}$ increases.

The optimal platform may also be written as $\gamma_i^S(s_i) = \gamma_i(y(F_{-i}(s_i)))$, where $\gamma_i(y) = iy$ and $y(P) = \frac{P}{\Lambda\alpha + P}x$. Doing so clarifies two properties. First, the platform is fully characterized by its *distance* $y(F_{-i}(s_i))$ from the DM, since a candidate will only position on her side of the DM. Second, the optimal distance only depends on the score s_i through the *probability* $F_{-i}(s_i)$ that her opponent $-i$ makes a lower score proposal than s_i. Since $y(P)$ is strictly increasing in P, this means that i's optimal platform is more distant from the DM the greater is the targetted score s_i and by implication the more likely she is to win.

Equilibrium Score Conditions

To derive the equilibrium score CDFs (F_i, F_{-i}), note that every score $s_i \in [0, \bar{s}]$ in the CDFs' common support must maximize $i's$ utility when the optimal platforms $\gamma_i^S(s_i)$ are chosen. Since i is indifferent over all scores in $[0, \bar{s}]$, substituting $\gamma_i^S(s_i)$ into equation A.1, differentiating with respect to s_i and setting it equal to zero over $[0, \bar{s}]$ yields a pair of differential equations that must be jointly satisfied in equilibrium:

$$f_{-i}(s) \cdot \left(\theta + \lambda_c \left((x + y(F_i(s)))^2 - (x - y(F_{-i}(s)))^2\right)\right) = \alpha \;\; \forall i \text{ and } s \in [0, \bar{s}] \tag{A.2}$$

Equation A.2 has a natural interpretation. The right side is $i's$ cost of increasing her own valence, which increases her score. The left side is $i's$ marginal ideological gain from increasing her score: with "probability" $f_{-i}(s)$ she goes from losing to winning the election, which shifts the outcome from an ideology that is distance $y(F_i(s))$ from the DM on the *opposite* side, to an ideology that is distance $y(F_{-i}(s))$ from the DM on the *same* side.

Necessary Conditions

Given the above, necessary conditions for SPNE are as follows. (A rigorous proof that discards the simplifying assumptions is in Appendix D).

Proposition A.1. *Necessary conditions for SPNE are as follows:*

1. ***(Ideological Optimality)*** *With probability* 1*, proposals take the form* $(s_i, \gamma_i(y(F_{-i}(s))))$*, where* $\gamma_i(y) = iy$*,* $y(P) = \frac{P}{\Lambda\alpha + P}x$*, and* $\Lambda = \frac{\lambda_v}{\lambda_c}$*.*
2. ***(Score Optimality)*** *Score CDFs* (F_i, F_{-i}) *must satisfy* $F_k(0) > 0$ *for at most one* k*, supp*$\{F_i\} \cap [0, \infty] =$*supp*$\{F_{-i}\} \cap [0, \infty] = [0, \bar{s}]$ *where* $\bar{s} \in (0, \infty]$*, and*

$$f_{-i}(s) \cdot \left(\theta + \lambda_c \left((x + y(F_i(s)))^2 - (x - y(F_{-i}(s)))^2\right)\right) = \alpha \;\; \forall i \text{ and } s \in [0, \bar{s}]$$

B Derivation of Equilibrium Strategies

Using the necessary conditions stated in Proposition A.1 we now explicitly derive the unique equilibrium. We first use that any pair of equilibrium score CDFs must be identical, i.e., $F_i(s) = F_{-i}(s) = F(s)$, implying that any equilibrium must be symmetric and unique (proof in Appendix D).

Proposition B.1. *In any SPNE,* $F_i(s) = F_{-i}(s) = F(s)$*, where* $F(0) = 0$ *and*

$$f(s)(\theta + 4\lambda_c x y(F(s))) = \alpha \;\; \forall s \in [0, \bar{s}]$$

Using this we can explicitly derive the *inverse* of the unique score CDF $F(s)$ satisfying Proposition B.1, which we denote as $s_F(P)$. Since $F(0)=0$, the inverse straightforwardly satisfies the boundary condition $s_F(0)=0$. Now using that $f(s)=\frac{1}{s_F'(F(s))}$ and that $y(P)=\frac{Px}{\Lambda\alpha+P}$, the differential equation may be rewritten as

$$s_F'(P)=\frac{1}{\alpha}\left(\theta+4\lambda_c x\frac{Px}{\Lambda\alpha+P}\right)\ \forall P\in[0,1] \tag{B.1}$$

Finally $s_F(P)=s_F(P)-s_F(0)=\int_0^P s_F\left(\hat{P}\right)d\hat{P}$; the first equality follows from the boundary condition, and the second from the fundamental theorem of calculus.

The preceding characterizes the unique strategies satisfying Proposition A.1 as a common score CDF $F(s)$ that the candidates mix over, and the platform $\gamma_i(y(F(s)))$ at which candidate i positions when generating score s (implying a spending level of $q_S(s)=\frac{s+\lambda_v[\gamma_i(y(F(s)))]^2}{\mu}$). However, these strategies may be expressed equivalently but more intuitively as a CDF $F_Y(y)$ over the *distance* of each candidate's platform from the DM (that is, their extremism), and a platform $\gamma_i(y)$ and level of spending $q_Y(y)$ when positioning distance y from the DM.

To derive these quantities from the original characterization, we first argue that $y_F(P)$ (the inverse of $F_Y(y)$) is exactly equal to $y(P)$ as defined in Proposition A.1, and hence

$$F_Y(y)=\alpha\Lambda\left(\frac{y}{x-y}\right).$$

First, since there is a positive association between platform extremism and score, $F_Y(y)=F(s_Y(y))$, where $s_Y(y)$ denotes the score of a candidate's platform when positioning distance y from the DM. This in turn implies that $y_F(P)=y_S(s_F(P))$. Second, letting $y_S(s)$ denote the inverse of $s_Y(y)$, by Proposition A.1 we have $y_S(s)=y(F(s))\rightarrow y_S(s_F(P))=y(P)$.

Next we derive $s_Y(y)$, which implies spending $q_Y(y)=\frac{1}{\mu}\left(s_Y(y)+\lambda_v y^2\right)$. Recall that $s_Y(y)$ is just the inverse of $y(F(s))$. We then have that $s_Y(y)=s_F(F_Y(y))$, recalling that $s_F(P)$ denotes the inverse of $F(s)$. Now differentiating both sides yields that $s_Y'(y)=s_F'(F_Y(y))f_Y(y)$, and it is easily derived that $f_Y(y)=\frac{\alpha\Lambda x}{(x-y)^2}$. Substituting in then yields

$$s_Y'(y)=\frac{x}{(x-y)^2}\Lambda(\theta+4\lambda_c xy) \tag{B.2}$$

Finally, $F(0)=0$ and $y(0)=0\rightarrow y(F(0))=0\rightarrow s_Y(y)=0$ (that is, when positioning at the DM's ideal a candidate's proposal has 0-score). Thus $s_Y(y)=s_Y(y)-s_Y(0)=\int_0^y s_Y'(y)\,dy$, where again the first equality comes from the boundary condition and the second from the fundamental theorem of calculus.

The following proposition both summarizes the preceding derivations and proves the derived strategies are sufficient as well as necessary for equilibrium (subsuming Proposition 1 in the main text). It is proved in Appendix D.

Proposition B.2. *There is a unique SPNE. The equilibrium is symmetric and can be equivalently described as follows.*

1. *Each candidate chooses the score of her proposal according to a common CDF $F(s)$, and positions at platform $\gamma_i(y(F(s))) = i\frac{F(s)}{\Lambda\alpha + F(s)}x$ when making a proposal with score s. The inverse $s_F(P)$ of $F(s)$ is equal to*

$$s_F(P) = \int_0^P \frac{1}{\alpha}\left(\theta + 4\lambda_c x \frac{\hat{P}x}{\Lambda\alpha + \hat{P}}\right) d\hat{P},$$

2. *Each candidate chooses the distance of her platform from the median voter according to a common CDF $F_Y(y) = \alpha\Lambda\left(\frac{y}{x-y}\right)$. When positioning at distance y, candidate i selects platform $\gamma_i(y) = iy$ and spends $q_Y(y) = \frac{1}{\mu}\left(s_Y(y) + \lambda_v y^2\right)$, where*

$$s_Y(y) = \int_0^y \frac{x}{(x-\hat{y})^2}\Lambda\left(\theta + 4\lambda_c x\hat{y}\right) d\hat{y},$$

C Comparative Statics

We now prove the comparative statics results in the main text. For the proofs we consider each quantity of interest separately.

C.1 Probabilistic Outcomes

Because the candidates mix over continuum of platforms and spending levels, most of the equilibrium outcomes we consider are probabilistic; in particular, the candidates' platforms, spending, valence, and the DM's utility. To analyze comparative statics we thus consider *first-order* stochastic changes in the outcome of interest (when such comparisons are possible). Recall that for two distributions over a univariate outcome z described by CDFs $F_Z(z)$ and $\hat{F}_Z(z)$, distribution F_Z first-order stochastically dominates distribution $\hat{F}_Z$ if and only if $F_Z(z) \leq \hat{F}_Z(z)\ \forall z$ and $F_Z(z) < \hat{F}_Z(z)$ for some z.

For the subsequent proofs we rely on the following straightforward observations. Consider an absolutely continuous CDF $F_Z(z;m)$ describing the distribution over some outcome z, and which is also a continuously differentiable function of some parameter m. Further suppose the CDF satisfies $F_Z(0;m) = 0\ \forall m$ and $F_Z(\bar{z}(m);m) = 1$ for some $\bar{z}(m) > 0$. Then the CDF has a well-defined inverse $z_F(P;m)$ over $P \in [0,1]$ satisfying $z(0;m) = 0$ and $z(1;m) = \bar{z}(m)$. In addition, $\frac{\partial F_Z(z;m)}{\partial m} < 0\ \forall z \in (0, \bar{z}(m))$ *or* $\frac{\partial z_F(P;m)}{\partial m} > 0\ \forall P \in (0,1)$ equivalently imply that the distribution is first-order stochastically increasing in m (that is, $F_Z(z;m')$ first-order stochastically dominates $F_Z(z;m)$ for $m' > m$), while the reverse signs imply that it is first-order stochastically decreasing in m.

We first consider the distribution over the candidates' platform extremism; these results are subsumed in Proposition 2 in the main text.

Proposition C.1. *The candidates' platform extremism is first-order stochastically increasing in (x, λ_c, μ), decreasing in (a, λ_v), and unaffected by θ.*

Proof: The CDF over each candidates' platform extremism $F_Y(y) = \alpha\Lambda\left(\frac{y}{x-y}\right)$, which is transparently first-order stochastically decreasing in $\alpha = \frac{a}{\mu}$ and $\Lambda = \frac{\lambda_v}{\lambda_c}$, increasing in x, and unaffected by θ. QED

We next consider comparative statics in the score CDF $F(s)$. This is necessary as an intermediate step to analyze statics in spending and valence (subsumed in Proposition 2 in the main text), as well as the DM's equilibrium utility if valence is interpreted literally (subsumed in Proposition 3 in the main text).

Proposition C.2. *The score of the candidates' platforms is first-order stochastically increasing in $(x, \lambda_c, \mu, \theta)$, and decreasing in (a, λ_v).*

Proof: It is simpler to work with the inverse $s_F(P) = \int_0^P \frac{1}{\alpha}\left(\theta + 4\lambda_c x \frac{\hat{P}x}{\Lambda\alpha + \hat{P}}\right) d\hat{P}$ for $P \in (0,1)$ and differentiate under the integral sign (since the model parameters do not enter the limits of integration). Doing so transparently yields that function and hence the distribution is decreasing in $\alpha = \frac{a}{\mu}$ and λ_v and increasing in θ, x, and λ_c. QED

We next consider comparative statics in the valence generated by each candidate; these results are subsumed in Proposition 2 in the main text.

Proposition C.3. *A candidate's valence is first-order stochastically increasing in $(x, \lambda_c, \mu, \theta)$ and decreasing in a. When $\frac{a}{\mu} \geq \frac{\lambda_c}{\lambda_v} \iff \alpha\Lambda \geq 1$, it is also decreasing in λ_v.*

Proof: A candidate with platform (s, y) spends $q = \frac{1}{\mu}(s + \lambda_v y^2)$ and thus has valence $v = \mu q = s + \lambda_v y^2$. Now letting $F_V(v)$ denote the CDF over a candidates' valence, it is straighforward to see that this relationship implies that the inverse of $F_V(v)$ is simply $v_F(P) = s_F(P) + \lambda_v [y_F(P)]^2$. Results about the effect of α, θ, x, and λ_c are then immediately implied by Propositions C.1 and C.2; $v_F(P)$ is strictly increasing in both $s_F(P)$ and $y_F(P)$, these parameters only influence $v_F(P)$ through $s_F(P)$ and $y_F(P)$, and their impact on these two functions have (weakly) the same sign.

To evaluate the effect of λ_v on $v_F(P)$ it is straightforward that $\frac{\partial}{\partial \lambda_v}\left(\lambda_v [y_F(P)]^2\right) < 0$ $\forall P \in (0,1]$ is a *sufficient* (but not necessary) condition for $v_F(P)$ to be decreasing in λ_v (in conjunction with Proposition C.2) and hence the desired result. Now $\lambda_v [y_F(P)]^2 = \lambda_v \left(\frac{P}{\Lambda\alpha+P}x\right)^2 = \frac{x^2\lambda_c}{\alpha}\cdot\Lambda\alpha\left(\frac{P}{\Lambda\alpha+P}\right)^2$. Since $\Lambda\alpha$ is strictly increasing in λ_v it suffices to show that $\frac{\partial}{\partial(\Lambda\alpha)}\left(\Lambda\alpha\left(\frac{P}{\Lambda\alpha+P}\right)^2\right) < 0 \; \forall P \in (0,1)$ which in turn is implied by $\frac{\partial}{\partial(\Lambda\alpha)}\left(\log\left(\Lambda\alpha\left(\frac{P}{\Lambda\alpha+P}\right)^2\right)\right) = -\frac{\Lambda\alpha-P}{\Lambda\alpha(\Lambda\alpha+P)} < 0$ (since log of a strictly positive number is a strictly increasing transformation with finite derivative), which is transparenty the case $\forall P \in (0,1)$ when $\Lambda\alpha \geq 1$. QED

We next consider comparative statics in the spending by each candidate; these results are also subsumed in Proposition 2 in the main text.

Proposition C.4. *A candidate's spending is first-order stochastically increasing in (x, λ_c, θ) and decreasing in a. When $\alpha\Lambda \geq 1$ it is also increasing in μ and decreasing in λ_v.*

Proof: A candidate with platform (s, y) spends $q = \frac{1}{\mu}(s + \lambda_v y^2)$. Letting $F_Q(q)$ denote the CDF over a candidates' spending, the preceding relationship implies that the inverse is $q_F(P) = \frac{1}{\mu}\left(s_F(P) + \lambda_v [y_F(P)]^2\right)$. Now it is clear that x, λ_c, θ, a, and λ_v only impact spending through valence $s_F(P) + \lambda_v [y_F(P)]^2$; hence the stated comparative statics are implied by Proposition C.3.

To evaluate the impact of μ, observe that

$$
\begin{aligned}
q_F(P) &= \frac{1}{\mu}\left(\int_0^P \frac{1}{\alpha}\left(\theta + 4\lambda_c x \frac{\hat{P}x}{\Lambda\alpha+\hat{P}}\right) d\hat{P} + \lambda_v [y_F(P)]^2\right) \\
&= \int_0^P \frac{1}{a}\left(\theta + 4\lambda_c x \frac{\hat{P}x}{\Lambda\alpha+\hat{P}}\right) d\hat{P} + \frac{\lambda_c}{a}\Lambda\alpha\left[\frac{P}{\Lambda\alpha+P}x\right]^2
\end{aligned}
$$

Recalling that $\alpha = \frac{a}{\mu}$, the first term is clearly increasing in μ. To see the second term is increasing in μ, observe that since $\Lambda\alpha$ is decreasing in μ it suffices to show that $\frac{\partial}{\partial(\Lambda\alpha)}\left(\Lambda\alpha\left(\frac{P}{\Lambda\alpha+P}\right)^2\right) < 0 \; \forall P \in (0,P)$ when $\alpha\Lambda \geq 1$, which is already shown in the proof of Proposition C.3. QED.

We last consider comparative statics in the DM's utility. We first analyze these statics when valence is interpreted "literally" (the third statement in main text Proposition 3).

Proposition C.5. *If valence is interpreted literally, then the median voter's equilibrium utility is first-order stochastically increasing in $(x, \lambda_c, \mu, \theta)$, and decreasing in (a, λ_v). It thus tracks the amount of spending (in the sense of obeying identical comparative statics with respect to all six individual model parameters) when $\Lambda\alpha \geq 1$.*

Proof: If valence is interpreted literally, the DM's equilibrium utility is equal to the score of the candidate she selects. Since she always selects the highest score, the distribution over her equilibrium utility is described by the CDF over the maximum score $[F(s)]^2$, and thus obeys identical comparative statics as the score CDF (characterized in Proposition C.2). Finally, by Proposition C.4 candidate spending also obeys the same comparative statics when $\alpha\Lambda \geq 1$. QED

We now analyze these statics when valence is interpreted "behaviorally" and excluded from the welfare calculation (this is the second statement in Proposition 3).

Proposition C.6. *If valence is interpreted behaviorally, the median voter's equilibrium utility is first-order stochastically decreasing in $(x, \lambda_c, \mu, \theta)$ and increasing in a. When $\alpha\Lambda \geq 1$ it is also increasing in λ_v and thus inversely tracks the extremism of platforms (in the sense of obeying opposite comparative statics with respect to all six individual model parameters).*

Proof: If valence is interpreted behaviorally, then the DM's utility for a proposal (s_i, y_i) is $-\lambda_v y_i^2$. Let $\ell_i = \lambda_v y_i^2$ denote the DM's *losses* from the proposal (so losses are the negative of utility), and let $F_L(\ell)$ denote the CDF over losses that the candidates' platforms offer the DM. Since the DM (incorrectly) chooses the candidate with the most extreme platform due to spending, she always suffers the maximum losses presented to her, and thus the CDF over her behavioral losses is $[F_L(\ell)]^2$. By implication, first-order stochastic increases (decreases) in the losses offered by the candidates' platforms imply corresponding first-order stochastic increases (decreases) in the DM's equilibrium losses, and first-order stochastic decreases (increases) in the DM's (behavioral) equilibrium utility. Finally, the inverse $\ell_F(P)$ of a candidate's loss CDF $F_L(\ell)$ is just $\lambda_v [y_F(P)]^2$. The parameters $(x, \lambda_c, \mu, \theta, a)$ thus only affect behavioral losses through the extremism of platforms, and so each of these parameters affect platform extremism and equilibrium losses in the same way. For the parameter λ_v, equilibrium losses are first order stochastically decreasing for $\alpha\Lambda \geq 1$ when $\frac{\partial}{\partial \lambda_v}\left(\lambda_v [y_F(P)]^2\right) < 0$, which is already shown in the proof of Proposition C.3. QED

C.2 Candidate Utility

The candidates' equilibrium utility is the only outcome of interest that is not probabilistic, since mixing implies that the candidates achieve their exact equilibrium utility with any proposal in the support of their strategy. In particular, candidate i achieves her equilibrium utility with proposal $\left(\bar{s}, i\left(\frac{F(\bar{s})}{\Lambda\alpha+F(\bar{s})}\right)x\right) = \left(s_F(1), i\left(\frac{1}{\Lambda\alpha+1}\right)x\right)$; that is, a proposal with the maximum score (which ensures winning the election for sure) alongside her optimal platform if she expects to win for sure. This yields the following comparative statics in the candidates' equilibrium utility.

Proposition C.7. *The candidates' equilibrium utility is decreasing in (x, λ_c, μ), increasing in (a, λ_v), and unaffected by θ. It thus inversely tracks the extremism of platforms.*

Proof: A candidates' utility from platform $\left(s_F(1), i\left(\frac{1}{\Lambda\alpha+1}\right)x\right)$ and thus her equilibrium utility is equal to

$$\begin{aligned} & -\alpha\left(s_F(1)+\lambda_v\left(\frac{x}{\Lambda\alpha+1}\right)^2\right)+\left(\theta-\lambda_c\left(x-\left(\frac{x}{\Lambda\alpha+1}\right)\right)^2\right) \\ = & \quad \theta-\alpha s_F(1)-x^2\lambda_c\frac{\Lambda\alpha}{\Lambda\alpha+1} = -x^2\lambda_c\left(\int_0^1\frac{4\hat{P}}{\Lambda\alpha+\hat{P}}d\hat{P}+\frac{\Lambda\alpha}{\Lambda\alpha+1}\right)\end{aligned}$$

We now make three straightforward observations. First, candidate utility is clearly decreasing in x and unaffected by θ. Second, the effect of (a, μ, λ_v) is mediated exclusively through $\Lambda\alpha$, which only affects the term inside the parentheses. Third, a sufficient condition for the utility to be decreasing in λ_c is that the term inside the parentheses is increasing in λ_c.

Finally, taking the derivative of this term inside the parentheses w.r.t. $\Lambda\alpha$ yields

$$-\int_0^1\frac{4\hat{P}}{\left(\Lambda\alpha+\hat{P}\right)^2}d\hat{P}+\frac{1}{(\Lambda\alpha+1)^2} < -\int_0^1\frac{4\hat{P}}{(\Lambda\alpha+1)^2}d\hat{P}+\frac{1}{(\Lambda\alpha+1)^2} = -\frac{1}{(\Lambda\alpha+1)^2} < 0.$$

This proves that utility is increasing in $\Lambda\alpha$ (holding λ_c fixed) and so increasing in (a, λ_v) and decreasing in μ. It also proves that the term inside the parentheses is increasing in λ_c, and thus utility is decreasing in λ_c. QED

D Accessory Proofs

Proof of Proposition A.1

We prove the proposition in a series of lemmas. We begin with a generalized version of the model that allows for asymmetries between the candidates; specifically, each candidate $i \in \{-1, 1\}$ has a marginal cost of spending $a_i > 0$ (so that $\alpha_i = \frac{a_i}{\mu}$), weight on ideology $\lambda_i > 0$, and officeholding benefit $\theta_i \geq 0$.

First, let $\bar{\Pi}_i (s_i, \gamma_i; \sigma_{-i})$ denote i's expected utility for proposal (s_i, γ_i) if a tie would be broken in her favor. Clearly this is i's expected utility from making a proposal at any s_i where $-i$ has no atom, and i can always achieve utility arbitrarily close to $\bar{\Pi}_i (s_i, \gamma_i; \sigma_{-i})$ by making $\varepsilon-$higher score proposals. So

$$\bar{\Pi}_i (s_i, \gamma_i; \sigma_{-i}) = -\alpha_i \left(s_i + \lambda_v \gamma_i^2\right) + F_{-i} (s_i) \cdot \left(\theta_i - \lambda_i (x_i - \gamma_i)^2\right) + \int\limits_{s_{-i} > s_i} - \lambda_i (x_i - \gamma_{-i})^2 \, d\sigma_{-i}.$$

Taking the derivative with respect to γ_i yields the first Lemma.

Lemma D.1. *At any score s_i where $F_{-i}(\cdot)$ has no atom, the proposal $\left(s_i, \left(\frac{F_{-i}(s_i)}{\Lambda_i \alpha_i + F_{-i}(s_i)}\right) x_i\right)$ is the strictly best score-s_i proposal, where $\Lambda_i = \frac{\lambda_v}{\lambda_i}$ is the voter's relative ideological strength. Defining the functions $\gamma_i (y) = iy$ and $y_i (P) = \frac{P}{\Lambda_i \alpha_i + P} |x_i|$, the optimal platform may be written as $\gamma_i (y_i (F_{-i} (s_i)))$.*

Proof: Straightforward and omitted. QED

Lemma D.1 states that at almost every score, proposer i's best combination of platform and spending to generate that score involves a platform exactly at $\left(\frac{F_{-i}(s_i)}{\Lambda_i \alpha_i + F_{-i}(s_i)}\right) x_i = \gamma_i (y_i (F_{-i} (s_i)))$.

The second lemma establishes that at least one of the proposers is always *active*, in the sense of making a proposal with strictly positive score (all positive-score proposals are positive-quality, but the reverse is not necessarily true). Intuitively, this holds because the proposers wish to move policy in opposite directions from the DM, and can beat negative-score proposals for "free" by proposing the DM's ideal and spending nothing.

Lemma D.2. *In equilibrium $F_k (0) > 0$ for at most one $k \in \{L, R\}$.*

Proof: Suppose not, so $F_i (0) > 0$ $\forall i$ in some equilibrium. Let U_i^* denote proposer i's equilibrium utility, which can be achieved by mixing according to her strategy conditional on making score-$s \leq 0$ proposal. Let $\bar{\gamma}^0$ denote the expected ideological outcome and $\bar{w}_i^0$ the probability proposer i wins conditional on both sides making score ≤ 0 proposals (these quantities may depend on the DM's tie-breaking rule when both make proposals with the same score). Now $\bar{w}_i^0 \leq 1$ for both candidates and $\theta_i \geq 0$ (winning office is weakly beneficial). In addition $x_{-1} < 0 < x_1$ implies $-\left(x_i - \bar{\gamma}^0\right) \leq -\left(x_i - 0\right)$ for at least one k, and both proposers have mean-variance preferences over ideology. Together these imply that $U_k^* \leq \bar{\Pi}_k\left(0, 0; \sigma_{-k}\right)$, which in turn is $< \bar{\Pi}_k\left(0, \gamma_k\left(y\left(F_{-k}\left(0\right)\right)\right); \sigma_{-k}\right)$ by Lemma D.1 and $F_{-k}\left(0\right) > 0$, so k has a strictly profitable deviation. QED

The third Lemma establishes that in equilibrium there is 0 probability of a tie at any score. The absence of score ties is an intuitive consequence of exactly opposing ideological interests and the fact that spending is "all pay" – at least one proposer will find it in her interests to spend a bit more to break the tie, and make an ideological proposal that is weakly better than the expected outcome from a tie.

Lemma D.3. *In equilibrium there is 0-probability of a tie at any score s.*

Proof: The absence of ties at scores $s \leq 0$ is immediately implied by Lemma D.2. To rule out ties at scores $s > 0$, suppose not, so each proposer's strategy generates an atom of size $p_i^s > 0$ at some $s > 0$. Now let $\bar{\gamma}_i^s$ denote i's expected ideology conditional on a score-s proposal, $\bar{\gamma}^s$ denote the expected ideological outcome conditional on a tie at score s, and $\bar{w}_i^s$ the probability proposer i wins conditional on a tie at score s. Proposer i achieves her equilibrium utility U_i^* by mixing according to her strategy conditional on a score-s proposal. Now using that proposers have mean-variance preferences, that $\theta_i \geq 0$, and that $-\left(x_i - \bar{\gamma}^s\right)^2 \leq -\left(x_i - 0\right)^2$ for at least one k, it is straightforward to show that U_k^* is $\leq$

$$
\begin{aligned}
&-\alpha_k\left(s + \lambda_v\left[\bar{\gamma}_k^s\right]^2\right) + \left(F_{-k}\left(s\right) - p_{-k}^s\right) \cdot \left(\theta_k - \lambda_k\left(x_k - \bar{\gamma}_k^s\right)^2\right) + p_{-k}^s\left(\bar{w}_k^s \theta_k - \lambda_k\left(x_k - 0\right)^2\right) \\
&+ \int\limits_{s_{-k} > s_k} - \lambda_k\left(x_k - \gamma_{-k}\right)^2 d\sigma_{-k}
\end{aligned}
\tag{D.1}
$$

We now argue k has a strictly profitable deviation. If k's proposal at score s is $(s, 0)$ with probability 1, then eqn D.1 is $\leq \bar{\Pi}_k\left(s, 0; \sigma_{-k}\right) < \bar{\Pi}_k\left(s, \gamma_k\left(y_k\left(F_{-k}\left(s\right)\right)\right); \sigma_{-k}\right)$ (by Lemma D.1 and $F_{-k}\left(s\right) \geq p_{-k}^s > 0$). If k sometimes proposes something else, then it is straightforward to show that eqn D.1 is $< \left(1 - \frac{p_{-k}}{F_{-k}(s)}\right) \bar{\Pi}_k\left(s, \bar{\gamma}_k^s; \sigma_{-k}\right) + \left(\frac{p_{-k}}{F_{-k}(s)}\right) \bar{\Pi}_k\left(s, 0; \sigma_{-k}\right)$, which is k's utility if she were to instead propose $(s, 0)$ with probability $\frac{p_{-k}}{F_{-k}(s)}$, and the expected ideology $\bar{\gamma}_k^s$ of her strategy at score s with the remaining probability (and always win ties). QED

Lemmas D.1 – D.3 jointly imply the "ideological optimality" portion of the Proposition; each proposer makes proposals of the form $(s_i, \gamma_i(y_i(F_{-i}(s_i))))$ with probability 1. Thus, proposer i can compute her expected utility from any proposal (s_i, γ_i) *as if* her opponent only makes proposals of the form $(s_{-i}, \gamma_{-i}(y_i(F_i(s_{-i}))))$, and the utility from making *any* proposal (s_i, γ_i) where $-i$ has no atom (or a tie would be broken in i's favor) is therefore $\bar{\Pi}_i^*(s_i, \gamma_i; F) =$

$$-\alpha_i\left(s_i + \lambda_v \gamma_i^2\right) + F_{-i}(s_i) \cdot \left(\theta_i - \lambda_i(x_i - \gamma_i)^2\right) + \int_{s_i}^{\infty} -\lambda_i\left(x_i - \gamma_{-i}\left(y_{-i}\left(F_i(s_{-i})\right)\right)\right)^2 dF_{-i}. \tag{D.2}$$

Proposer i's utility from making the *best* proposal with score s_i is $\bar{\Pi}_i^*(s_i, \gamma_i(y_i(F_{-i}(s))); F)$, which we henceforth denote $\bar{\Pi}_i^*(s_i; F)$.

Fourth, we establish that equilibrium score CDFs must satisfy natural properties arising from the all pay component of the contest, and that these properties yield a pair of differential equations characterizing any equilibrium score CDFs $\{F_i, F_{-i}\}$.

Lemma D.4. *The support of the equilibrium score CDFs over $[0, \infty]$ is common, convex, and includes 0; that is, supp$\{F_i\} \cap [0, \infty] =$supp$\{F_{-i}\} \cap [0, \infty] = [0, \bar{s}]$ where $\bar{s} \in (0, \infty]$. In addition, for all i and $s \in [0, \bar{s}]$, $\{F_i, F_{-i}\}$ satisfy the differential equations*

$$f_{-i}(s) \cdot \left(\theta_i + \lambda_i\left(\left(x_i - \gamma_{-i}\left(y_{-i}\left(F_i(s)\right)\right)\right)^2 - \left(x_i - \gamma_i\left(y_i\left(F_{-i}(s)\right)\right)\right)^2\right)\right) = \alpha_i$$

Proof: We first argue $\hat{s} > 0$ in support of $F_i \to F_{-i}(s) < F_{-i}(\hat{s})\ \forall s < \hat{s}$. Suppose not; so $\exists s < \hat{s}$ where $-i$ has no atom and $F_{-i}(s) = F_{-i}(\hat{s})$. Then $\bar{\Pi}_i(\hat{s}, \gamma_i; F) - \bar{\Pi}_i(s, \gamma_i; F) = -\alpha_i(\hat{s} - s) < 0$, implying i's best score-s proposal is strictly better than her best score-$\hat{s}$ proposal, a contradiction. We now argue this yields the desired properties. First, an $\hat{s} > 0$ in i's support but not $-i$ implies $\exists\ \delta > 0$ s.t. $F_{-i}(s - \delta) = F_{-i}(s)$. Next, if the common support were not convex or did not include 0, then there would $\exists \hat{s} > 0$ in the common support s.t. neither proposer has support immediately below, and $F_i(s) < F_i(\hat{s})\ \forall i,\ s < \hat{s}$ would imply both proposers have atoms at $\hat{s}$, contradicting Lemma D.3. Finally $\bar{s} > 0$ follows immediately from Lemma D.2.

To see the differential equations, observe that $\hat{s} > 0$ in supp$\{F_i\}$ implies all $s \in [0, \hat{s}]$ are also in supp$\{F_i\}$, implying $\bar{\Pi}_i(s; F) \geq U_i^*\ \forall s \in [0, \hat{s}]$ and hence $\lim_{s \to \hat{s}^-}\left\{\bar{\Pi}_i(s; F)\right\} \geq U_i^*$. Equilibrium also requires $\bar{\Pi}_i(s; F) \leq U_i^*\ \forall s$ (otherwise i would havea strictly proftiable deviation). Hence $\bar{\Pi}_i(s; F) = U_i^*\ \forall s \in [0, \bar{s}]$. This further implies the F's must absolutely continuous over $(0,\infty)$ (given our initial assumptions), and therefore $\frac{\partial}{\partial s}\left(\Pi_i^*(s;F)\right)=0$ for almost all $s\in[0,\bar{s}]$. Finally this straightforwardly yields the differential equations for score optimality, with the boundary conditions implied by Lemma D.4. QED

Finally, the preceding Lemmas together yield Proposition A.1, observing that when symmetry is imposed on the candidates ($x_i = ix$, $a_i = a$, $\lambda_i = \lambda_c$, $\theta_i = \theta$, implying that $\alpha_i = \alpha$, $\Lambda_i = \Lambda$, and $y_i(P) = y(P) = \frac{P}{\Lambda\alpha+P}x$) the differential equations in Lemma D.4 reduce to

$$f_{-i}(s)\cdot\left(\theta+\lambda_c\left((x+y(F_i(s)))^2-(x-y(F_{-i}(s)))^2\right)\right)=\alpha$$

Proof of Proposition B.1

We prove $F_i(s)=F_{-i}(s)=F(s)\ \forall s\geq 0$ (and hence $\forall s$ since then $F_i(0)=F_{-i}(0)=0$ from Lemma D.2) by contradiction. First we explicitly write out the coupled system of differential equations from score optimality, which states that $\forall s\in[0,\bar{s}]$ and $\forall i$:

$$\begin{aligned}
f_{-i}(s)\cdot\left(\theta+2\lambda_c x\sum_j y(F_j(s))+\lambda_c\left([y(F_i(s))]^2-[y(F_{-i}(s))]^2\right)\right) &= \alpha \quad \text{and}\\
f_i(s)\cdot\left(\theta+2\lambda_c x\sum_j y(F_j(s))-\lambda_c\left([y(F_i(s))]^2-[y(F_{-i}(s))]^2\right)\right) &= \alpha
\end{aligned}$$

It follows immediately that $F_i(s)>F_{-i}(s)$ at $s\geq 0\rightarrow f_i(s)>f_{-i}(s)$. Restated more intuitively, if $F_i(s)-F_{-i}(s)$ is strictly positive at some $s\geq 0$ then it is also strictly increasing, which further implies that $F_i(s)-F_{-i}(s)>0\rightarrow F_i(\hat{s})-F_{-i}(\hat{s})\geq F_i(s)-F_{-i}(s)\ \forall\hat{s}>s$. Now suppose we have non-symmetric score CDFs; then there $\exists\hat{s}\in[0,\bar{s})$ s.t. $F_i(\hat{s})>F_{-i}(s)$; then by the preceding $\lim_{s\to\bar{s}}\{F_i(s)-F_{-i}(s)\}\geq F_i(\hat{s})-F_{-i}(\hat{s})>0$, contradicting atomless common support. QED

Proof of Proposition B.2

The explicit derivation of strategies is fully contained in Appendix D. It remains only to show that strategies satisfying the necessary conditions in Proposition A.1 and in which the score CDFs are identical ($F_i(s)=F_{-i}(s)=F(s)$ with $F(0)=0$) are also sufficient for equilibrium. First, the differential equations and boundary conditions imply that $F(s)$ is atomless, so all proposals (s_i,γ_i) yield utility $\bar{\Pi}_i^*(s_i,\gamma_i;F)$ (the utility when ties are broken in i's favor), and the strictly best proposal $(s_i,\gamma_i(y(F_{-i}(s))))$ with score s_i yields utility $\bar{\Pi}_i^*(s_i;F)$.

Thus it suffices to show that $\bar{\Pi}_i^*(s;F) = \max_{s_i}\left\{\bar{\Pi}_i^*(s_i;F)\right\}$ $\forall s \in [0,\bar{s}]$. By construction $\bar{\Pi}_i^*(s;F) = \hat{U}_i^*$ $\forall s \in [0,\bar{s}]$. For $s > \bar{s}$ we have $\bar{\Pi}_i(s;F) - \bar{\Pi}_i(\bar{s};F) = -(\alpha-1)(s-\bar{s}) < 0 \rightarrow \bar{\Pi}_i(s;F) < \hat{U}_i^*$). For $s < 0$ we have $\bar{\Pi}_i^*(s;F) = \bar{\Pi}_i^*(0;F) \rightarrow \bar{\Pi}_i^*(s;F) = \hat{U}_i^*$. QED.

Details for Footnote 5

To see why spending q_i may equivalently represent the sum of positive campaigning p_i by i to improve her own image and negative campaigning n_i to worsen her opponent's image, suppose that the candidates may engage in these two distinct strategies. Then the DM's utility for electing candidate i is $\mu(p_i - n_{-i}) - \lambda_v \|y_i\|^2$ and her utility for electing candidate $-i$ is $\mu(p_{-i} - n_i) - \lambda_v \|y_{-i}\|^2$. The DM thus prefers candidate i to candidate $-i$ i.f.f.

$$\mu(p_i + n_i) - \lambda_v \|y_i\|^2 \geq \mu(p_{-i} + n_{-i}) - \lambda_v \|y_{-i}\|^2$$

and for the purposes of influencing the DM's behavior positive and negative campaigning are equivalent. In our model, the distinction between positive and negative campaigning would only affect equilibria if they had different marginal costs. The distinction is also meaningful when evaluating voter welfare if valence is interpreted literally, since the candidates view the two strategies equivalently but one increases voter welfare while the other reduces it.

Details for Footnote 7

We explain how the distribution over the final policy outcome depicted in Figure 2 in the main text is derived. Observe that the distance of the final policy outcome from the DM is described by the CDF $[F_Y(y)]^2$ since the DM always chooses candidate with the most distant platform. Thus the density of the distance of the final policy outcome is $2F_Y(y)\,f_y(y)$. Finally, for each distance $y \in \left[0, \frac{x}{\Lambda\alpha+1}\right]$ with total density $2F_Y(y)\,f_y(y)$ this distance is equally likely to result from a platform of the left or right candidate; thus platforms $\gamma \in \left[-\frac{x}{\Lambda\alpha+1}, \frac{x}{\Lambda\alpha+1}\right]$ have density $F_Y(|\gamma|)\,f_y(|\gamma|)$.

E The Marginality Hypothesis

We now examine the model when one candidate is purely office-motivated and prove Proposition 3. Applying the revised parameter assumptions that $\theta_i = \theta$, $\lambda_{-1} = 0$, $\theta_i = \theta$, $a_1 \leq a_{-1}$ (where player -1 is the "challenger" and player 1 is the "incumbent"), we use Lemmas D.1–D.4, simplify, and rearrange to yield revised necessary conditions for equilibrium.

1. **(Ideological Optimality)** With probability 1, player (-1)'s proposals take the form $(s_{-1}, 0)$ and player (1)'s proposals take the form

$$\left(s_1, \left(\frac{\lambda_1}{\lambda_v \frac{\alpha_1}{F_{-1}(s_1)} + \lambda_1}\right) x_1\right)$$

2. (**Score Optimality**) Score CDFs (F_i, F_{-i}) must satisfy $F_k(0) > 0$ for at most one k, $\text{supp}\{F_i\} \cap [0, \infty] = \text{supp}\{F_{-i}\} \cap [0, \infty] = [0, \bar{s}]$ where $\bar{s} \in (0, \infty]$, and

$$f_{-i}(s) = \frac{\alpha_i}{\theta + \lambda_i x_i \cdot \phi_i (F_{-i}(s); \alpha_i)} \quad \forall i \text{ and } s \in [0, \bar{s}]$$

where

$$\phi_i(Q; \alpha_i) = 1 - \left(1 - \frac{\lambda_i}{\lambda_v \frac{\alpha_i}{Q} + \lambda_i}\right)^2$$

Observe that for $\lambda_i > 0$ and $\alpha_i > 0$, we have $\phi_i(0; \alpha_i) = 0$, and $\phi_i(Q; \alpha_i) < 1$ strictly increasing in Q and strictly decreasing in α_i.

We first argue that for $0 < \alpha_1 \leq \alpha_{-1}$ and $\lambda_{-1} = 0 < \lambda_1$ equilibrium requires that $F_1(0) = 0 < F_{-1}(0)$ (the incumbent is "always active" in the sense of offering a platform that might win, while the challenger is "sometimes inactive" in the sense of sometimes offering a platform that will definitely lose). Observe from the differential equations that since $\lambda_{-1} = 0$ we have $f_{-1}(s) = \frac{\alpha_1}{\theta + \lambda_1 x_1 \cdot \phi_1(F_{-1}(s); \alpha_i)} < f_1(s) = \frac{\alpha_{-1}}{\theta} \quad \forall s \in [0, \bar{s}]$; then if $F_{-1}(0) \leq F_1(0)$ we must have $F_{-1}(\bar{s}) < F_1(\bar{s})$, contradicting $F_{-1}(\bar{s}) = F_1(\bar{s})$ from full atomless support of both CDFs over $[0, \bar{s}]$.

Having established that $F_1(0) = 0$ and also $f_1(s) = \frac{\alpha_{-1}}{\theta}$ we then have that $F_1(s) = \frac{\alpha_{-1}}{\theta} s$; in words, the incumbent's score CDF is distributed uniformly over $\left[0, \frac{\theta}{\alpha_{-1}}\right]$ (as in the standard all pay contest), establishing that $\bar{s} = \frac{\theta}{\alpha_{-1}}$ in equilibrium. Thus, $F_{-1}(s)$ over $[0, \bar{s}]$ is the unique solution to the first order differential equation and boundary condition

$$f_{-1}(s) = \frac{\alpha_1}{\theta + \lambda_1 x_1 \cdot \phi_1 (F_{-1}(s); \alpha_1)} \; \forall s \in [0, \bar{s}] \text{ and } F_{-1}(\bar{s}) = 1 \text{ with } \bar{s} = \frac{\theta}{\alpha_{-1}}$$

Next we argue that $F_{-1}(s)$ is first-order stochastically increasing in $\alpha_1 \in (0, \alpha_{-1}]$ over $[0, \bar{s})$ (noting that $\bar{s}$ is constant in α_1 when $\alpha_1 \leq \alpha_{-1}$). In other words, for $0 < \tilde{\alpha}_1 < \alpha_1 \leq \alpha_{-1}$ we have $F_{-1}(s) < \tilde{F}_{-1}(s) \; \forall s \in [0, \bar{s})$, where $\tilde{F}_{-1}(s)$ and $F_{-1}(s)$ denote the unique solutions to the differential equation and boundary condition for costs $(\tilde{\alpha}_1, \alpha_1)$, respectively. To see this, observe from the differential equation and the aforementioned properties of $\phi_1(Q;\alpha_1)$ that $\tilde{F}_{-1}(s) \leq F_{-1}(s) \to \tilde{f}_{-1}(s) < f_{-1}(s)$. Thus, if there exists any $s \in [0,\bar{s}]$ where $\tilde{F}_{-1}(s) \leq F_{-1}(s)$ then $\tilde{F}_{-1}(\bar{s}) < F_{-1}(\bar{s})$, a contradiction.

Last we argue that for $0 < \tilde{\alpha}_1 < \alpha_1 \leq \alpha_{-1}$, the incumbent wins strictly less often with costs α_1 than with costs $\tilde{\alpha}_1$ and also takes first-order stochastically more moderate platforms. First observe that the incumbent's probability of victory in the contest is $\int_0^{\bar{s}} F_{-1}(s) f_1(s)\, ds = \int_0^{\bar{s}} F_{-1}(s) \frac{\alpha_{-1}}{\theta} ds$, so a first-order stochastic increase in $F_{-1}(s)$ yields a strictly lower probability of victory. Next recall that at each score $s \in [0,\bar{s}]$ candidate (1)'s platform is $\left(\frac{\lambda_1}{\lambda_v \frac{\alpha_1}{F_{-1}(s)} + \lambda_1}\right) x_1$ which is strictly decreasing in α_1 and increasing in $F_{-1}(s)$; thus, since $F_{-1}(s)$ first order stochastically dominates $\tilde{F}_{-1}(s)$, the incumbent takes a more moderate platform *at each score* $s \in [0,\bar{s}]$ when her costs are α_1 vs. $\tilde{\alpha}_1$. Finally, the incumbent uses the same score CDF $F_1(s) = \frac{\alpha_{-1}}{\theta} s$ for both α_1 and $\tilde{\alpha}_\cdot$, yielding the desired result.