Productive Policy Competition and Asymmetric Extremism*

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Abstract

Viable public policy alternatives are not simply willed into existence; they must be developed by individuals and groups with the expertise and willingness to do so. We analyze a model in which competing actors with differing ideologies and abilities develop policies for consideration by a decisionmaker. We show that the process exhibits unequal participation, inefficiently unpredictable and extreme policies and outcomes, wasted effort, and an apparent advantage for extreme policies. Imbalances can arise both because of unequal intrinsic extremism or unequal skill at policy development but in either case benefit the decisionmaker, in contrast to a literature showing the benefits of balanced competition. Our model provides a rationale for why extreme actors may come to dominate policymaking that is rooted in the nature of productive policy development, and highlights the difficulty in assessing the normative implications of such dominance.

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The reasonable man adapts himself to the world: the unreasonable one persists in trying to adapt the world to himself. Therefore all progress depends on the unreasonable man.

– George Bernard Shaw, *Man and Superman* (1903)

Ideological conflict is a key driver of policymaking in representative democracies; citizens, parties, and interest groups compete via elections to elect politicians who share their ideological preferences, who in turn compete via the rules and procedures of policymaking to enact public policies that reflect those preferences. Correspondingly, political scientists have devoted considerable attention to these two processes, in order to better understand why some public policies become law and not others.

However, public policy scholars have long recognized the critical importance of an intermediate stage of the policy process – how viable public policy alternatives are initially *developed*. For example, in his sweeping work on the policy process, Kingdon (1984) describes the development of concrete policy alternatives as a necessary precondition for change – "before a subject can attain a solid position on a decision agenda, a viable alternative [must be] available for decisionmakers to consider" (p142), and recounts a Presidential staffer's perspective on policy development as follows (p132):

> Just attending to all the technical details of putting together a real proposal takes a lot of time. There's tremendous detail in the work. It's one thing to lay out a statement of principles or a general proposal, but its quite another thing to staff out all the technical work that is required to actually put a real detailed proposal together.

Similarly, a key element of Heclo's (1974) account of social welfare policymaking in 1980s Britain and Sweden was the "competition for power" that occurred within policy communities via "effort to develop more knowledgeable means of addressing the policy problem" (Sabatier (1988), p. 130).[1]

Given the certain costs and uncertain rewards to policy development, who will "invest the resources – time, energy, reputation, and sometimes money" (Kingdon (1984), p122) to do it? When

[1] See Callander (2011) for an analysis of a second key element – learning via experimentation.

and why? And how does ideological conflict between participants in the process influence which policies are ultimately adopted? To examine these questions we study a generalization of the "competitive policy development" model introduced in Hirsch and Shotts (2015). In the original model, two identical policy developers with opposing ideological preferences develop competing policies, at cost to themselves and with no guarantee of adoption, for consideration by a centrist decisionmaker. We extend this model to study *asymmetries* between the policy developers; in their underlying ideological extremism, ability at developing high-quality policies, or both. In so doing, we uncover new insights about the nature of competitive policy development, and surprising normative implications about the causes and consequences of extreme behavior and imbalanced political participation.

Policy Development as a Contest Before discussing our new results, we first revisit the model's central elements. Two competing policy developers simultaneously craft policy alternatives; each consists of an ideological orientation and a level of "quality." Quality is modelled as a policy-specific public good that all participants in the process value, and is intended to capture the sorts of "criteria for survival" that Kingdon identifies as necessary for public policies, including technical and administrative feasibility, efficacy, equity, efficiency, and cost-effectiveness (Kingdon (1984), Ch. 6). The ideology of a policy may be chosen freely, but generating its quality requires a costly up-front investment. Once produced quality is attached to the policy in question, and cannot be preserved if its ideological orientation is altered. A centrist decisionmaker then chooses from the available policies, including those created by the developers and potentially some preexisting alternatives.

Much like the "policy entrepreneurs" in Kingdon's theory, our developers intrinsically value quality, but are predominantly motivated by *ideology* when developing their respective policies. Specifically, "they want to promote their values, or affect the shape of public policy" (Kingdon (1984), p123), and do so by developing policies that – through a combination of ideological concessions and quality investments – are sufficiently appealing to be chosen by the decisionmaker. When crafting their respective policies, the developers therefore face a tradeoff between generating a policy more reflective of their ideal ideology – which must then be higher quality to remain appealing – vs. something more moderate – which may then be lower quality and still remain competitive. Crucially, the developers also fear that should their policy fail to be adopted, they will instead have to live under an ideologically-distant policy developed by their competitor.

Policy development in the model thus plays out as a kind of "contest," related to but distinct from the standard "all-pay contest" (Siegel (2009)). In the all-pay contest, participants expend costly up-front effort to try to win a fixed prize, which is awarded to the participant who exerts the most effort; this model has been fruitfully applied to electoral competition and lobbying (e.g., Hillman and Riley (1989) and Meirowitz (2008)). In our model, participants expend costly up-front effort to enhance a public good (quality) that makes one's own policy more appealing to everyone; the policy that is most appealing to a decisionmaker with known preferences is implemented, and the "prize" is the ideological benefit of living under one's own (vs. one's opponent's) endogenously-chosen policy.

The Hirsch and Shotts (2015) model studies policy developers who are equally ideologically extreme and capable at developing high-quality policies, and yields three main insights. First, in equilibrium there will be a *positive association* between a policy's extremism and its quality – the reason is that strategic developers know that extreme policies have no hope of adoption *unless* they are also higher-quality to "compensate" for their greater extremism. Second, *ideological polarization* between the developers is not an unalloyed bad – conflict over "the shape of public policy" driven by conflicting ideologies motivates quality investments that benefit the decisionmaker (despite the resulting policies' greater ideological extremism). Finally, *better technology* or skill at producing high-quality policies is not an unalloyed good; both developers will partially exploit such skill to craft more extreme but still competitive policies that better reflect their ideological interests.

General Properties The first contribution of the model herein – in which the developers may differ in both their *intrinsic ideological extremism* and their *ability at generating quality* – is to identify six general properties of "policy development as a contest." The first four are also present in the symmetric model of Hirsch and Shotts (2015) – our analysis thus confirms they are robust arbitrary asymmetries between the developers. The final two are specific to the asymmetric model.

First, policy development as a contest is *unpredictable* – when crafting policies, the developers are uncertain about exactly what their competitor will develop, whose policy will prevail, and what the final outcome will be. More precisely, the unique Nash equilibrium of the model is in *mixed strategies*, in which each developer randomizes over a continuum of potential policies, each with a different combination of ideology and quality. This uncertainty is necessary to preserve each developers' willingness to craft a new policy; if a developer knew that his policy would fail to be adopted he would decline to exert effort crafting it, and if he knew that it would definitely be adopted (because it would surely be more appealing to the decisionmaker than his competitor's policy) then he would modestly scale back on his quality investment to save on costs while preserving his advantage.

Second, policy development as a contest is *divergent* – whenever a developer exerts effort crafting a new policy (which we term being "active"), its ideology diverges from the decisionmaker's ideal. The reason is that it is always worthwhile for a developer to invest in strictly more quality than is necessary to produce a centrist policy to instead produce a more ideologically-extreme policy that is equally competitive.

Third, it is *biased toward extremism* – *in equilibrium* the decisionmaker is more likely to choose an extreme policy crafted by a given developer than a moderate one crafted by that same developer. This surprising pattern results from the strategic connection between a policy's level of quality and its ideology. Specifically, *when* a developer chooses to craft a more extreme policy, it is in his interest to not only produce enough additional quality to *preserve* that policy's appeal to the decisionmaker, but to also *enhance* that appeal (since a more extreme policy, if adopted, will also benefit him more).

Fourth, it is *inegalitarian*, in the sense that the benefits and costs of competition are distributed unequally among the participants. The decisionmaker always strictly benefits from competition, and would never want to prohibit one developer from participating even if he is very extreme. This is true even though all of the quality invested in the ultimately-losing policy is wasted, since it cannot be transferred between policies. In contrast, both developers are always strictly harmed by competition, in the sense that each would prefer the other to be blocked from participation – this is true even when one or both developers are making significant investments in quality.

Fifth, it exhibits *asymmetric participation*. Specifically, one developer is always active crafting a new policy, while the other developer sometimes declines to do so. Since new policies always diverge from the decisionmaker's ideal, a further implication is that the final policy always diverges from the decisionmaker's ideal as well. This asymmetry arises from differences in the developers' underlying extremism – which affects their willingness to participate in policy development – and abilities at generating quality – which affects their success at competing via policy development.

Sixth, it is *inefficient* in a variety of ways. For one, the expected policy outcome generically differs from both the decisionmaker's ideal, as well as the ideology that would maximize social welfare. In addition, there is substantial uncertainty about the ideology of the final policy outcome which harms all participants in the process due to risk aversion. Finally, any effort spent to improve the quality of the losing policy is ultimately wasted.

Asymmetric Extremism Having established some fundamental properties of policy development as a contest, we next examine the consequences of *asymmetric extremism* in the developers' underlying ideologies. Both the causes and consequences of asymmetric extremism have generated substantial interest among scholars of American politics, due to evidence that the Republican party has moved away from the median voter faster than the Democratic party (e.g. Grossmann and Hopkins (2016); McCarty (2015)). Asymmetric extremism is also of first-order importance for policymaking within institutions like legislatures and bureaucracies (Hitt, Volden and Wiseman (2017); Ting (2003)), since decisionmakers often have preferences that are better aligned with one internal faction over others due to shifting electoral outcomes and political appointments (Lewis (2008)).

When the developers' ideologies are asymmetrically extreme (relative to the decisionmaker), their willingness to engage in policy development is also unsurprisingly asymmetric; one always crafts a new policy while the other sometimes declines to do so. Intuition suggests that the more moderate developer will be more active, by virtue of his better alignment with the decisionmaker. However, in fact the reverse is true; the *more extreme* developer will be more active, since his greater motivation to "affect the shape of public policy" makes him the "stronger" player in the contest over policy (Hillman and Riley (1989)). How will the extremist craft policy as compared to the moderate? He will naturally craft a more extreme policy (in a first order stochastic sense). However, because he is more motivated, he will *also* craft a higher quality policy (again in a first order stochastic sense). In fact, his policy will be *so much* higher quality despite its greater extremism that it will also be better for the decisionmaker (again in a first order stochastic sense). Thus, in equilibrium the decisionmaker's choices will *appear biased toward the extremist despite* his policy's greater extremism.

What then happens as an extremist developer becomes *yet more* extreme? He will become *even more* active in policy development, crafting a yet more extreme but even higher quality policy that is even better for the decisionmaker. While intuitive, this differs starkly from the standard asymmetric all-pay contest, where the strategy of the stronger player is *does not change* if he becomes even stronger (Siegel (2009)). In addition, the extremist's competitor *also* changes his policy, despite no change in his underlying preferences or abilities. First, he further reduces his likelihood of participation, expecting that any effort spent on policy development is more likely to be wasted. Second, he further moderates his policy (first order stochastically), expecting to fail at exploiting quality to move the ideological outcome in his direction. Finally, he becomes strictly worse off, since the growing quality of the extremist's policy will be insufficient to "compensate" for its greater extremism. This final property also differs starkly from the standard asymmetric all-pay contest, where the payoff of the "weaker" player is unaffected by the preferences and abilities of the stronger one.

Finally, while an increasingly-extreme developer crafts an increasingly extreme policy, *and also* pushes the moderate developer out of competing, the decisionmaker nevertheless becomes *increasingly* *better off* in equilibrium. Moreover, this effect is strong, in the sense that *any* level of decisionmaker welfare can be achieved if *only one* developer has sufficiently extreme preferences. Put more intuitively, unilateral extremism results in less (and in the limit no) observable competition, but a (and in the limit infinitely) better-off decisionmaker. The reason is that an increasingly extreme developer, despite crafting an increasingly extreme policy, *also* becomes increasingly fearful of losing to a competing centrist policy; this motivates him to keep investing in quality even when real competition is vanishingly unlikely to materialize. Our finding that asymmetric extremism benefits the decisionmaker is much stronger than the analogous result in Hirsch and Shotts (2015) that *symmetric extremism* benefits the decisionmaker, because symmetrically-extreme developers remain equally motivated to change policy. Indeed, it contrasts starkly with the asymmetric all-pay contest, where the decisionmaker is always harmed by asymmetries because the only consequence of the "stronger" player becoming even stronger is to discourage the weaker player (Hillman and Riley (1989)).[2]

Summarizing, the model with asymmetric extremism provides a novel explanation for how apparent ideological extremism may come to dominate policymaking in a particular domain that is rooted in the nature of productive policy development, rather than political dysfunction, capture, or some other systemic failure. It illustrates how striking imbalances in political participation can arise naturally from differing motivations and abilities of participants in the policy process, and may even have beneficial effects since extreme preferences can motivate investments in "good policy." It further shows how and why a centrist decisionmaker vs. ideologically divergent policy developers may come to hold very different views about the efficacy of the policy process at promoting public welfare. Finally, it demonstrates that asymmetric ideological extremism *alone* is not enough to produce the level of dysfunction that many attribute to contemporary US policymaking (Mann and Ornstein (2016)) – rather, such dysfunction also requires the presence of alternative "destructive"

[2]Moreover, if our model is changed so the developers *only* care about policy when they win, the benefits of symmetric extremism remain but of asymmetric extremism vanish; see Appendix E.

methods of influence (Gieczewski and Li (2022); Hirsch and Kastellec (2022)) and an increasingly asymmetric willingness or ability to employ such methods (Helmke, Kroeger and Paine (2022)).

Asymmetric Ability We last examine the consequences of *asymmetric ability* at generating quality. At the national level such asymmetries may arise as parties fluctuate in their success at cultivating a community of experts (Rich (2004)), and several observers have argued that there is presently a deficit of expertise in the Republican party.[3] More broadly, asymmetries in expertise and resources are a central feature of policy domains dominated by interest group politics or subject to formal rule-making (Yackee and Yackee (2006)) – particularly when the primary axis of conflict is between poorly funded public interest groups (such as environmental organizations) and well-resourced businesses.

We find that asymmetric ability is *observationally equivalent* to asymmetric extremism. Specifically, a developer who is no more intrinsically extreme than his competitor, but who is more capable at producing high quality policies, will be more active, and develop a policy that is more extreme but also higher quality and better for the decisionmaker. Conversely, a developer who is no less ideologically extreme than his competitor, but who is less capable of producing high quality policies, will be less active, and develop a policy that is more moderate but also lower quality and worse for the decisionmaker. As a more-expert developer becomes increasingly expert, his policy becomes increasingly extreme but also higher quality and better for the decisionmaker, while his competitor moderates his policy and becomes increasingly unlikely to develop one. The competitor becomes increasingly worse off, and the decisionmaker becomes increasingly better off despite the growing imbalance in participation and extremism of the expert's policy.

A key implication of the model with asymmetric ability is thus that asymmetric extremism may be not only a "cause" (when it describes the underlying preferences of political actors) but also a "consequence" (when it describes the observed behavior of political actors). Because ideologically-

[3] https://www.politico.com/magazine/story/2017/06/24/intellectual-conservatives-lost-republican-trump-215259/

motivated actors exploit quality investments to further their ideological aims, they will behave *as if* they differ in their intrinsic extremism even if they only differ in their policy development ability. This has far reaching implications for how political scientists measure the policy preferences of elite actors from their observable behaviors like votes and bill sponsorship (Clinton (2012)).

Finally, the model with asymmetric ability has implications for the design of institutions that encourage "effective" policymaking (Volden and Wiseman (2014)). In our model, a more-expert developer is only better at generating a public good – but this nevertheless has distributional consequences, benefitting himself (with more ideologically appealing outcomes) and the decisionmaker (with higher quality policies) at the expense of an ideological competitor. By implication, reform proposals that aim to improve policymaking by enhancing the "non-partisan" capacity of legislatures and bureaucracies (Reynolds (2020)) must be designed with careful attention to exactly *how* the decision rights over new resources are allocated – and with the expectation that such reforms may generate distributional effects and increasing polarization as a byproduct.

The Model

Two developers, labelled -1 (left) and 1 (right), develop competing policies for consideration by a decisionmaker (DM), labelled player 0. A policy (y, q) consists of an *ideology* $y \in \mathbb{R}$ and a level of *quality* $q \in [0, \infty) = \mathbb{R}^+$. All players are purely policy-motivated, in the sense that their final policy payoffs depend only on the ideology and quality of the final policy. The utility of player i for a policy (y, q) is $U_i(y, q) = \lambda q - (y - x_i)^2$. The parameter x_i is player i's ideological ideal point, the decisionmaker is located at 0, the left developer is distance $|x_{-1}|$ to her left, and the right developer is distance $|x_1|$ to her right. A developer's distance $|x_i|$ from the decisionmaker reflects his ideological extremism. A policy's quality q is a public good that all players value at weight λ; higher λ thus means that the players collectively care more about policy quality vs. ideology.

The game proceeds in two stages. In the first, the developers simultaneously select the ideology and quality their respective policies (y_i, q_i). Endowing a policy with quality q_i costs $c_i(q_i) = a_i q_i$

up front, which reflects the initial time and energy needed to improve the policy's quality. The parameter a_i is developer i's marginal cost of increasing quality, and reflects his *ability* at doing so. $\alpha_i = \frac{a_i}{\lambda}$ denotes the ratio of a developer's marginal cost of quality to its marginal benefit, which captures the *weighted marginal cost* of quality once its intrinsic value is taken into account. We assume that this is greater than 1 for both developers, implying that neither would invest in quality for its own sake. In the second stage the DM chooses a final policy to implement. This may be one of the two policies created by the developers, or any other policy from a (possibly empty) set of outside options $\mathbb{O}$ no better for the DM than her ideal point with 0-quality (which yields utility 0). This reflects the idea that the DM has the power to freely *choose* policy, but not to *develop* it.

Theoretical Antecedents The classical approach to studying policy development supposes that a policy *outcome* results from the combination of a policy *choice* and an unknown *state of the world*. This approach has been widely applied to study many institutional environments including legislatures and bureaucracies (Gailmard and Patty (2012)); its central tension is that privately-informed experts worry their that expertise will be "expropriated" to implement policy outcomes contrary to their ideological interests. Our model, in contrast, is part of a growing literature that captures an alternative conceptualization of expertise, in which actors make *policy-specific* investments that they use to achieve a particular ideological goal (e.g. Hirsch and Shotts (2012); Hitt, Volden and Wiseman (2017); Lax and Cameron (2007); Londregan (2000); Ting (2011); Turner (2017)). This approach yields very different strategic tensions than the classical one; rather than fear their information will be expropriated, experts "attempt to exploit their monopoly power over investments to compel decision makers to accept policies that promote their interests" (Hirsch and Shotts (2015)).[4] Particularly

[4]Effectively in between these approaches is work by Callander (2011), which models the "unknown state" governing the relationship between policies and outcomes as an entire function that is the realized path of a Brownian motion. In this approach, information about how to successfully achieve a particular outcome is more relevant for achieving "nearby" ones than "distant" ones.

noteworthy are works by Lax and Cameron (2007) and Hitt, Volden and Wiseman (2017), who also study competitive valence models, but in which developers craft policies in a predetermined order. These models are better suited to studying institutions with structured agenda procedures like the US Supreme Court or House of Representatives, and yield very different patterns of competition that more closely resemble "entry deterrence" models of market competition. However, they share many of our substantive insights about the distributional effects of extremism and ability.

Our model is also theoretically related to a large literature studying "political contests" such as lobbying and elections (e.g. Ashworth and Bueno de Mesquita (2009); Munster (2006); Serra (2010); Wiseman (2006)). Foundational work by Tullock (1980) modelled lobbying as a process by which competing groups exert wasteful effort to increase their chance of securing "politically-contestable rents." Important follow-on work by Hillman and Riley (1989) studied political contests in which the "rents" fall to the group exerting the most effort, and groups could value policy control differently. This model is now known as the *all-pay contest* due to its close relationship to the all-pay auction format, in which a prize is awarded to the highest bidder, but all bidders pay their bids (Baye, Kovenock and de Vries (1996); Siegel (2009)). Our model is closely related to the all-pay contest in two ways; the costs of generating quality is paid by a developer regardless of whether his policy is ultimately implemented, and the policy preferred by the decisionmaker is implemented with certainty (rather than probabilistically). However, the key distinction (and analytical challenge) is that the developers' payoffs from winning and losing are *endogenous* to the policies that they develop, as befits a setting where the participants are "ideologically motivated" rather than "rent-seeking."[5] This crucial property differentiates our model from even the most general formulation of the asymmetric all-pay contest (Siegel (2009)), and is responsible for several distinctive results – among them that asymmetries may benefit the decisionmaker despite reducing participation.

[5]In the terminology of Baye, Kovenock and Vries (2012) the model features a rank-order spillover.

Empirical Domain Our intended empirical domain is policymaking settings that are "healthy" in two particular senses – (1) there exists some common ground between competing actors in the form of policy attributes that they all value, and (2) they are both able and willing to channel their ideological disagreements into productive investments in these attributes. A growing political science literature applies similar models to a range of institutional settings, thereby implicitly or explicitly supposing that these settings (at least sometimes) exhibit these features. An early example is Londregan (2000), who posited that competing branches of the Chilean government "weigh policy alternatives in terms of ideology, about which they disagree, and on the basis of shared public policy values, such as the desire for efficiency." Subsequent work uses such models to examine intra and inter-court bargaining (Clark and Carrubba (2012); Lax and Cameron (2007)) (with opinion attributes like "persuasiveness, clarity, and craftsmanship" valued by all judges); Congressional delegation to the bureaucracy (Huber and McCarty (2004); Ting (2011)) (with "effective implementation" in the sense of "whether regulations are enforced, revenues are collected, benefits are distributed, and programs are completed" valued by legislators and bureaucrats); judicial oversight of the bureaucracy (Turner (2017)) (with "policy precision" valued by both risk-averse judges and bureaucrats); and legislative policymaking (Hirsch and Shotts (2012); Hitt, Volden and Wiseman (2017)) (with the "costs and benefits [of policies] across an array of societally valued criteria" being valued similarly by all legislators).

Clarifying these features also helps to demarcate the *boundaries* of our model's empirical domain. First, in some issue areas, disagreement between competing actors may become so pathological that participants value the "quality" of ideologically-distant policies *negatively*. Consider for example the politics of reproductive rights; pro-life voters likely place an intrinsic negative value on policy attributes that pro-choice voters associate with quality, such as population coverage and cost-effectiveness. Policies in such issue domains are arguably better thought of "all ideology" because actors on opposite sides of the ideological spectrum strongly disagree over their desirability, even if aspects of them like accessibility and cost-effectiveness are costly to design. Second, there may be some issue domains where shared notions of quality are indeed present, but superseded by other (possibly strategic) considerations. For example, when policymaking is dynamic, implementing a high-quality policy "today" might improve one actor's control over policymaking "tomorrow"; this can give a competing actor the incentive to sabotage the policy (in the sense of damaging quality that they intrinsically value) to improve their prospects for future control (Gieczewski and Li (2022); Hirsch and Kastellec (2022)). The applicability of our model requires that such destructive means of gaining policy influence be absent, relatively costly to employ (as compared to the productive means we study), or prohibited by either formal rules or shared norms of governance.

Preliminary Analysis

The Monopolist's Problem To begin the formal analysis, it is helpful to first consider the model with only one developer, i.e. a "monopolist" (see also Hitt, Volden and Wiseman (2017) and Hirsch and Shotts (2018)). W.l.o.g. suppose he is the right developer ($i = 1$). The "monopolist's problem" is depicted in the left panel of Figure 1; ideology is on the x-axis and quality is on the y-axis. The shaded region depicts the set of policies that the decisionmaker would be willing to implement in lieu of (y_0, q_0), which we use to denote the best policy she can implement without the developer's help, i.e., her "outside option." To clarify incentives we temporarily allow this policy to be strictly better than $(0, 0)$ (the decisionmaker's ideal point with 0-quality), but recall that in the main model we have assumed that the decisionmaker's best outside option is no better than this policy.[6]

[6]This exposition further assumes that the decisionmaker's best outside option (y_0, q_0) is *no worse* than $(0, 0)$ (as in Hirsch and Shotts (2018)). With a monopoly developer, this assumption proxies for an "open rule" in which the decisionmaker can choose any 0-quality policy in lieu of the developer's policy. The monopoly analysis herein is thus effectively a generalization of the "open rule" model studied in Hitt, Volden and Wiseman (2017) Prop. 3 allowing for positive-quality status quos. Alternatively, when the rule is "closed" and there is a monopoly developer, there are additional cases to consider in which the developer "crafts" a 0-quality policy that the decisionmaker cannot

The developer's job is to choose both *whether* to craft a new policy that the decisionmaker is willing to implement (in the shaded region), and if so exactly *which* policy (y, q) to develop. This problem can be understood using the inequality

$$\underbrace{\arg\max}_{\left\{(y,q):\lambda q - y^2 \geq \lambda q_0 - y_0^2\right\}} \left\{\left(\lambda q - (y - x_1)^2\right) - a_1 q\right\} \geq \lambda q_0 - (y_0 - x_1)^2 \tag{1}$$

The policy (y, q) that maximizes the left hand side is optimal *if* the developer chooses to be active (invest effort in developing a new policy), and depends on both the developer's ideology x_1 and ability a_1. *Whether* he will be active in turn depends on whether the left hand side (his utility from the developing the optimal policy) exceeds the right hand side (his utility from the decisionmaker's "outside option" (y_0, q_0)). Importantly, the outside option appears on both sides of the inequality because it functions as both a *constraint on* and a *motive for* policy development. It is a constraint because the developer must craft something at least as good as (y_0, q_0) for it to be adopted; the higher is the decisionmaker's indifference curve (in green) passing through (y_0, q_0) the harder it is to "beat." It is a motive because the developer must live with (y_0, q_0) if he doesn't develop something else; while all outside options on the same green curve are equally difficult to beat, those further to the left are worse for the developer, and so more strongly incentivize policy development.

To solve this problem and aid in the subsequent analysis, it is helpful to reparameterize policies (y, q) in terms of their ideology y and the *utility they give the decisionmaker* – we henceforth call this a policy's *score* and denote it s. Now observe that $s = \lambda q - y^2$, implying that a score-s policy with ideology y must have quality $q = \frac{s+y^2}{\lambda}$. Recalling that $\alpha_i = \frac{a_i}{\lambda}$ is the marginal cost of generating quality weighted by the marginal benefit and substituting into (1) then yields the revised problem:

$$\underbrace{\arg\max}_{\{(s,y):s\geq s_0\}} \left\{\underbrace{-(\alpha_1 - 1)\, s}_{\text{score effect}} + \underbrace{2x_1 y - x_1^2 - \alpha_1 y^2}_{\text{ideology effect}}\right\} \geq s_0 + 2y_0 x_1 - x_1^2, \tag{2}$$

where s and s_0 are the score of the developer's new policy and the decisionmaker's outside option. Now it is easy to see that *if* the developer crafts a new policy, it will optimally be *no better* for the

access on her own in lieu of developing a new one; see Hitt, Volden and Wiseman (2017) Prop. 2.

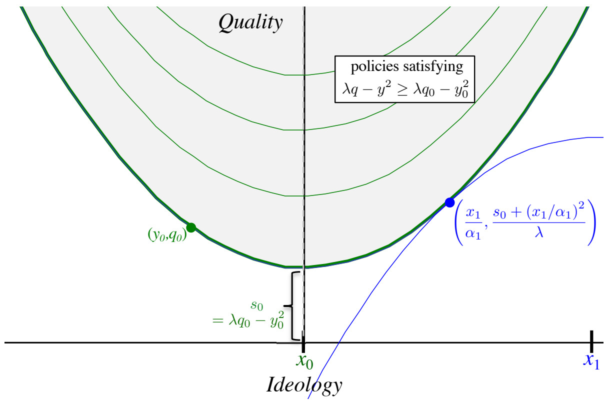

Figure 1: *The Developer's Problem.* The left panel depicts the problem faced by a "monopolist" when the decisionmaker's outside option is (y_0, q_0); the green curves are the decisionmaker's indifference curves, the shaded region depicts the policies the decisionmaker weakly prefers to her outside option, and the blue curve depicts the policies that the developer is indifferent over crafting conditional on acceptance. The right panel depicts the competitive problem, where the decisionmaker chooses her favorite between the policies crafted by the two developers or her best outside option.

decisionmaker than her outside option ($s^* = s_0$), since quality is insufficiently valuable to generate for its own sake ($\alpha_1 - 1 > 0$). The ideology y^* of the optimal policy then trades off the ideological benefit $2x_1y$ of a more extreme policy against the cost $\alpha_1 y^2$ of "compensating" the decisionmaker for it with additional quality, yielding $y^* = \frac{x_1}{\alpha_1}$; this is a weighted average (by $\frac{1}{\alpha_1}$) of the developer's and decisionmaker's ideal ideologies. Finally, the decisionmaker's equilibrium utility under monopoly is no greater than 0, under our assumption that her best outside option is no better than $(0, 0)$.

The Competitive Problem When there is a second developer, the decisionmaker's best outside option may no longer be an exogenous policy, but instead the policy (s_{-1}, y_{-1}) developed by the competitor. The setup of the competitive problem is depicted in the right panel of Figure 1. A developer's policy is a two-dimensional "bid" (s_i, y_i) consisting of a policy's score s_i and ideology y_i. After seeing the two policies, the decisionmaker chooses the one with the highest score (i.e., on the highest indifference curve in Figure 1) or her best outside option. The developers thus compete in a "contest" over policy-development, where a policy's likelihood of winning is determined by its score, but its ideology affects both its up-front cost to develop and the value of winning with it.

Now recall that a monopolist will choose to either develop *no* policy, or develop one *no better* than the decisionmaker's outside option. Applying this insight to the competitive model straightforwardly implies that there is no pure strategy equilibrium. Were each developer i to expect a *specific* policy (s_{-i}, y_{-i}) from his competitor $-i$, each would treat the other's policy like the outside option in the monopoly model, and equilibrium would require that the developers be crafting policies with the exact same score. But if both policies had the exact same score, then at least one developer would have a strict incentive to break this "tie"; either by developing a slightly higher-score policy, or by dropping out of policy development entirely. In the Appendix, we show that the model consequently has a unique equilibrium in mixed strategies; the developers randomize both over whether they develop a new policy, as well as the exact ideology and quality of the policy that they develop.

Deriving Equilibrium We conclude the preliminary analysis by sketching out how to derive the unique equilibrium (see Appendix A for details); this section may be skipped with no loss of substantive insight (a similar treatment can be found in Hirsch and Shotts (2015) Sections I.A-I.C).

First, each developer's mixed strategy can be written as pair of functions: (1) a *cumulative distribution function* $F_i(s)$ describing the probability that developer i crafts a policy with score less than or equal to s, and (2) an *ideology function* $y_i(s)$ describing the exact ideology that developer i targets when crafting a policy with score s. Next, the ideology functions $y_i(s)$ are shown to be continuous and strictly increasing over $[0, \bar{s}]$, meaning that each score a developer targets with positive probability is associated with a unique ideology. Finally, the CDFs $F_i(s)$ are shown to be *continuous* and *strictly increasing* over a common interval $[0, \bar{s}]$ with $F_k(0) > 0$ for at most one k – each developer thus *randomizes smoothly* over crafting a policy with a score in this interval, and one developer always crafts a policy strictly better for the decisionmaker than $(0, 0)$. Intuitively, these properties arise because (i) developing a higher-score policy must always be rewarded with a higher probability of victory (since it is costly), (ii) no developer may craft a positive-quality policy that he thinks may result in a "tie" (since he could endow that policy with just a little more quality to break the tie), and (iii) it is "as if" the decisionmaker's best outside option is exactly $(0,0)$ (even if in reality it is strictly worse) because this is the most competitive "free" policy to craft, and the developers wish to move ideology in opposite directions from it.

Equilibrium then requires that each developer i be *indifferent* over every crafting every policy in the support of his mixed strategy; formally, every policy (s,y) that satisfies both $s \in [0,\bar{s}]$ and $y = y_i(s)$ must maximize the expression:

$$\underbrace{F_{-i}(s)}_{\text{Pr win}}\underbrace{\left(\left(s+y^2\right)-\left(y-x_i\right)^2\right)}_{\text{utility from winning}}-\underbrace{\alpha_i\left(s+y^2\right)}_{\text{effort cost}}+\underbrace{\int_{s_i}^{\bar{s}}\left(\lambda\left(s_{-i}+\left[y_{-i}(s)\right]^2\right)-\left(y_{-i}(s)-x_i\right)^2\right)f_{-i}(s_{-i})\,ds_{-i}}_{\text{expected utility when losing}}$$

The preceding in turn simplifies to:

$$-\left(\alpha_i - F_{-i}(s)\right)s + F_{-i}(s)\cdot\left(2x_i y - x_i^2\right) - \alpha_i y^2 + \int_{s}^{\bar{s}}\left(s_{-i} + 2x_i y_{-i}(s) - x_i^2\right) f_{-i}(s_{-i})\,ds_{-i} \qquad (3)$$

The competitive problem exhibits two key differences with the monopoly problem in eqn. (2). First, developer i only enjoys the benefit of crafting a policy with score s *when it wins* – this increases the effective marginal cost of crafting a policy with score s (from to $\alpha_i - 1$ to $\alpha_i - F_i(s)$), and reduces the probability that he enjoys the ideological benefit (from 1 to $F_{-i}(s)$). Second, there is an additional term from when his opponent's policy has higher a score and is instead selected. Equation 3 illustrates two crucial properties of the competitive model: (1) a developer's choice of score and ideology *cannot be disentangled* (because a policy's score s determines the probability $F_{-i}(s)$ that developer i will enjoy the ideological benefit from crafting it), and (2) developing a higher score policy than s_0 is *strategically productive* because it increases the chance that the policy "wins."

Finally, differentiating eqn. (3) and setting equal to zero (first with respect to y, and then with respect to s) yields the ideology functions $y_i(s)$ and a system of differential equations characterizing any equilibrium score CDFs $F_i(s)$ (a more complete statement of the following is in Appendix A):

Proposition A.1 *In equilibrium,* $y_i(s) = \frac{x_i}{\alpha_i}F_{-i}(s)$, $F_k(0) > 0$ *for at most one developer* $k \in \{-1,1\}$, $F_L(\bar{s}) = F_R(\bar{s}) = 1$, *and* $\alpha_i - F_{-i}(s) = f_{-i}(s)\cdot 2x_i\left(y_i(s) - y_{-i}(s)\right)$ $\forall s \in [0,\bar{s}]$ and $i \in \{-1,1\}$.

The key difficulty in solving the system in Proposition A.1 is that it is *coupled*, since each developer i's objective function contains both *his opponent's* score CDF $F_{-i}(s)$ (which determines the probability that a score s policy will be successful) and *his own* score CDF $F_i(s)$ (which determines the ideology of the marginal score-s policy $y_{-i}(s) = \frac{x_{-i}}{\alpha_{-i}} F_i(s)$ that he will defeat if he is increases his score). This mutual dependence in the system of differential equations is not present in the asymmetric all-pay contest, but arises naturally from our key assumptions that the developers both choose their policy's ideology and care about ideology when they lose. A key contribution of our analysis relative to Hirsch and Shotts (2015) is to derive a closed-form solution to this coupled system (see Appendix B for details), which permits constructive proofs of equilibrium existence and uniqueness, and a comparative statics analysis of equilibrium policies, outcomes, and utilities.

Equilibrium

In equilibrium each developer mixes smoothly over a continuum of policies with ideologies in between their own ideal point and that of the decisionmaker. A developer's mixed strategy can be written as a pair of functions $(q_i(\delta), G_i(\delta))$ that describe: (1) the level of quality $q_i(\delta)$ that developer i produces when she crafts a policy whose ideology is *distance* δ from the decisionmaker, (2) a *cumulative distribution function* $G_i(\delta)$ that describes the *probability* developer i crafts a policy with ideology weakly closer to the decisionmaker than δ. The equilibrium values of these functions are as follows (see Appendix B for a detailed derivation).

Proposition 1. *For each developer $i \in \{-1, 1\}$, define the strictly **decreasing** function*

$$\epsilon_i(p) = \int_p^1 \frac{|x_i|}{\alpha_i - q} dq = |x_i| \log\left(\frac{\alpha_i - p}{\alpha_i - 1}\right).$$

Let $p_i(\epsilon) = \alpha_i - (\alpha_i - 1) e^{\frac{\epsilon}{|x_i|}}$ denote the well-defined inverse of $\epsilon_i(p)$, and let k denote the developer with the smallest value of $\epsilon_i(0)$.

- *When developer i crafts a policy whose ideology is distance δ from the decisionmaker, he targets* *ideology $i\delta$ and attaches quality $q_i(\delta) = \frac{\delta^2 + s_i(\delta)}{\lambda}$, where*

$$s_i(\delta) = 2\int_{\epsilon_i\left(\frac{i\delta}{x_i/\alpha_i}\right)}^{\epsilon_k(0)} \left(\sum_{j\in\{-1,1\}} \frac{|x_j|}{\alpha_j} p_j(\epsilon)\right) d\epsilon$$

- *The probability that developer i crafts a policy closer to the decisionmaker than δ is*

$$G_i(\delta) = p_{-i}\left(\epsilon_i\left(\frac{i\delta}{x_i/\alpha_i}\right)\right) = \alpha_{-i} - (\alpha_{-i} - 1)\left(\frac{x_i - i\delta}{x_i - x_i/\alpha_i}\right)^{\left|\frac{x_i}{x_{-i}}\right|},$$

- *Developer $-k$ is always active, but developer k is inactive with probability $p_{-k}(\epsilon_k(0)) = \alpha_{-k} - (\alpha_{-k} - 1)\left(\frac{\alpha_k}{\alpha_k - 1}\right)^{\left|\frac{x_k}{x_{-k}}\right|}$*

Although the equilibrium strategies are straightforward to express, they are somewhat hard to interpret from the equations alone. We therefore describe the structure of equilibrium, alongside six key properties, with the aid of an example. Figure 2 depicts equilibrium strategies and outcomes when the developers are equally skilled ($\alpha_1 = \alpha_{-1}$), but the right developer is more extreme ($|x_1| > |x_{-1}|$).

Properties of Equilibrium

1. Uncertainty Although the game is of complete and perfect information, in the unique equilibrium each developer is uncertain about exactly which ideology their competitor will target, how much they will invest in its quality, and whose policy will be chosen. This uncertainty remains regardless of how asymmetric the developers are – there is always some chance that the winning policy comes from either developer. This fundamental unpredictability arises from the need for both developers to remain competitive in the policy process when each can ensure victory over a known competing policy by investing sufficiently in quality and/or making sufficient ideological concessions.

2. Asymmetric Participation The unique equilibrium generically exhibits asymmetric participation in the policy process. Specifically, one of the two developers (in the example the right developer) is always *active* in the sense of developing a new policy with strictly positive quality. Moreover, any such policy is strictly better for the decisionmaker than $(0, 0)$ (her ideal point with 0-quality), even if

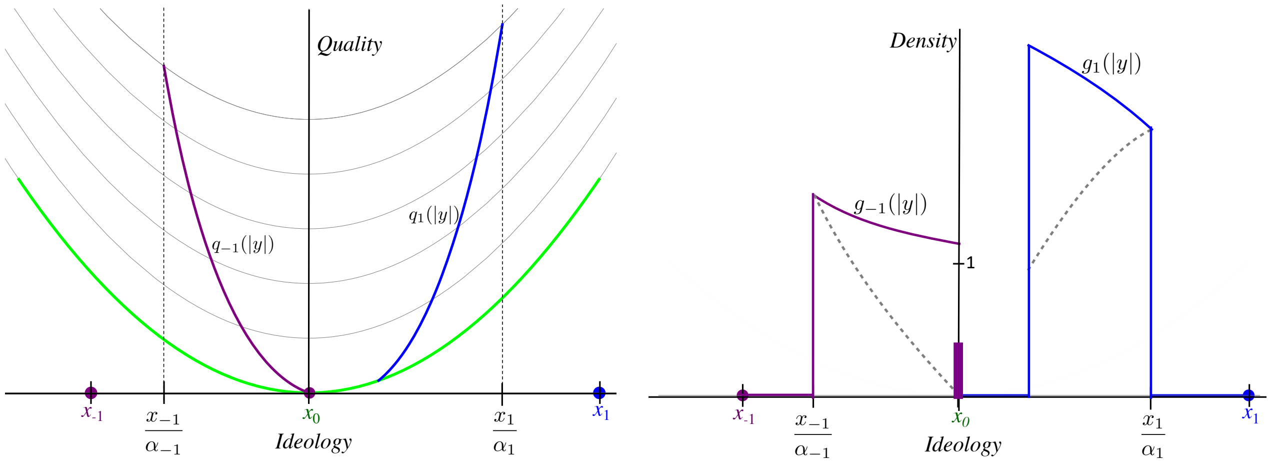

Figure 2: *Equilibrium with Asymmetric Extremism.* The left panel depicts the ideology and quality of the policies that the left (purple) and right (blue) developers randomize over. The decisionmaker's indifference curves are in gray. The right panel depicts the probability distributions (PDFs) governing the ideology of the left (purple) and right (blue) developer's policies. *When* a developer chooses to craft a new policy, its ideology is continuously distributed over an interval with the depicted density. In the figure the left developer sometimes chooses to craft *no* new policy. This is depicted in the left panel by the purple dot at the origin (the DM's ideal point with zero quality), and the probability this occurs is depicted in the right panel by the height of the thick purple segment. Finally, the density over the ideology of the final policy chosen by the DM is depicted by the gray dashed lines.

her *actual* best outside option is strictly worse. The other developer (in the example the left developer, and more generally developer k defined in Proposition 1) is only *sometimes* active; with strictly positive probability he develops nothing. This asymmetry in participation arises from differences in the developers' underlying extremism and/or ability at crafting high quality policies.

3. Ideological Divergence The equilibrium exhibits ideological divergence in the policies developed. Specifically, *when* a developer chooses to craft a new policy, its ideology strictly diverges from the DM's ideal. (In Figure 2, all positive-quality policies have divergent ideologies). Active participation in the policy process is thus always accompanied by an attempt to extract "ideological rents" in the form of a policy closer to one's ideal than the DM's ideal. This property arises because quality is insufficiently valuable to the developers to create it for its own sake.

4. Unequal Benefits and Costs In equilibrium, new policies always diverge from the DM's ideal; selecting them thus entail an ideological cost for the DM. It is therefore not obvious whether, and when, the DM actually benefits from policy development. Indeed, when there is only a single developer the DM does not benefit at all, since a monopolist extracts *all* the benefits of quality in the form of ideological rents. Competition, however, strictly benefits the DM – *with certainty* the DM will receive at least one new policy that is strictly better for her than $(0, 0)$. This holds regardless of the developers' characteristics, and even when one developer is very unlikely to be active. This is visible in Figure 2 by observing that all positive-quality policies are above the DM's indifference curve through the origin, combined with the fact that at least one developer is always active.

It is also not obvious whether the developers benefit from or are harmed by competition, since a competitor develops ideologically-unappealing policies, but gains support for them by making productive investments in quality. Despite this, competition turns out to strictly harm the developers, in the sense that that each would strictly prefer to be the only one developing a new policy. The reason is that the developers invest in enough quality to compensate the DM for her ideological losses, but not enough to compensate their competitor.

5. Inefficiency Although competition benefits the decisionmaker, it is also inefficient in several ways. First, the expected ideology of the final policy generically differs from both the DM's ideal and the ideology that maximizes aggregate utility. Second, the ideology of the final policy is uncertain ex-ante, which harms all participants in the process due to risk aversion (its distribution is depicted by the dashed grey lines in the right panel of Figure 2). Finally, because the developers must make their quality investments before they know which policy will be chosen, all of the effort invested in the losing policy is wasted. These inefficiencies arise from the fact that policy is developed and chosen via a "contest," rather than an orderly process designed by the decisionmaker, with an ex-ante commitment to whose policy she will choose under what conditions.

6. A Bias Toward Extremism In equilibrium, the developers invest more in quality when developing a more extreme policy to ensure it remains competitive ($q_i'(\delta) > 0$). More surprising is that they actually invest *so much* more in the quality of more-extreme policies that such policies are also more appealing overall to the DM *when they are developed*, and therefore more likely to be chosen. (In the left panel of Figure 2, the developers' quality functions are steeper than the DM's indifference curves). The decisionmaker will thus appear to be *biased* towards extremism, in the sense that she will be more likely to choose a particular developer's policy the more *extreme* it is. In addition, the ideology of the final policy (distributed according to the gray dashed line in the right panel of Figure 2) will be even more extreme than the developers' initial policies.

The reason for these counterintuitive effects is that the developers are *strategic* when they choose whether to craft a more extreme policy. Specifically, a developer chooses his policy's combination of ideology and quality by trading off the benefit of winning with a more-extreme policy against the cost of producing the additional quality needed to preserve that policy's competitiveness. *When* he chooses to craft a policy that is better for the DM, ideological concessions become a costlier way to gain the DM's support relative to quality investments because the policy is more likely to actually become the final outcome. Reversing the statement, when a developer chooses to craft a policy that makes fewer concessions to the DM (i.e., that is more ideologically extreme), that policy must be more appealing to the DM, and therefore more likely to be chosen.

Having fully characterized equilibrium, we now study several special cases of the general model; proofs of all results may be found in Appendices C-D.

The Politics of Asymmetric Extremism

We first turn to the politics of *asymmetric extremism* by studying equilibrium when the developers are equally capable ($\alpha_1 = \alpha_{-1}$) but one is more ideologically extreme ($|x_i| \neq |x_{-i}|$) – we call the more extreme developer "the extremist" and the other "the moderate."

Proposition 2. *If the developers are equally skilled but i is more extreme ($|x_i| > |x_{-i}|, \alpha_i = \alpha_{-i}$),*

- *the extremist always develops a new policy, while the moderate only sometimes does*

- *the extremist's policy is first-order stochastically more extreme than the moderate's policy, but also first-order stochastically higher quality and better for the decisionmaker*
- *the extremist's policy is strictly more likely to be chosen*

Recall that an example of equilibrium with asymmetric extremism is depicted in Figure 2.

Proposition 2 first characterizes the form of asymmetric participation – it is the *extremist* who always develops a new policy, while the moderate (despite being better aligned with the decisionmaker) only sometimes does so. The extremist also develops a first-order stochastically more extreme policy than the moderate. Surprisingly, however, his policy actually performs better than the moderate's policy, because it is so much higher quality so as to be first-order stochastically better for the decisionmaker *despite* its greater extremism. What explains the extremist's dominance of the policy process despite his ideologically-extreme policy proposal? Simply put, it is because the extremist is *more motivated* – motivated to invest in enough quality to compensate the decisionmaker for a more extreme policy, and also motivated to craft a combination of ideology and quality that will prevent opponent's policy from being chosen.

We next examine what happens to the developers' policies when one's developer's underlying ideology *becomes* more extreme.

Proposition 3. *If developer i becomes intrinsically more ideologically extreme (higher $|x_i|$), his* ***own strategy*** *and his* ***opponent's strategy*** *are affected in the following ways:*

(Own strategy)

- *if he previously did not always develop a policy, he becomes strictly more likely to do so*
- *his policy becomes first-order stochastically more extreme*
- *his policy becomes first-order stochastically higher quality, better for the decisionmaker, and strictly more likely to be chosen*

(Opponent's strategy)

- *if he did not always develop a policy, he becomes strictly less likely to do so*
- *his policy become first-order stochastically more moderate*
- *there is no unambiguous first-order stochastic change to his policy's quality or appeal to the decisionmaker, but his policy becomes strictly less likely to be chosen*

Although the above comparative statics apply to any configuration of preferences and costs, they are easiest to discuss in the special case of purely asymmetric extremism ($\alpha_i = \alpha_{-1}$).

When a particular developer i's underlying preferences become unilaterally more extreme, the effect on his own behavior is quite natural. If he is initially the moderate, then he becomes strictly more likely to be active, since greater *balance* in the developers' ideological extremism results in greater overall participation. Regardless of whether he is initially the moderate or the extremist, his policy also becomes first-order stochastically more extreme, but also higher quality, better for the decisionmaker, and strictly more likely to be chosen. This reflects the fact that his more-extreme preferences both increase his motivation to *win with a more extreme policy*, but also to just to *win at all*; he therefore "overcompensates" for the greater extremism of his policy with quality to further increase his policy's appeal. While intuitive, this property is notably distinct from the standard asymmetric all-pay contest, where the strategy of the stronger player *does not change* when he becomes even stronger (Siegel (2009)), because it is entirely driven by the need to encourage the weaker player's participation in the contest.

More subtly, when developer i becomes intrinsically more extreme, his *competitor's policy* also changes, despite no change in the competitor's preferences and abilities. If developer i is initially the extremist and becomes yet more extreme, his competitor becomes strictly *less* likely to be active by virtue of his lower relative ideological motivation; thus, a greater imbalance in ideological extremism results in a greater imbalance in political participation. Moreover, as developer i becomes more extreme, his competitor also *moderates* his policy (first-order stochastically). This is driven *not* by the competitor's desire to make his policy more competitive when facing a more extreme developer, but rather his acceptance of the fact that he is less likely to win against such a developer and move the ideological outcome in his direction. Finally, there is no unambiguous first-order stochastic change in his policy's appeal to the decisionmaker (meaning that it does not become unambiguously better or worse overall); however, it does become unconditionally less likely to be chosen.

We last examine the effect of a developer i becoming more extreme on the other players' welfare, beginning with his competitor.

Proposition 4. *If developer i becomes intrinsically more ideologically extreme (higher $|x_i|$), the equilibrium utility of his competitor $-i$ decreases*

The effect of unilateral extremism on a competitor's welfare is thus unambiguous – despite the greater quality of the extremist's policy the competitor is harmed; the greater quality is insufficient to compensate the competitor for the policy's greater extremism. While intuitive, this effect also differs starkly from the standard asymmetric all-pay contest, in which the payoff of the "weaker" player is unaffected by the characteristics of the stronger one (Siegel (2009)).

Finally, the effect of unilateral extremism on the decisionmaker's welfare is quite striking.

Proposition 5. *Unilateral changes in extremism have the following effects on the decisionmaker.*

- *If the developers are symmetrically capable and extreme ($|x_i| = |x_{-i}|, \alpha_i = \alpha_{-i}$) and developer i becomes intrinsically more extreme, the decisionmaker's utility locally increases*
- *As a developer becomes intrinsically more extreme ($|x_i| \to \infty$), the competitor's probability of developing a policy approaches 0, but the decisionmaker's utility approaches infinity*

While characterizing the precise local effect of a more-extreme developer is difficult, the broader relationship is simple and striking; the decisionmaker strongly benefits from unilateral extremism.

If the developers begin symmetrically capable and extreme and one becomes more extreme, the decisionmaker benefits despite the resulting imbalance in participation. And as a developer becomes increasingly extreme, the decisionmaker becomes increasingly – and even unboundedly – better off; even though his competitor also becomes vanishingly likely to participate. Unilateral extremism thus strongly benefits the decisionmaker, even though it decreases (and in the limit eliminates) observable competition. This contrasts strikingly with the standard all-pay contest, in which asymmetries can only harm the decisionmaker because a "stronger" player becoming stronger does not change his own strategy, but does discourage his weaker competitor (Hillman and Riley (1989)). In our model, this "discouragement effect" is still present, but in addition, an increasingly extreme developer crafts an increasingly appealing policy because he becomes increasingly fearful of losing out to a competing policy (even though such a policy also becomes vanishingly unlikely to materialize).[7] This beneficial effect of unilateral extremism is strong enough to outweigh the cost of the discouragement effect.[8]

[7]However, if our model is altered so that each developer receives a fixed payoff from losing, then an increasingly extreme developer crafts no better policies for the decisionmaker once he is the stronger player, so the decisionmaker is harmed by asymmetries. See Appendix E for details.

[8]A broader contest theory literature studies the implications of the discouragement effect (Chowdhury, Esteve-Gonzalez and Mukherjee (2022)). In most "perfectly discriminating" contests (where the outcome is fully determined by the players' strategies, like the all pay contest) decisionmakers are harmed by asymmetries because of the discouragement effect – ours is a notable exception. However, if there is enough "noise" in the outcome, then the decisionmaker can also benefit from large asymmetries in an otherwise standard model. For example, in a Tullock (1980) contest, total effort approaches zero as one player's prize value approaches ∞ (meaning the decisionmaker is harmed from large asymmetries) unless there are weakly decreasing returns to scale ($r \leq 1$); this generates sufficient uncertainty in the outcome to tamp down the strength of the discouragement effect.

The Politics of Asymmetric Ability

We last examine the politics of *asymmetric ability* by studying equilibrium when the developers are equally extreme ($|x_i| = |x_{-i}|$) but one is more skilled at producing quality ($\alpha_i < \alpha_{-i}$); we call the more capable developer "the expert" and the other "the amateur."

Proposition 6. *If the developers are equally extreme but i is more skilled ($|x_i| = |x_{-i}|, \alpha_i < \alpha_{-i}$), then the equilibrium pattern of competition is identical to the pattern described in Proposition 2 in which the developers are equally capable but i is more extreme ($|x_i| > |x_{-i}|, \alpha_i = \alpha_{-i}$).*

Asymmetric ability and extremism are thus effectively *observationally equivalent*, as depicted in Figure 3. That is, the expert exploits his greater ability at crafting high quality policies to craft a more competitive but also more extreme policy, consistent with the finding in Hitt, Volden and Wiseman (2017) that "more effective lawmakers" (i.e., with lower α_i) "are more likely to offer successful proposals." The amateur reacts by both disengaging from policy development, and by moderating his policy when he crafts one. The key empirical implication is that observably-extreme *behavior* by one political faction may not actually reflect greater underlying extremism, but rather greater ability at crafting "good policies" that are appealing on non-ideological grounds.

The observational equivalence between asymmetric extremism and ability also extends to several consequences of one developer becoming more skilled.

Proposition 7. *If developer i becomes more skilled (lower α_i)*

- *his own strategy and his opponent's strategy are affected in the same ways as when he becomes more ideologically extreme (higher $|x_i|$)*
- *the equilibrium utility of his competitor decreases.*

A developer becoming more skilled thus increases his own activity (if he was the amateur) and makes his policy more extreme, higher quality, and better for the decisionmaker. It further decreases his

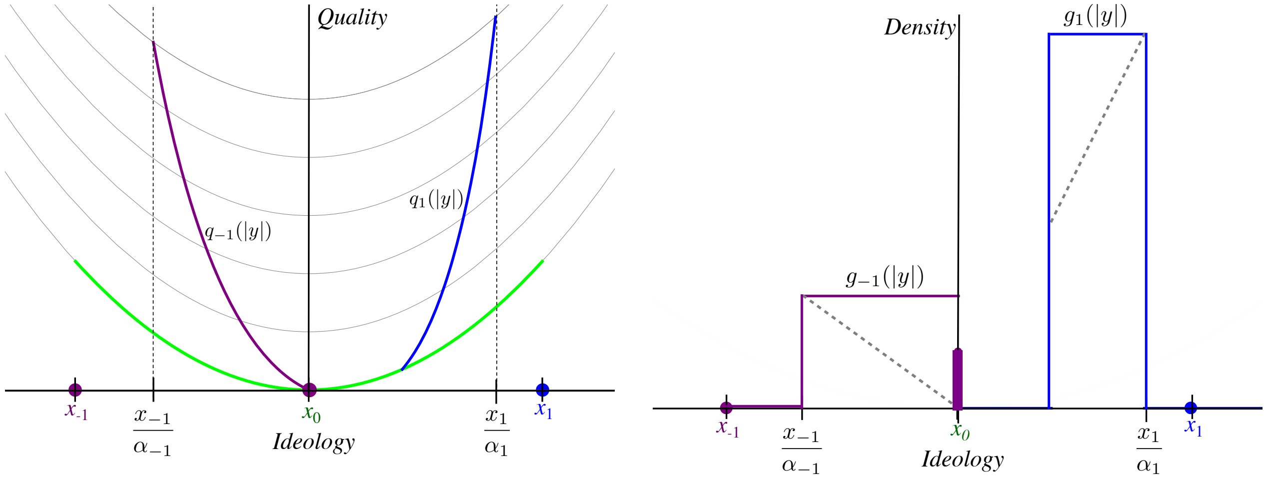

Figure 3: *Equilibrium Strategies with Asymmetric Ability.* The left panel depicts the ideology and quality of the policies that the left and right developers randomize over. The right panel depicts the PDFs governing the ideology of the left (purple) and right (blue) developer's policies; the density over the ideology of the final policy is depicted by the gray dashed lines.

competitor's activity (if he was the amateur) and makes his policy more moderate. Finally, it harms his competitor. That the isomorphism between greater skill and extremism extends to developer welfare is surprising, given that the skill in question is making *common value policy investments that benefit everyone*. Indeed, it is a striking demonstration of how "good policy" considerations cannot really be considered separately from "ideological" ones even if are *theoretically* distinct, because of how strategic actors will exploit their skill at crafting "good policy" to achieve their ideological goals.

We conclude by examining how unilateral changes in ability effect the decisionmaker's welfare.

Proposition 8. *Unilateral changes in ability have the following effects on the decisionmaker.*

- *If the developers begin symmetric ($|x_i| = |x_{-i}|, \alpha_i = \alpha_{-i}$) and developer i becomes more skilled, then the decisionmaker's utility locally increases.*

- *As a developer becomes increasingly skilled ($\alpha_i \to 1$), the competitor's probability of developing a policy approaches 0, but the decisionmaker's utility approaches a strictly positive bound; this bound is strictly increasing in the inactive developer's extremism and ability.*

The decisionmaker thus benefits when a developer becomes unilaterally more skilled even though he crafts a more extreme policy and his competitor participates less. As one developer becomes increasingly skilled ($\alpha_i \to 1$), the effect again resembles that of a developer becoming increasingly extreme ($|x_i| \to \infty$), but with some notable differences. As before, the "weaker" developer (here the amateur, previously the moderate) is eventually driven out, but the decisionmaker still benefits from his *potential* participation (in the sense that her utility is bounded away from her utility under monopoly). The decisionmaker's utility doesn't increase unboundedly, however; rather, it approaches a strictly positive value that is generally higher than her utility under symmetry.[9] The benefit of an asymmetrically skilled developer once again stems from our central but realistic assumption that the developers care about policy even when they lose. Finally, this value depends on the traits of the amateur; even when the amateur is effectively driven out of development the expert still reacts to him becoming more extreme or capable by crafting a more appealing policy. The key empirical implication is that the characteristics of a seemingly irrelevant participant in the policy process can still critically influence behavior and outcomes. Interestingly, this property does not require an agenda procedure whereby the developers craft policies *sequentially* (as in Hitt, Volden and Wiseman (2017) and Lax and Cameron (2007)), so that one tries to "deter" the other from participating.

Discussion and Conclusion

We have analyzed a model of strategic policy development by competing actors with differing ideologies and abilities. We show that the process exhibits several intuitive patterns including unequal participation, inefficiently unpredictable and extreme policies and outcomes, wasted effort, and an apparent bias toward extreme policies. We further explore the politics of asymmetric extremism and ability, and find that despite increasingly imbalanced policies and outcomes, and even seemingly absent competition against an extreme and/or capable faction, a moderate decisionmaker nevertheless

[9] Specifically, if the developers begin symmetric and developer i becomes arbitrarily skilled, the decisionmaker will be better off than under symmetry as long as $\alpha_{-i} \geq \underline{\alpha} \approx 1.0435$.

strongly benefits from the potential for competition (although at the expense of the weaker faction).

Our model provides a novel rationale for how ideological extremism may come to dominate policymaking; one rooted in the nature of *productive* policy competition rather than dysfunction, bias, capture, or some other systemic failure. It further shows that extremism of outcomes (regardless of the cause) may not necessarily come at the expense of ideological centrists; rather, the level of dysfunction commonly attributed to contemporary US policymaking also requires a willingness to use *destructive* means of competition (like policy sabotage) that are absent from our model. Our model further illustrates how asymmetrically extreme *behavior* may result from asymmetric *ability* rather than asymmetric preferences, which has implications for how political scientists measure the underlying preferences of political actors from their observed behavior. Finally, it shows how ostensibly nonpartisan "good policy" considerations and partisan ideological ones are inextricably linked because of how strategic actors exploit ability at crafting good policy to gain ideological influence, with implications for the design of political institutions to encourage effective policymaking.

We have modelled costly policy development as an individual or group making a policy-specific investment in quality that is valued by all participants in the policy process, in contrast to a large literature that models policy expertise as the acquisition of private knowledge about an "unknown state of the world." Our model is intentionally sparse and lacking in institutional detail in order to be applicable to the wide variety of settings in which competitive policy development occurs, including but not limited to legislatures, bureaucracies, and courts. Broadening the interpretation of disagreement beyond ideology in a classical left-right sense, the model is applicable to any environment where there are a mixture of competing and common interests, freedom among several individuals or groups to make proposals, and a decisionmaker who must make a single final choice.

Our model may thus be fruitfully applied to a variety of policymaking environments in addition to those previously mentioned. One is decisionmaking in small groups like Presidential cabinets (see also Hirsch and Shotts (2018)); when facing a specific crisis or immediate policy dilemma, a chief executive typically solicits proposals for how to proceed from his or her cabinet secretaries. These secretaries have a vested interest in good policy and the success of the administration, but also competing personal beliefs and/or interests for their respective departments (Kearns (2005)). Another is policy competition between government agencies such as branches of the military. A nation facing a security crises may seek to achieve a specific military goal; the joint chairman of the various service branches can solicit proposals from them. The branches all value successful military outcomes, but also seek to further their parochial interests (Zimmerman et al. (2019)). A third is policymaking in one-party states. An important principle of decisionmaking in communist regimes is "democratic centralism," or the idea that diverse interests within the party should be free to develop proposals for how to deal with a policy problem without fear of reprisal, but a central leader then has the authority to implement a final policy decision that all party members must obey (Angle (2005)).

Finally, our analysis suggests two broad avenues for follow-on work. The first is to include additional model elements to tailor it to specific settings. For example, what if there are not just two but many potential policy developers as in a legislature? It is straightforward to show that when ability is common there is always an equilibrium in which only the two most ideologically-extreme developers are active. However, there may be other equilibria with broader participation (Baye, Kovenock and de Vries (1996)), and ability may be very unevenly distributed among potential policy developers (Hitt, Volden and Wiseman (2017)). Or, what if the developers can also engage in unproductive or destructive activities alongside policy development? They may try to bribe the decisionmaker, engage in unproductive advertising, lobbying, or grassroots mobilization; harm the reputation of their competitor or their policy; or even sabotage its functioning. What if the policy developers are not individuals but teams (as in McCarty (2020)) – for example, aligned legislators and interest groups – who have common ideological interests, but must figure out how to distribute the costs of developing high quality policies that achieve those interests among them?

The second avenue (following the classical literature on policy expertise) is to consider how political institutions can be *designed* to encourage effective policymaking. What if the developers can actually be chosen by the decisionmaker, as in the literature on legislative committee composition (Krehbiel (1992))? What if there are existing policy developers – when would the decisionmaker want to subsidize their activities, and how? What if it is not the identities of the developers under consideration but that of the decisionmaker, as in the Presidential appointment of an agency head to consider proposals from career staff and outside groups (Lewis (2008))? Finally, what if the decisionmaker is not a unitary actor but a collective choice body, as in a legislature? What sorts of collective choice rules will best encourage the development of high quality legislation? We hope to explore these and other avenues in future work.

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Productive Policy Competition and Asymmetric Extremism

Online Appendix

This Appendix is divided into five parts. Appendix A is a general analysis of the model concluding with a statement of necessary and sufficient conditions for equilibrium. Appendix B derives the closed-form characterization of the equilibrium strategies given in main text Proposition 1. Appendix C analyzes properties of equilibrium using this characterization. Appendix D describes where to locate results in the main text propositions in the general model analysis in Appendices B-C. Appendix E analyzes a variant of the model in which developers have a fixed payoff from losing to isolate which properties of the main model are distinctively the result of purely policy-motivated developers.

Contents

A General Equilibrium Characterization

We begin with a (slightly) more general formulation of the model than stated in the main text. Two developers labelled -1 (left) and 1 (right) craft competing policies for consideration by a decisionmaker (DM), labelled player 0. A policy (γ, q) consists of an *ideology* $\gamma \in \mathbb{R}$ and a level of *quality* $q \in [0, \infty) = \mathbb{R}^+$. Utility over policies takes the form

$$U_i(\gamma, q) = \lambda q - (\gamma - X_i)^2,$$

where X_i is player $i's$ ideological ideal point, and λ is the weight all players place on quality. The developers' ideal points are on either side of the decisionmaker ($X_{-1} < X_D < X_1$).

The game is as follows. First, the developers simultaneously craft policies (γ_i, q_i); crafting a policy with quality q_i costs $c_i(q_i) = a_i q_i$, where $a_i > \lambda$. Second, the DM chooses one of the two policies or something else from an exogenous set of outside options $\mathbb{O}$, where $\mathbb{O}$ may contain the DM's ideal point with no quality $(0, 0)$ and/or policies that are strictly worse (and can be empty).

A.1 Preliminary Analysis

The game is a multidimensional contest in which the scoring rule applied to "bids" (γ, q) is just the DM's utility $U_D(\gamma, q) = \lambda q - (X_D - \gamma)^2$. To facilitate the analysis we thus reparameterize policies (γ, q) to be expressed in terms of (s, y), where $y = \gamma - X_D$ is the (signed) distance of a policy's ideology from the DM's ideal, and $s = \lambda q - y^2$ is the DM's utility for a policy or its *score*. The implied quality of a policy (s, y) is then $q = \frac{s+y^2}{\lambda}$. Using this we re-express the developers' utility and cost functions in terms of (s, y). Note that the decisionmaker's ideal point with 0-quality has exactly 0 score, and is the most competitive "free" policy to craft.

Definition A.1.

1. *Player i's utility for policy (s, y) is*

$$V_i(s, y) = U_i\left(y + X_D, \frac{s + y^2}{\lambda}\right) = -x_i^2 + s + 2x_i y$$

where $x_i = X_i - X_D$ is the (signed) distance of i's ideal from the DM.

2. *Developer i's cost to craft policy (s, y) is*

$$c_i\left(\frac{s + y^2}{\lambda}\right) = \frac{a_i}{\lambda}\left(s + y^2\right) = \alpha_i\left(s + y^2\right)$$

where $\alpha_i = \frac{a_i}{\lambda}$ is i's weighted marginal cost of generating quality.

Definition 1 reparameterizes policies into score and ideological distance (henceforth just ideology) (s, y), and the five primitives (X_i, a_i, λ) into four parameters (x_i, α_i) describing the developers' (signed) ideal ideological distance from the DM $x_i = X_i - X_D$ (henceforth just ideal ideology) and weighted marginal costs of generating quality $\alpha_i = \frac{a_i}{\lambda}$ (henceforth just costs).

A.1.1 Necessary and Sufficient Equilibrium Conditions

In the reparameterized game, a developer's pure strategy (s_i, y_i) is a two-dimensional element of $\mathbb{B} \equiv \left\{(s, y) \in R^2 \mid s + y^2 \geq 0\right\}$. A mixed strategy σ_i is a probability measure over the Borel subsets of $\mathbb{B}$, and let $F_i(s)$ denote the CDF over scores induced by i's mixed strategy σ_i.[10]

We now derive necessary and sufficient equilibrium conditions in a series of four lemmas. Let $\bar{\Pi}_i(s_i, y_i; \sigma_{-i})$ denote i's expected utility for crafting a policy (s_i, y_i) with $s_i \geq 0$ *if* a tie would be broken in his favor. Clearly this is i's expected utility from crafting a policy with any $s_i > 0$ where $-i$ has no atom, and i can always achieve utility arbitrarily close to $\bar{\Pi}_i(s_i, y_i; \sigma_{-i})$ by crafting $\varepsilon-$higher

[10] For technical convenience we restrict attention to strategies generating score CDFs that can be written as the sum of an absolutely continuous and a discrete distribution.

score policies. Now $\bar{\Pi}_i(s_i, y_i; \sigma_{-i}) =$

$$-\alpha_i\left(s_i + y_i^2\right) + F_{-i}(s_i) \cdot V_i(s_i, y_i) + \int\limits_{s_{-i} > s_i} V_i(s_{-i}, y_{-i})\, d\sigma_{-i}. \tag{A.1}$$

The first term is the up-front cost of generating the policy's quality. The second term is the probability i's policy is selected, times his utility for it. The third term is i's utility should he lose, which requires integrating over all the policies in the support of his opponent's mixed strategy with score higher than s_i. Taking the derivative with respect to y_i and setting equal to 0 yields the first Lemma.

Lemma A.1. *At any score $s_i > 0$ where $F_{-i}(\cdot)$ has no atom, the policy $(s_i, y_i^*(s_i))$, where $y_i^*(s_i) = F_{-i}(s_i) \cdot \frac{x_i}{\alpha_i}$, is the strictly best score-s_i policy.*

Proof: Straightforward. QED

Lemma A.1 states that at almost every score $s_i > 0$, developer i's unique best combination of ideology and quality to generate that score is just a weighted average of the developer's and DM's ideal ideologies $\frac{x_i}{\alpha_i}$, multiplied by the probability $F_{-i}(s_i)$ that i's opponent crafts a lower-score policy. Note that i's optimal ideology does not depend directly on his opponent $-i$'s ideologies, since a policy's ideology (holding score fixed) only matters conditional on winning. The optimal ideology also depends on the exact score s_i only *indirectly* through probability $F_{-i}(s_i)$ the policy wins the contest, since i's utility conditional on winning is additively separable in score and ideology.

The second lemma establishes that at least one of the developers is always *active*, in the sense of crafting a policy with strictly positive score (all positive-score policies are positive-quality, but the reverse is not necessarily true).

Lemma A.2. *In equilibrium $F_k(0) > 0$ for at most one k.*

Proof: Suppose not, so $F_i(0) > 0\ \forall i$ in some equilibrium. Let U_i^* denote developer i's equilibrium utility, which can be achieved by mixing according to his strategy conditional on crafting a score-$s \leq 0$ policy. Let $\bar{y}^0$ denote the expected ideological outcome and $\bar{s}^0$ the expected score outcome conditional on both sides crafting score ≤ 0 policies. Since $x_L < 0 < x_R$, we have $V_k\left(\bar{s}^0, \bar{y}^0\right) \leq V_k(0,0)$ for at least one k, which implies k has a profitable deviation since $U_k^* \leq \bar{\Pi}_k(0,0;\sigma_{-k}) < \bar{\Pi}_k\left(0, y_k^*(0); \sigma_{-k}\right)$ (since $F_{-k}(0) > 0$). QED

The third Lemma establishes that in equilibrium there is 0 probability of a tie at a positive score.

Lemma A.3. *In equilibrium there is 0-probability of a tie at scores $s > 0$.*

Proof: Suppose not, so each developer's strategy generates an atom of size $p_i^s > 0$ at some $s > 0$. Developer i achieves his equilibrium utility U_i^* by mixing according to his strategy conditional on a score-s policy. Let $\bar{y}^s$ denote the expected ideological outcome conditional on both sides crafting score-s policies; then $V_k(s, \bar{y}^s) \leq V_k(s, 0)$ for at least one k, who has a profitable deviation. If k's policy at score s is $(s, 0)$, then $U_k^* \leq \bar{\Pi}_k(s, 0; \sigma_{-k}) < \bar{\Pi}_k\left(s, y_k^*(s); \sigma_{-k}\right)$ (since $F_{-k}(s) > 0$). If k sometimes crafts something else, then $U_k^* < \left(1 - \frac{p_{-k}}{F_{-k}(s)}\right) \bar{\Pi}_k\left(s, E[y_k|s]; \sigma_{-k}\right) + \left(\frac{p_{-k}}{F_{-k}(s)}\right) \bar{\Pi}_k(s, 0; \sigma_{-k})$, which is k's utility if he were to instead craft $(s, 0)$ with probability $\frac{p_{-k}}{F_{-k}(s)}$, and the expected ideology $E[y_k|s]$ of his strategy at score s with the remaining probability (and always win ties). QED

Lemmas A.1 – A.3 jointly imply that in equilibrium, developer i can compute his expected utility *as if* his opponent only crafts policies of the form $\left(s_{-i}, y_{-i}^*(s_{-i})\right)$. The utility from crafting *any* policy (s_i, y_i) with $s_i > 0$ where $-i$ has no atom (or a tie would be broken in i's favor) is therefore

$$\bar{\Pi}_i^*(s_i, y_i; F) = -\alpha_i\left(s_i + y_i^2\right) + F_{-i}(s_i) \cdot V_i(s_i, y_i) + \int_{s_i}^{\infty} V_i\left(s_{-i}, y_{-i}^*(s_{-i})\right) dF_{-i}. \tag{A.2}$$

Developer i's utility from crafting the *best* policy with score s_i is $\bar{\Pi}_i^*\left(s_i, y_i^*(s_i); F\right)$, which we henceforth denote $\bar{\Pi}_i^*(s_i; F)$.

Fourth and finally, we establish that equilibrium score CDFs must satisfy the following natural properties arising from the all pay component of the contest.

Lemma A.4. *Support of the equilibrium score CDFs over $\mathbb{R}^+$ is common, convex, and includes* 0.

Proof: We first argue $\hat{s} > 0$ in support of $F_i \rightarrow F_{-i}(s) < F_{-i}(\hat{s})\ \forall s < \hat{s}$. Suppose not; so $\exists s < \hat{s}$ where $-i$ has no atom and $F_{-i}(s) = F_{-i}(\hat{s})$. Then $\bar{\Pi}_i(\hat{s}, y_i; F) - \bar{\Pi}_i(s, y_i; F) = -(\alpha_i - F_{-i}(\hat{s})) \cdot (\hat{s} - s) < 0$, implying i's best score-s policy is strictly better than his best score-$\hat{s}$ policy, a contradiction. We now argue this yields the desired properties. First, an $\hat{s} > 0$ in i's support but not $-i$ implies $\exists\ \delta > 0$ s.t. $F_{-i}(s - \delta) = F_{-i}(s)$. Next, if the common support were not convex or did not include 0, then there would $\exists\ \hat{s} > 0$ in the common support s.t. neither developer has support just below, so $F_i(s) < F_i(\hat{s})\ \ \forall i,\ s < \hat{s}$ would imply both developers have atoms at $\hat{s}$, a contradiction. QED

We conclude by combining the preceding lemmas to provide a preliminary characterization of all equilibria in the form of necessary and sufficient conditions.

Proposition A.1. *Necessary conditions for SPNE are as follows:*

1. ***(Ideological Optimality)*** *With probability* 1*, policies are either*

   (a) *negative score* $s_i \leq 0$ *and* 0*-quality* $\left(s_i + y_i^2 = 0\right)$

   (b) *positive score* $s_i > 0$ *with ideology* $y_i = y_i^*(s_i) = \left(\frac{x_i}{\alpha_i}\right) F_{-i}(s_i)$.

2. ***(Score Optimality)*** *The profile of score CDFs* (F_i, F_{-i}) *satisfy the following boundary conditions and differential equations.*

   - ***(Boundary Conditions)*** $F_k(0) > 0$ *for at most one developer* k*, and there* $\exists \bar{s} > 0$ *such that* $\lim_{s \rightarrow \bar{s}} \{F_i(s)\} = 1\ \forall i$.

   - ***(Differential Equations)*** *For all* i *and* $s \in [0, \overline{s}]$,

$$\alpha_i - F_{-i}(s) = f_{-i}(s) \cdot 2x_i \left(y_i^*(s) - y_{-i}^*(s)\right)$$

The above and $F_i(s) = 0\ \forall i, s < 0$ *are sufficient for equilibrium.*

Proof: (*Score Optimality*) A score $\hat{s} > 0$ in the common support implies $[0, \hat{s}]$ in the common support (by Lemma A.4) implying $\lim_{s \to \hat{s}^-} \left\{ \bar{\Pi}_i (s; F) \right\} \geq U_i^*$. Equilibrium also requires $\bar{\Pi}_i (s; F) \leq U_i^*$ $\forall s$ so $\bar{\Pi}_i (s; F) = U_i^*$ $\forall s \in [0, \bar{s}]$, further implying the F's are absolutely continuous over $(0, \infty)$ (given our initial assumptions), and therefore $\frac{\partial}{\partial s} (\Pi_i^* (s; F)) = 0$ for almost all $s \in [0, \bar{s}]$. This straightforwardly yields the differential equations for score optimality, with the boundary conditions implied by Lemma A.4. (*Ideological Optimality*) At most one developer k crafts ≤ 0-score policies with positive probability, so $F_{-k} (0) = 0$. Such policies lose for sure and never influence a tie, and therefore must be 0-quality with probability 1, yielding property (a). Atomless score CDFs $\forall s > 0$ implies $(s, y_i^* (s))$ is the strictly best score-s policy (by Lemma A.1), yielding property (b). (*Sufficiency*) Necessary conditions imply all $(s, y_i^* (s))$ with $s \in (0, \bar{s}]$ yield a constant U_i^*. $F_{-k} (0) = 0$ implies k's strictly best score-0 policy is $(0, y_k^* (0)) = (0, 0)$ and yields $\bar{\Pi}_k (0; F)$, and $F_k (s) = 0$ for $s < 0$ implies k has a size $F_k (0)$ atom here. Thus both developers' mixed strategies yield U_i^*, and neither can profitably deviate to $s \in (0, \bar{s}]$. To see neither can profitably deviate to $s > \bar{s}$, observe $\Pi_i^* (s; F) - \Pi_i^* (\bar{s}; F) = -(\alpha_i - 1)(s - \bar{s}) < 0$. To see k cannot profitably deviate to $s_k \leq 0$, $F_{-k} (0) = 0$ implies such policies lose and never influence a tie, and so yield utility $\leq U_k^*$. To see $-k$ cannot profitably deviate to $s_{-k} \leq 0$, observe all such policies result in either $(0, y_{-k})$ or $(0, 0)$ when $s_k \leq 0$ (since the DM's other choices are $(0, 0)$ and $\mathbb{O}$), and thus yield utility $\leq \max \left\{ \bar{\Pi}_{-k} (0, 0; F), \bar{\Pi}_{-k} (0, y_{-k}; F) \right\}$ which is $\leq U_{-k}^*$. QED

A.1.2 Preliminary Observations about Equilibria

Proposition A.1 implies that all equilibria have a simple form. At least one developer (henceforth labelled $-k$) is always active – thus, competition not only strictly benefits the DM in expectation, but with probability 1. The other developer (henceforth labelled k) may also always be active ($F_k (0) = 0$), or be inactive with strictly positive probability ($F_k (0) > 0$). Inactivity may manifest as crafting the DM's ideal point with no quality $(0, 0)$, or as "position-taking" with more distant 0-quality policies that lose for sure ($s_k < 0$ and $s_k + y_k^2 = 0$). However, any equilibrium exhibiting the latter is payoff-equivalent to one exhibiting the former; we thus focus on the former for comparative statics.[11] When either developer i is active, he mixes smoothly over the ideologically-optimal policies $\left(s, \frac{x_i}{\alpha_i} F_{-i}(s)\right)$ with scores in a common mixing interval $[0, \bar{s}]$ according to the CDF $F_i(s)$.[12]

The differential equations characterizing the equilibrium score CDFs arise intuitively from the developers' indifference condition over $[0, \bar{s}]$. The left hand side is $i's$ net marginal cost of crafting a higher-score policy given a fixed probability $F_{-i}(s)$ of winning the contest; the developer pays marginal cost $\alpha_i > 1$ for sure, but with probability $F_{-i}(s)$ his policy is chosen and he enjoys a marginal benefit of 1 (because he values quality). The right hand side represents $i's$ marginal ideological benefit of increasing his score. Doing so increases by $f_{-i}(s)$ the probability that his policy wins, which changes the ideological outcome from his opponent's optimal ideology $y^*_{-i}(s)$ at score s to his own optimal ideology $y^*_i(s)$ at score s.

B Closed Form Equilibrium Characterization

The first and most critical step in generating a unique closed form equilibrium characterization and analytically examining its properties is to use the coupled system of differential equations that characterize any pair of equilibrium score CDFs $(F_L(s), F_R(s))$ to derive a simple *functional relationship* that must hold between them.

Lemma B.1. *In any SPNE,* $\epsilon_i(F_{-i}(s)) = \epsilon_{-i}(F_i(s))\ \forall s \geq 0$*, where*

$$\epsilon_i(p) = \int_p^1 \frac{|x_i|}{\alpha_i - q} dq = |x_i| \log\left(\frac{\alpha_i - p}{\alpha_i - 1}\right)$$

Proof: Rearranging the differential equation in score optimality yields $\frac{f_{-i}(s)\cdot|x_i|}{\alpha_i - F_{-i}(s)} = \frac{f_i(s)\cdot|x_{-i}|}{\alpha_{-i} - F_i(s)}$

[11] Profiles with "position-taking" are equilibria if the position-taking does not invite a deviation by $-k$ to negative scores; whether this is the case depends on k's score-CDF below 0 and the DM's outside options $\mathbb{O}$. When $(0,0) \in \mathbb{O}$ the necessary conditions are also sufficient.

[12] Technically, the proposition does not state that the support interval is also bounded ($\bar{s} < \infty$), but this is later shown indirectly through the analytical equilibrium derivation.

$\forall s \in [0, \bar{s}] \rightarrow \int_s^{\bar{s}} \frac{f_{-i}(s)\cdot|x_i|}{\alpha_i - F_{-i}(s)} ds = \int_s^{\bar{s}} \frac{f_i(s)\cdot|x_{-i}|}{\alpha_{-i} - F_i(s)} ds$ $\forall s \in [0, \bar{s}]$; a change of variables and the boundary condition $F_i(\bar{s}) = 1$ yields $\int_s^{\bar{s}} \frac{f_{-i}(s)\cdot|x_i|}{\alpha_i - F_{-i}(s)} ds = \int_{F_{-i}(s)}^{1} \frac{|x_i|}{\alpha_i - q} dq = \epsilon_i(F_{-i}(s))$. The relationship holds trivially for $s > \bar{s}$. QED

We refer to the property in Lemma B.1 as the *engagement equality.* To see why, observe that the decreasing function $\epsilon_i(p)$ captures i's relative willingness to deviate from a policy that wins with probability p to one that wins for sure (since the marginal ideological benefit of moving the ideological outcome in his direction is $|x_i|$, and the net marginal cost of increasing score on a policy winning the contest with probability q is $\alpha_i - q$). We call this function i's *engagement at probability* p. The engagement equality $\epsilon_i(F_{-i}(s)) = \epsilon_{-i}(F_i(s))$ states that at every score $s \geq 0$ both developers must be equally engaged given the resulting probabilities of winning the contest, and therefore equally willing to deviate to the maximum score $\bar{s}$. It is easily verified that $\epsilon_i(1) = 0$ $\forall i$ and $\epsilon_i(p)$ is strictly increasing in $|x_i|$ and decreasing in α_i $\forall p \in [0, 1)$.

Usefully, the engagement equality implies a simple functional relationship between the developers' score CDFs that must hold in equilibrium regardless of their exact values. Letting

$$p_i(\epsilon) = \alpha_i - (\alpha_i - 1) e^{\frac{\epsilon}{|x_i|}}$$

denote the inverse of $\epsilon_i(p)$ (which is decreasing in p, increasing in $|x_i|$, and decreasing in α_i) equilibrium then requires that $F_i(s) = p_{-i}(\epsilon_i(F_{-i}(s)))$ $\forall s \in [0, \bar{s}]$.

B.1 Identity of developer k and probabilities of participation

We first use the engagement equality to derive the identity of the sometimes-inactive developer k and the probability $F_k(0)$ that he is sometimes inactive, and perform comparative statics on $F_k(0)$.

Proposition B.1. *In equilibrium* $k \in \arg\min_i \{\epsilon_i(0)\}$ *and*

$$F_k(0) = p_{-k}(\epsilon_k(0)) = \alpha_{-k} - (\alpha_{-k} - 1)\left(\frac{\alpha_k}{\alpha_k - 1}\right)^{\left|\frac{x_k}{x_{-k}}\right|}.$$

The probability k is inactive $F_k(0)$ is decreasing in his distance from the DM $|x_k|$ and his opponent's quality costs α_{-k}, and increasing in his opponent's distance from the DM $|x_{-k}|$ and his own quality costs α_k. In addition, $\lim_{|x_k|\to 0}\{F_k(0)\} = \lim_{|x_{-k}|\to\infty}\{F_k(0)\} = \lim_{\alpha_k\to\infty}\{F_k(0)\} = \lim_{\alpha_{-k}\to 1}\{F_k(0)\} = 1$.

Proof: Suppose $\epsilon_k(0) < \epsilon_{-k}(0)$; then $F_k(0) = 0$ and the engagement equality would imply $F_{-k}(0) < 0$, a contradiction. Since $F_i(0) = 0$ for some i we must have $F_{-k}(0) = 0$ and $F_k(0) = p_{-k}(\epsilon_k(0)) > 0$. Comparative statics and limit statements follow from previous observations on $\epsilon_i(\cdot)$ and $p_i(\cdot)$. QED

The sometimes-inactive developer is thus the one with the lowest engagement at probability 0 – that is, who is least willing to participate in the contest entirely.

B.2 Equilibrium Score CDFs

With the engagement equality and the identity of the sometimes-inactive developer k we may next characterize the equilibrium score CDFs $F_i(s)$ satisfying Proposition A.1, which are shown constructively to be unique.

Proposition B.2. *The unique score CDFs over $s \geq 0$ satisfying Proposition A.1 are $F_i(s) = p_{-i}(\epsilon(s))\ \forall i$, where $\epsilon(s)$ is the inverse of*

$$s(\epsilon) = 2\int_{\epsilon}^{\epsilon_k(0)} \sum_j \frac{|x_j|}{\alpha_j} p_j(\hat{\epsilon})\, d\hat{\epsilon}.$$

The inverse score CDFs are $s_i(F_i) = s(\epsilon_{-i}(F_i))\ \forall i$, and the score targetted at each ideology is $s\left(\epsilon_i\left(\frac{y}{x_i/\alpha_i}\right)\right)$. The function $s(\epsilon)$ is strictly increasing in x_i and strictly decreasing in α_i $\forall \epsilon \in [0, \epsilon_k(0))$, and the maximum score is $\bar{s} = s(0)$.

Increasing a developer's extremism $|x_i|$ or decreasing his costs α_i first-order stochastically increases his own score CDF, but has ambiguous effects on his opponent's score CDF.

Proof: From the engagement equality $\epsilon_i(F_{-i}(s)) = \int_{F_{-i}(s)}^{1} \frac{|x_i|}{\alpha_i - q} dq = \epsilon(s)\ \forall i, s$ for some $\epsilon(s)$. We characterize the unique $\epsilon(s)$ implying score CDFs $F_i(s) = p_{-i}(\epsilon(s))$ and optimal ideologies

$y_i(s) = \frac{x_i}{\alpha_i} p_i(\epsilon(s))$ that satisfy score optimality. First observe that $\epsilon'(s) = f_i(s)\epsilon'_{-i}(F_i(s)) = -\frac{f_{-i}(s)|x_i|}{\alpha_i - F_{-i}(s)}$. Next the differential equations may be rewritten as $\frac{\alpha_i - F_{-i}(s)}{f_{-i}(s)\cdot|x_i|} = 2\sum_j y_j(s)$. Substituting the preceding observations into both sides yields $\frac{1}{\epsilon'(s)} = -2\sum_j \frac{x_j}{\alpha_j} p_j(\epsilon(s))$, and rewriting in terms of the inverse $s(\epsilon)$ yields $s'(\epsilon) = -2\sum_j \frac{|x_j|}{\alpha_j} p_j(\epsilon)$. Lastly $\epsilon_k(F_{-k}(s)) = \epsilon(s)$ and $F_{-k}(0) = 0$ imply the boundary condition $s(\epsilon_k(0)) = 0$ so $s(\epsilon) = \int_{\epsilon}^{\epsilon_k(0)} -s'(\hat{\epsilon})\, d\hat{\epsilon} = 2\int_{\epsilon}^{\epsilon_k(0)} \sum_j \frac{|x_j|}{\alpha_j} p_j(\hat{\epsilon})\, d\hat{\epsilon}$. Now $s(\epsilon)$ is increasing in $|x_i|$ and decreasing in α_i given previous observations about $p_j(\hat{\epsilon})$. QED

The maximum score $\bar{s}$ thus changes continuously with the parameters of both developers even when one is dominant. This contrasts with the standard 2-player all pay contest, where the mixing interval is unaffected by the parameters of the stronger player. Increasing a developer's extremism $|x_i|$ or decreasing his costs α_i first-order stochastically increases his own score CDF, but has ambiguous effects on his opponent's score CDF. To see this, suppose that the always-active developer $-k$ becomes even more extreme or able. Then his opponent k becomes less likely to be active, but also the range of scores $[0, \bar{s}]$ over which he mixes *when* he is active increases. He thus has a higher probability of crafting a very high-score policy, even while he is simultaneously less likely to enter the contest.

B.3 Derivation of Strategies in Proposition 1

Finally we transform the preceding characterization of ideologically optimal policies and equilibrium score CDFs into the more intuitive characterization of equilibrium strategies provided in main text Proposition 1.

First, recall that a policy (s, y) has quality $q = \frac{y^2+s}{\lambda}$. Next, when a developer crafts a policy that is distance δ from the decisionmaker, its ideology is $i\delta$; consequently, the score at which developer i crafts policy $i\delta$ is $s\left(\epsilon_i\left(\frac{i\delta}{x_i/\alpha_i}\right)\right)$ by Proposition B.2. Combining the preceding, the quality associated with a policy that is distance δ from the decisionmaker is $q_i(\delta) = \frac{(i\delta)^2 + s\left(\epsilon_i\left(\frac{i\delta}{x_i/\alpha_i}\right)\right)}{\lambda}$, which simplifies to the expression in the proposition.

Next, the probability distribution over the extremism of each developer's policy can be simply derived from the engagement equality as follows.

Proposition B.3. *Let $G_i(y) = \Pr(|y_i| \leq \delta)$ denote the probability that i's policy is closer to the DM than δ. Then*

$$G_i(\delta) = p_{-i}\left(\epsilon_i\left(\frac{i\delta}{x_i/\alpha_i}\right)\right) = \alpha_{-i} - (\alpha_{-i} - 1)\left(\frac{x_i - i\delta}{x_i - x_i/\alpha_i}\right)^{\left|\frac{x_i}{x_{-i}}\right|},$$

which is first-order stochastically increasing in i's extremism $|x_i|$, decreasing in his costs α_i, decreasing in his opponent's extremism $|x_{-i}|$, and increasing in his opponent's costs α_i.

Proof: Developer i's ideology at score s is $y_i^*(s) = \frac{x_i}{\alpha_i} F_{-i}(s)$ (from ideological optimality), so $F_{-i}(s_i^*(y)) = \frac{y}{x_i/\alpha_i}$ where $s_i^*(y)$ is the inverse of $y_i^*(s)$. That is, the probability $-i$ crafts a policy with score $\leq s_i^*(y)$ is $\frac{y}{x_i/\alpha_i}$. Now the probability $G(\delta)$ that i crafts a policy closer to the DM than y is $F_i(s_i^*(i\delta))$, which is $= p_{-i}(\epsilon_i(F_{-i}(s_i^*(i\delta)))) = p_{-i}\left(\epsilon_i\left(\frac{i\delta}{x_i/\alpha_i}\right)\right)$ from the engagement equality. Comparative statics are straightforward. QED

C Additional Quantities and Comparative Statics

In this section we calculate and examine the general properties of additional equilibrium quantities; these propositions form the basis for the main-text propositions that study properties of the model in the special cases of pure asymmetric extremism and pure asymmetric ability.

C.1 Probabilities of Victory

We next use the engagement equality to derive the developers' probabilities of victory.

Proposition C.1. *In equilibrium the probability developer k loses the contest is*

$$\int_0^1 p_{-k}(\epsilon_k(p))\, dp = \int_0^1 \left(\alpha_{-k} - (\alpha_{-k} - 1)\left(\frac{\alpha_k - p}{\alpha_k - 1}\right)^{\left|\frac{x_k}{x_{-k}}\right|}\right) dp$$

which is decreasing in his distance from the DM $|x_k|$ and his opponent's quality costs α_{-k}, and increasing in his opponent's distance from the DM $|x_{-k}|$ and his own quality costs α_k.

Proof: The probability k loses the contest is $\int_0^{\bar{s}} f_{-k}(s) F_k(s)\, ds$; applying the engagement equality this is $\int_0^{\bar{s}} p_{-k}(\epsilon_k(F_{-k}(s))) f_{-k}(s)\, ds$, and applying a change of variables of $F_{-k}(s)$ for p (recalling $F_{-k}(0) = 0$) yields the result. QED

The probability k loses thus obeys the same comparative statics as his probability of inactivity. Somewhat paradoxically, he becomes *less* likely to win when his preferences are closer to the DM or his opponent's are more distant. More intuitively, he becomes more likely to win if he is more able or his opponent less able.

C.2 Conditions for First-Order Stochastic Dominance

In the standard asymmetric two-player all-pay contest there is always an unambiguously weaker player, who makes bids that are first-order stochastically worse for the DM. In the present contest, in contrast, there may be no unambiguously weaker player in this sense.

Proposition C.2. *Developer i is dominated ($F_{-i}(s) < F_i(s)\ \forall s \in (0,\bar{s})$) i.f.f. he is less engaged at every probability p ($\epsilon_i(p) < \epsilon_{-i}(p)\ \forall p \in (0,1)$). Equivalently, he is dominated i.f.f. both $\int_0^1 \frac{|x_i|}{\alpha_i - q} dq \leq \int_0^1 \frac{|x_{-i}|}{\alpha_{-i} - q} dq$ and $\frac{|x_i|}{\alpha_i - 1} \leq \frac{|x_{-i}|}{\alpha_{-i} - 1}$, where the latter condition is stronger than the former i.f.f. i has a cost advantage.*

Proof: Lemma B.1 and the engagement function $\epsilon_i(p)$ strictly decreasing when $p \in [0,1)$ immediately implies $sign(\epsilon_{-k}(F_{-k}(s)) - \epsilon_k(F_{-k}(s))) = sign(F_k(s) - F_{-k}(s))\ \forall s \in [0,\bar{s})$, which straightforwardly yields the first statement. Now let $\delta(p) = \epsilon_{-k}(p) - \epsilon_k(p)$, so $\delta(0) \geq 0 = \delta(1)$. We argue $\delta'(1) \leq 0$ is necessary and sufficient. For necessity, $\delta'(1) > 0 = \delta(1) \rightarrow \delta(p) < 0$ in a neighborhood below 1. For sufficiency, it is easily verified that $\delta'(p) = \frac{|x_k|}{\alpha_k - p} - \frac{|x_{-k}|}{\alpha_{-k} - p}$ crosses 0 at most once when the developers are asymmetric; thus $\delta(0) \geq 0 = \delta(1) \geq \delta'(0)$ implies $\delta(p)$ strictly quasi-concave over $[0,1]$ and $\delta(p) > \min\{\delta(0), \delta(1)\} \geq 0$ for $p \in (0,1)$.

We last argue $\delta(0) \geq 0$ and $\alpha_k > \alpha_{-k} \rightarrow \delta'(1) < 0$. Observe that $\alpha_k < \alpha_k$ and $\delta'(0) = \frac{x_k}{\alpha_k} - \frac{x_{-k}}{\alpha_{-k}} \leq 0 \rightarrow \delta'(1) = \frac{|x_k|}{\alpha_k}\left(\frac{1}{1 - 1/\alpha_k}\right) - \frac{|x_{-k}|}{\alpha_{-k}}\left(\frac{1}{1 - 1/\alpha_{-k}}\right) < 0$. If $\delta'(0) \leq 0$ we are done; if $\delta'(0) > 0$ then $\delta'(1) \geq 0 \rightarrow \delta'(p) > 0\ \forall p \in [0,1) \rightarrow \delta(1) > 0$, a contradiction. QED

Clearly, a developer k who is *both* less extreme ($|x_k| \leq |x_{-k}|$) and less able ($\alpha_k \geq \alpha_{-k}$) (with one strict) satisfies both conditions and is therefore dominated. However, when one developer is more extreme while the other is more able, then lower engagement at probability 0 is necessary but not sufficient for the more able developer to be dominated.

C.3 Developer Payoffs

Using Proposition B.2, the developers' equilibrium payoffs are as follows.

Proposition C.3. *Developer i's equilibrium utility is $\Pi_i^*(\bar{s}; F^*) = -\left(1 - \frac{1}{\alpha_i}\right) x_i^2 - (\alpha_i - 1)\bar{s}$, which is decreasing in his own costs α_i as well as either players' extremism $|x_j| \forall j$, and increasing in his opponent's costs α_{-i}.*

Proof: A developer's equilibrium utility is straightforward since $(\bar{s}, y_i^*(\bar{s}))$ is in the support of their strategy and wins for sure. Comparative statics of a developer i's parameters on his opponent $-i$'s utility, as well as of x_i on his own utility, follow immediately from previously-shown statics on $\bar{s} = s(\epsilon)$. Taking the derivative with respect to α_i, substituting in $\frac{\partial}{\partial \alpha_i}\left(\frac{p_i(\epsilon)}{\alpha_i}\right) = \frac{x_i p_i'(\epsilon)}{(\alpha_i - 1)\alpha_i^2}$, $\frac{\partial \epsilon_k(0)}{\partial \alpha_k} = -\frac{|x_k|}{\alpha_k(\alpha_k - 1)}$, $-\frac{p_i'(\epsilon) x_i}{\alpha_i - p_i(\epsilon)} = 1$, performing a change of variables, and rearranging the expression yields $-\mathbf{1}_{i=k} \cdot 2\int_0^{\epsilon_k(0)} \frac{|x_{-k}|}{\alpha_{-k}}\left(p_{-k}(\epsilon) - \left(\alpha_k \log\left(\frac{\alpha_k}{\alpha_k - 1}\right)\right)^{-1} p_{-k}(\epsilon_k(0))\right) d\epsilon - \left(\frac{|x_i|}{\alpha_i}\right)^2 \left(1 + 2\int_{p_i(\epsilon_k(0))}^1 \left(\frac{\alpha_i p}{\alpha_i - p} - 1\right)\right) dp.$ The first term is negative since $p_{-k}(\epsilon) > p_{-k}(\epsilon_k(0))$ for $\epsilon < \epsilon_k(0)$ and $\frac{1}{\alpha_k} < \int_0^1 \frac{1}{\alpha_k - p} dp = \log\left(\frac{\alpha_k}{\alpha_k - 1}\right)$. The second term is also negative since $1 + 2\int_{p_i(\epsilon_k(0))}^1 \left(\frac{\alpha_i p}{\alpha_i - p} - 1\right) > (1 - p_i(\epsilon_k(0))) + 2\int_{p_i(\epsilon_k(0))}^1 \left(\frac{\alpha_i p}{\alpha_i} - 1\right) = \int_{p_i(\epsilon_k(0))}^1 (2p - 1)\, dp \geq 0$. QED

A developer's equilibrium utility has two components. The first $-\left(1 - \frac{1}{\alpha_i}\right) x_i^2$ is his utility if he could craft a policy as a "monopolist" (and the DM's outside option included $(0, 0)$). The second $-(\alpha_i - 1)\bar{s}$ is the cost generated by competition, which forces him to craft a policy that leaves the DM strictly better off than the best "free" policy $(0, 0)$ in order to maintain influence. This competition cost is increasing in i's marginal cost α_i of generating quality (holding $\bar{s}$ fixed) as well as the maximum score $\bar{s}$, which in turn is increasing in both developers' ideological extremism and decreasing in their costs everywhere in the parameter space. A developer is thus strictly harmed when his competitor becomes more extreme or able. This is distinct from all pay contests without spillovers (Siegel (2009)), where the equilibrium utility of the "sometimes inactive" player is pinned at his fixed value for losing.

A developer also worse off when his *own* preference become more distant from the decisionmaker. Finally, a developer is worse off when his costs of producing quality increase – even though there is a countervailing effect of reducing the intensity of competition (and indeed, the competition cost $(\alpha_i - 1)\bar{s}$ alone is not generically monotonic in α_i).

C.4 Decisionmaker Payoffs

Lastly, again using Proposition B.2 the DM's equilibrium utility and the developers' average scores (which bound the DM's utility from below) are as follows.

Proposition C.4. *The DM's equilibrium utility is* $U^*_{DM} = \int_{\epsilon_k(0)}^{0} s(\epsilon) \cdot \frac{\partial}{\partial \epsilon}\left(\prod_j p_j(\epsilon)\right) d\epsilon =$

$$2\int_0^{\epsilon_k(0)} \left(1 - \prod_j p_j(\epsilon)\right) \cdot \left(\sum_j \frac{|x_j|}{\alpha_j} p_j(\epsilon)\right) d\epsilon$$

Developer i's average score is $E[s_i] = \int_{\epsilon_k(0)}^{0} s(\epsilon) \cdot \frac{\partial}{\partial \epsilon}(p_{-i}(\epsilon))\, d\epsilon =$

$$2\int_0^{\epsilon_k(0)} (1 - p_{-i}(\epsilon)) \cdot \left(\sum_j \frac{|x_j|}{\alpha_j} p_j(\epsilon)\right) d\epsilon$$

Proof: $F_i(s) F_{-i}(s)$ is the CDF of $\max\{s_i, s_{-i}\}$ so the DM's utility is $\int_0^{\bar{s}} s \cdot \frac{\partial}{\partial s}\left(\prod_j F_j(s)\right) ds = \int_0^{\bar{s}} s \cdot \frac{\partial}{\partial s}\left(\prod_j p_j(\epsilon(s))\right) ds$. A change of variables from s to ϵ yields the first expression and integration by parts and rearranging yields the second. Nearly identical steps yield i's average score. QED

Direct comparative statics on the DM's utility U^*_{DM} are difficult because changing a developer's parameters has mixed effects on his opponent's score CDF. We thus consider two special cases; breaking symmetry, and the limiting cases of extreme imbalance.

Proposition C.5. *When the developers are symmetric ($|x_i| = |x_{-i}|$ and $\alpha_i = \alpha_{-i}$), the DM's utility is locally increasing eithers' extremism or ability.*

Proof: First differentiating the DM's utility U_{DM}^{*} with respect to $|x_{-k}|$ and applying symmetry yields $\frac{2}{\alpha}\int_0^{\epsilon(0)}\left(\left(1-3\left(p\left(\epsilon\right)\right)^2\right)\cdot x\frac{\partial p(\epsilon)}{\partial x}+\left(1-\left(p\left(\epsilon\right)\right)^2\right)p\left(\epsilon\right)\right)d\epsilon$ which is $\geq 2\frac{x}{\alpha}\int_0^{\epsilon(0)}\left(1-3\left(p\left(\epsilon\right)\right)^2\right)\frac{\partial p(\epsilon)}{\partial x}d\epsilon$. Now substituting $\frac{\partial p(\epsilon)}{\partial x}=-\log\left(\frac{\alpha-p(\epsilon)}{\alpha-1}\right)p'\left(\epsilon\right)$ and a change of variables yields $2\frac{x}{\alpha}\int_0^1\left(1-3p^2\right)\log\left(\frac{\alpha-p}{\alpha-1}\right)dp=2\frac{x}{\alpha}\int_0^1\left(\frac{p-p^3}{\alpha-p}\right)dp>0$. Next differentiating U_{DM}^{*} w.r.t. α_{-k} and applying symmetry yields $2x\int_0^{\epsilon(0)}\left(\left(1-\left(p\left(\epsilon\right)\right)^2\right)\frac{\partial}{\partial\alpha}\left(\frac{p(\epsilon)}{\alpha}\right)-\frac{2}{\alpha}\left(p\left(\epsilon\right)\right)^2\frac{\partial p(\epsilon)}{\partial\alpha}\right)d\epsilon$. Finally, substituting $\frac{\partial}{\partial\alpha}\left(\frac{p(\epsilon)}{\alpha}\right)=\frac{x}{(\alpha-1)\alpha^2}p'\left(\epsilon\right)$, $\frac{\partial p(\epsilon)}{\partial\alpha}=-\left(\frac{1-p(\epsilon)}{\alpha-1}\right)$, $-\frac{p'(\epsilon)x}{\alpha-p(\epsilon)}=1$, rearranging the expression, and another change of variables yields $\frac{2x^2}{(\alpha-1)\alpha^2}\int_0^1\left(2p^2\left(\frac{\alpha-\alpha p}{\alpha-p}\right)-\left(1-p^2\right)\right)dp<0$. QED

The DM thus strictly benefits locally if developers between the players is broken by one becoming more extreme or able – even though the other also becomes less active. The effect of extreme asymmetries is as follows.

Proposition C.6. *The DM's utility exhibits the following limiting behavior*

$$0=\lim_{\alpha_i\to\infty}U_{DM}^{*}=\lim_{x_i\to 0}U_{DM}^{*}<\lim_{x_i\to\infty}U_{DM}^{*}=\infty$$

and $\lim_{\alpha_i\to 1}U_{DM}^{*}=2x_k\int_0^1\left(\frac{1-p}{\alpha_k-p}\right)\cdot\left(\frac{x_k}{\alpha_k}p+x_{-k}\right)dp$, *which is strictly increasing in* x_k *and strictly decreasing in* α_k*.*

Proof: Observe that $E\left[s_{-k}\right]\leq U_{DM}^{*}\leq\bar{s}$. For the first two limiting statements it is easily verified that $\bar{s}\to 0$ as $\alpha_k\to\infty$ or $x_k\to 0$. For the third limiting statement observe that $E\left[s_{-k}\right]\geq\frac{|x_{-k}|}{\alpha_{-k}}p_{-k}\left(\epsilon_k\left(0\right)\right)\cdot 2\int_0^{\epsilon_k(0)}\left(1-p_k\left(\epsilon\right)\right)d\epsilon$ which $\to\infty$ as $|x_{-k}|\to\infty$ since the first term $\to\infty$ and the remaining terms are non-decreasing. For the fourth limiting statement, using the definition in Proposition C.4 and that $\lim_{\alpha_{-k}\to 1}\left\{p_{-k}\left(\epsilon\right)\right\}=1\ \forall\epsilon\in\left[0,\epsilon_k\left(0\right)\right]$ yields a limit of $2\int_0^{\epsilon_k(0)}\left(1-p_k\left(\epsilon\right)\right)\cdot\left(\frac{x_k}{\alpha_k}p_k\left(\epsilon\right)+x_{-k}\right)d\epsilon$. Observing that $-\frac{p_k'(\epsilon)x_k}{\alpha_k-p_k(\epsilon)}=1$, substituting into the expression, and applying a change of variables yields the expression, which straightforwardly obeys the stated comparative statics. QED

If an extreme imbalance is the result of one developer's incompetence *or* ideological moderation, the DM's utility approaches 0, her utility if $-i$ were a "monopolist" (and the DM's outside options included $(0,0)$). (Developer $-i$'s utility also approaches his utility if he were a monopolist). However, if extreme imbalance is the result of one developer's greater ability to produce quality (specifically, if his marginal cost of producing quality approaches its intrinsic value), then the DM's utility is bounded away from 0. In this case the DM strictly benefits from the potential for competition, even though actual competition is almost never observed (since $F_{-i}(0) = F_k(0)$ approaches 1). Finally, unilateral ideological extremism benefits the decisionmaker in a strong sense; the DM can achieve arbitrarily high utility with a developer whose preferences are sufficiently *distant* from her own.

D Main Text Propositions

In this Appendix we describe where to locate the results collated in main text Propositions 2-8 in the general analysis contained in Appendices B-C.

Proposition 2 To see the first bullet point, observe that Proposition B.1 on activity implies that the moderate is the sometimes-inactive developer k and is inactive with strictly positive probability.

To see the second bullet point, first observe that the greater (first-order stochastic) extremism of the extremists policy is an implication of the ideology comparative statics stated in Proposition B.3, which states that as a developer becomes unilaterally more extreme his policy's ideology becomes more extreme and his opponent's policy's ideology simultaneously becomes more moderate. Next observe that the greater (first-order stochastic) overall appeal to the decisionmaker of the extremists policy follows from the necessary and sufficient conditions for score-dominance in Proposition C.2 – a developer being more extreme and able with at least one strict is a sufficient condition for score dominance. Finally, the statement on quality is an immediate implication of the extremist crafting a more ideologically extreme but also higher score policy (first order stochastically).

Lastly, the third bullet point is an immediate implication of score-dominance.

Proposition 3 The first bullet point under both "own strategy" and "opponent's strategy" follow from Proposition B.1 on activity. The second bullet point under both "own strategy" and "opponent's strategy" follow from Proposition B.3 on ideology. The third bullet point under "own strategy" is a joint implication of the ideology comparative statics in Proposition B.3 and the comparative statics on "own score" in Proposition B.2. The third bullet point under "opponent strategy" also follows from Proposition B.2 and the subsequent discussion.

Proposition 4 Follows immediately from Proposition C.3 characterizing the developers' payoffs.

Proposition 5 The first bullet point just restates Proposition C.5. The second bullet point follows from Proposition B.1 (on activity) and Proposition C.6 (on the decisionmaker's welfare).

Proposition 6 Follows from Propositions B.1, B.3, and C.2 according to a nearly identical argument as in the proof of Proposition 2.

Proposition 7 The first statement follows from Propositions B.1-B.3 by a nearly identical argument as in the proof of Proposition 3. The second statement follows immediately from Proposition C.3.

Proposition 8 The first bullet point just restates Proposition C.5. The second bullet point follows from Proposition B.1 (on activity) and Proposition C.6 (on the decisionmaker's welfare).

E No Spillovers Variant

In this Appendix we examine a variant of the model that lacks "rank order spillovers," in which the developers only care about policy *when they win*. For algebraic simplicity, we assume that if they lose they receive utility "as if" the policy $(0, 0)$ is implemented. Borrowing from the main analysis, it is easily verified that a developer's i's expected utility when he develops a policy (s, y) with $s \geq 0$ where either his opponent has no atom or a tie would be broken in his favor is equal to:

$$- (\alpha_i - F_{-i}(s)) s + F_{-i}(s) \cdot 2x_i y - \alpha_i y^2 - x_i^2 \tag{E.1}$$

Necessary and Sufficient Conditions for Equilibrium Using a similar series of steps as in Appendix A.1 it is straightforward to show that any equilibrium must take an identical form as in the main model; a pair of score CDFs $F_i(s)\ \forall i$ that are continuously increasing over a common interval $[0,\bar{s}]$ satisfying (i) $F_k(0)>0$ for at most one k, (ii) $F_i(\bar{s})=1\ \forall i$, (iii) $y_i(s)=F_{-i}(s)\frac{x_i}{\alpha_i}$, and (iv) the following expression (which is eqn. E.1 with $y_i(s)$ substituted in) constant $\forall s\in[0,\bar{s}]$ and $\forall i$:

$$-\left(\alpha_i-F_{-i}(s)\right)s+\left[F_{-i}(s)\right]^2\frac{x_i^2}{\alpha_i}-x_i^2 \tag{E.2}$$

Deriving Equilibrium For each developer any $s\in[0,\bar{s}]$ and $\bar{s}$ must yield the same utility, i.e.

$$-\left(\alpha_i-F_{-i}(s)\right)s+\left[F_{-i}(s)\right]^2\frac{x_i^2}{\alpha_i}-x_i^2=-\left(\alpha_i-F_{-i}(s)\right)s+\left[F_{-i}(s)\right]^2\frac{x_i^2}{\alpha_i}-x_i^2$$

Applying $F_j(\bar{s})=1\ \forall i$ and simplifying yields the implicit characterization:

$$\frac{x_i^2}{\alpha_i\left(\alpha_i-1\right)}\cdot\left(1-\left[F_{-i}(s)\right]^2\right)=\bar{s}-\left(\frac{\alpha_i-F_{-i}(s)}{\alpha_i-1}\right)\cdot s \tag{E.3}$$

It is easily verified that for a given value of $\bar{s}$ this expression uniquely defines a continuously increasing $F_{-i}(s)\ \forall s\in[0,\bar{s}]$; it remains only to identify the value of $\bar{s}$ that will satisfy the boundary condition $F_k(0)>0$ for at most one k. Letting $\bar{s}_i=\frac{x_i^2}{\alpha_i(\alpha_i-1)}$, it is easily verified that the boundary condition at 0 is satisfied if and only if $k\in\arg\min_i\left\{\frac{x_i^2}{\alpha_i(\alpha_i-1)}\right\}$ and $\bar{s}=\bar{s}_k$; thus equilibrium is unique. Finally, substituting into E.3 and simplifying yields a characterization of the unique score CDFs for the always active developer $-k$:

$$x_k^2\cdot\left[F_{-k}(s)\right]^2=\alpha_k\left(\alpha_k-F_{-k}(s)\right)\cdot s \tag{E.4}$$

and the sometimes inactive developer k:

$$x_{-k}^2\cdot\left(1-\left[F_k(s)\right]^2\right)=\alpha_{-k}\left(\left(\alpha_{-k}-1\right)\bar{s}_k-\left(\alpha_{-k}-F_k(s)\right)s\right) \tag{E.5}$$

Properties of Equilibrium Although the variant without spillovers does not fit precisely into the "all-pay contest" framework of Siegel (2009) due to its multidimensionality, it nevertheless exhibits most of the characteristic properties of the standard two player asymmetric all pay contest (in contrast to the main model). First, evaluating eqn. E.2 for the weaker player k (that is, the one with the lower value of $\bar{s}_i = \frac{x_i^2}{\alpha_i(\alpha_i-1)}$) at $s = 0$ yields her equilibrium expected utility $-x_k^2$. Thus, as in the standard 2-player asymmetric all pay contest, the equilibrium utility of the weaker player k is *invariant to* the characteristics of the stronger player.

Second, it is clear from the characterization of the equilibrium score CDF $F_{-k}(s)$ of the stronger player $-k$ in eqn. E.4 that the score CDF of the stronger player is *invariant to her own characteristics* (x_{-k}, α_{-k}), and depends entirely on the characteristics of her weaker competitor (x_k, α_k).

Third, it is easily verified from the implicit characterization of the equilibriums score CDF $F_k(s)$ of the weaker player k in eqn. E.5 that at any $s \in [0, \bar{s})$ we have $F_k(s)$ strictly increasing in x_{-k} and strictly decreasing in α_{-k}; that is, the weaker player's score CDF is first order stochastically decreasing in the stronger player's extremism and increasing in the weaker player's cost. It is also easily verified that $F(s) \to 1 \ \forall s \in [0, \bar{s})$ as $x_k \to \infty$ or $\alpha_k \to 1$. Thus, the discouragement effect is present in the model, and as one developer becomes arbitrarily extreme or capable the other developer becomes almost always inactive. Combining these observations with the previous observation that the score CDF of the stronger player $-k$ is invariant to her own characteristics immediately yields that the decisionmaker is harmed by asymmetries, since $-k$ becoming more extreme or capable decreases k's score CDF but has no effect on her own.

Finally, when the developers are symmetrically extreme and capable ($|x_L| = |x_R| = x$ and $\alpha_L = \alpha_R = \alpha$), the unique symmetric equilibrium score CDF $F(s)$ is characterized by the equation:

$$x^2 \cdot [F(s)]^2 = \alpha(\alpha - F(s)) \cdot s$$

which is clearly first-order stochastically increasing in extremism x and decreasing in costs α. Thus, the variant without spillovers exhibits the benefit of greater *symmetric extremism and ability* in Hirsch and Shotts (2015), but does not exhibit the benefit of greater *asymmetric extremism and ability* in the main model.