Veto Players and Policy Development[1]

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Abstract

We analyze the effects of veto players when the set of available policies isn't exogenously fixed, but rather determined by policy developers who work to craft new high-quality proposals. If veto players are moderate, there is active competition between developers on both sides of the ideological spectrum. However, more extreme veto players induce asymmetric activity, as one side disengages from development. With highly-extreme veto players, policy development ceases and gridlock results. We also analyze effects on centrists' utility. Moderate veto players dampen productive policy development and extreme ones eliminate it entirely, either of which is bad for centrists. But some effects are surprisingly positive; somewhat-extreme veto players can induce policy developers who dislike the status quo to craft moderate, high-quality proposals. Our model accounts for changing patterns of policymaking in the U.S. Senate, and suggests that if polarization continues centrists will become increasingly inclined to eliminate the filibuster.

Many political organizations use decisionmaking procedures that empower *veto players* – individuals or groups who can block policy change. For example, executives often have constitutionally-granted veto powers (Cameron, 2000) and supermajority procedures in legislatures and commissions generate implicit "pivots" (Crombez (1996); Brady and Volden (1997); Krehbiel (1998); Tsebelis (2002)). Despite the ubiquity of such procedures, commentators are of two minds about their consequences. Consider for example the filibuster in the U.S. Senate; critics frequently complain about the minority's ability to obstruct, but defenders argue that additional hurdles encourage constructive deliberation (Arenberg and Dove, 2012). To understand how veto players affect policymaking, it is important to consider both possibilities; i.e., to allow for constructive policymaking as well as gridlock.

We study this question in *competitive* policymaking environments, in which different actors can generate new policy proposals. To do so we build on Hirsch and Shotts (2015), which models policymaking as a "contest" (Tullock (1980); Baye, Kovenock and de Vries (1996)) in which a decisionmaker considers policies crafted by policy-motivated actors, known as *policy developers*. Rather than promising payments or furnishing general policy-relevant information, developers gain support for their policies by making costly, up-front policy-specific investments in their *quality*. In the original model, competition between developers benefits a unitary decisionmaker because it prevents a developer from extracting all of the benefits of her quality investments in the form of ideological concessions. Here we consider how the inclusion of veto players affects this process.

Veto players create additional hurdles to policy change; their presence would therefore seem to harm a decisionmaker who would otherwise have full freedom of choice. Missing from this simple intuition, however, is that the decisionmaking context can affect the set of policies from which a decisionmaker chooses. In our model, policy developers anticipate veto players' actions, which affects both which policies they craft and how much effort they exert crafting them. The exact nature of these effects is subtle. On the one hand, veto players can discourage policy development, because developers anticipate that it is more difficult to achieve policy change. On the other hand, policy developers may invest more in quality to satisfy veto players' demands. For example, writing about the Affordable Care Act, Washington Post columnist Ruth Marcus argued that "a product that can secure the votes of 60 senators is more likely to be one that can achieve a national consensus as well. It is no accident that the Senate health care bill is better than its House counterpart" (Marcus, 2010). Given these countervailing effects, it is not obvious whether, and under what circumstances, the presence of veto players will benefit a centrist decisionmaker despite constraining his freedom of choice. This matters because centrists often play a key role in determining institutional procedures; for example, in the 118th Congress, Democrats were unable to reform the filibuster due to opposition from Senators Manchin and Sinema.

We first examine how veto players affect patterns of activity in competitive policy development, and show that they lead to asymmetric participation between otherwise-symmetric policy developers. The reason is that veto players permit the maintenance of a non-centrist status quo. When the status quo favors the interests of one developer, she is less-motivated to develop a replacement policy, while the competing developer is more motivated to do so. As is typical in asymmetric contests, this leads to asymmetric participation (Hillman and Riley (1989)); the favored developer is sometimes or always inactive, while the disfavored developer always crafts a new policy. Our model thus embodies a pattern frequently seen in real-world politics – the faction with the greatest interest in change works to develop a policy alternative, while the faction benefitting from the status quo is less constructively engaged in policy development.

We next study how patterns of participation are affected by the veto players' ideological extremity; in a legislature with spatial preferences, this proxies for the stringency of a supermajority hurdle (which determines the relevant "pivots" for policy change). If veto players are moderate, then both developers are likely to craft new policies for consideration, because each faces relatively modest hurdles to moving policy in her desired direction. However, as veto players become more extreme, developers' pattern of activity becomes increasingly asymmetric. This occurs because hurdles to change disproportionately impact the developer favored by the status quo; both directly – by making it harder for her to get her preferred policy enacted – and indirectly – by protecting her from the possibility that the other side will successfully enact a very unappealing policy. If veto players become too extreme, the pattern of activity again becomes symmetric; both developers decline to develop policies, and gridlock results.

We last examine when a centrist decisionmaker would benefit from eliminating the veto players. If they are highly moderate or highly extreme, the decisionmaker is better off eliminating them; in the former case they dampen productive competition, while in the latter case they discourage policy development entirely. However, if veto players are only somewhat extreme, the decisionmaker may benefit from their presence; the disfavored developer is willing to make substantial quality investments to cater to an opposing veto player, and these benefits to the decisionmaker may outweigh the costs of limiting his freedom of choice. Under these conditions, the developer favored by the status quo is also unlikely to develop a competing policy. Our model thus identifies surprising conditions under which the presence of veto players benefits a decisionmaker – when their extremism is "just right." It also counterintuitively predicts that veto players are most beneficial when their presence inhibits *observable* competition. Thus, the absence of observable competition in policy development is not prima facie evidence of dysfunctional politics or formal agenda control, but could instead simply reflect competing groups' differential motivation to change a lopsided status quo.

Finally, we discuss the changing impact of the filibuster in the Senate as its members have become increasingly polarized over time. The Senate represents a good application of the model because of its relatively open agenda procedures – individual Senators' right to unlimited debate ensures that majority leaders must satisfy a broad constituency when scheduling floor time, thereby permitting proposals to come from a variety of sources. We first show that the relevant veto players in the Senate – the "filibuster pivots" implied by the 60-vote threshold to invoke cloture – have polarized faster than the chamber's main policy developers – the party medians. The evolution of the Senate thus mimics the comparative static in our model of the veto players becoming more extreme. We then argue that our model provides a novel explanation for shifting patterns of policymaking; since the 1970s, the Senate has evolved from the "textbook Congress" with both sides generating policy options (because veto players were relatively moderate), to highly asymmetric policymaking with the majority developing policies and the minority engaging in obstruction (as veto players became more extreme), and finally to the contemporary gridlocked Senate (as veto players became very extreme). Further, our model provides a novel rationale for centrists' historical support of the filibuster as an institution; that it empowered somewhat-extreme veto players who encouraged the development of reasonably-centrist and high-quality policies. Finally, the model rationalizes the erosion of support in the present era, as Senators' increasingly stringent demands to invoke cloture now promote excessive gridlock.

Veto Constraints and Supermajority Rules

A large literature studies veto constraints and supermajority rules in both static and dynamic settings (Crombez (1996), Brady and Volden (1997), Krehbiel (1998), Cameron (2000), Tsebelis (2002)). A key insight undergirding pivot-based models is that in a unidimensional setting with sufficient structure on preferences, analysis of a collective choice body can be simplified by focusing on *pivots* who lack formal veto power (as in general social choice models, see Austen-Smith and Banks (1999)) because they can be modeled as *de facto* veto players backed by a coalition of other members.

Our work joins a subset of this literature that proposes rationales for supermajority rules despite their potentially pernicious effects (Dziuda and Loeper, 2018). Such rationales include static ones rooted in the effects on policymaking at a particular moment in time, as well as dynamic ones rooted in the effects on repeated policymaking in changing circumstances. Among dynamic rationales, the most relevant to the Senate center on the relationship between the future distribution of power and decisions about procedures and policy in the present (e.g., Dixit, Grossman and Gul (2000), Messner and Polborn (2004), Eguia and Shepsle (2015), Gibilisco (2015), Invernizzi and Ting (2024)). Our model, in contrast, adds to the literature on static rationales; others include protecting minorities (Aghion and Bolton, 2003), maximizing campaign contributions (Diermeier and Myerson, 1999), aggregating policy-relevant information (Persico, 2004), and counterbalancing the power of formal (i.e., explicitly privileged by procedures) agenda setters (Peress (2009); Krehbiel and Krehbiel (2023)). We are closest in spirit to the latter – policies have a unidimensional spatial component, institutional rules generate pivotal actors, and these "pivots" help counterbalance developers' *informal* agenda setting power. However, the distinction between formal agenda power (to make proposals) and informal agenda power (to craft them) is crucial, both for applying the model to settings with relatively open agenda procedures like the Senate, and for generating novel predictions about policymaking activity.

Although these works examine why decisionmakers may *want* to institute procedural constraints, they largely sidestep the puzzle of how they credibly do so; what stops them from doing away with such procedures when it suits their short-term interests? This is examined in a related literature on endogenous institutional rules, i.e., on models of legislators "choosing how to choose." One answer applicable to the Senate (whose standing rules require a 2/3 supermajority to invoke cloture on rule changes) is to suppose that legislators can impose binding constraints on future rule changes (Barbera and Jackson (2004)). This is somewhat unsatisfactory given the recent use of the "nuclear option," where a simple majority used a complex procedural maneuver to eliminate filibusters on Supreme Court nominees (Min Kim, Everett and Schor, 2017). Another is to explicitly model how the median might credibly give away power in a dynamic stationary bargaining model, where legislators cannot punish each other for reneging on a common understanding of procedures. For example, Diermeier, Prato and Vlaicu (2015) assume that switching to majoritarian procedures involves frictions that prevent an impatient median from immediately "getting her way."

A third answer is to explicitly model the stability of procedures in a dynamic nonstationary bargaining model where legislators *can* punish each other for deviating from agreed-upon procedures; this is akin to Diermeier's (1995) explanation for the stability of closed rules. In a similar fashion, the threat of reverting to majoritarian procedures could sustain supermajority rules designed to elicit high-quality legislation from policy developers. This possibility is salient in the Senate; when describing potential consequences of employing the nuclear option, a former staffer and parliamentarian wrote "This is a slippery slope. It will almost inevitably lead to strict majority rule of debate and amendment, turning the Senate into a smaller and less significant shadow of the House of Representatives"(Arenberg and Dove, 2012, p. 173).

Policy Quality

An important feature of our model is that policies have an endogenous quality dimension, so it is possible to "buy" votes by developing high-quality policies.[1] From an analytical perspective, quality is simply a second dimension of policy that all key actors agree upon. For example, although policymakers may disagree about how much should be done to mitigate climate change, if the government decides to pursue a green industrial policy, people across the ideological spectrum would prefer to financially support well-run companies developing feasible technologies rather than badly-run ones working on infeasible technologies. More broadly, policies' quality characteristics may include cost savings, promotion of economic growth, or efficient and effective administration. A narrower interpretation of quality when participants are risk-averse over ideology is that it is simply reduced uncertainty about a policy's ideological outcome (e.g., Huber and McCarty (2004); Turner

[1] A large literature analyzes the choice between two fixed policies: Dion et al. (2016) study filibusters as a war of attrition where parties have private information about costs of fighting, Gibbs (2023) analyzes signaling to external constituencies, Anesi and Bowen (2021) study vote buying via transfers, and Chan et al. (2018) and Anesi and Safronov (2023) analyze how decision rules affect information acquisition.

(2017)). Regardless of the interpretation, a central presumption of our model is that that enhancing a policy's quality requires costly up-front effort to acquire the requisite information and expertise, and translate it into a concrete proposal.

A second key feature of our approach is that the return on a developer's costly investment is *policy-specific*, rather than general to policies anywhere in the ideological spectrum; in so doing we build on Londregan (2000); Bueno De Mesquita and Stephenson (2007); Lax and Cameron (2007); Ting (2011); and Hirsch and Shotts (2012, 2018). Our approach contrasts with a large literature building on Crawford and Sobel (1982) in which the return to an expert's effort is both "general" (knowledge about the full mapping between all possible policies and outcomes) and "simple" (because each possible mapping is described by a unique scalar). For example, in climate policy, a single piece of information—such as the value of a temperature "tipping point"—would inform how much each actor prefers to invest in mitigation, even though actors would still disagree about the appropriate level of investment when fully informed. In this classical approach, an expert's willingness to work is degraded by his fear that a decisionmaker will learn and exploit his knowledge to implement an undesirable policy outcome. In contrast, in models like ours an expert can exert informal agenda power or "real authority" (Aghion and Tirole, 1997) by crafting a high-quality policy, because a decisionmaker must actually implement her policy to enjoy the benefits of its quality.[2] Most closely related to our work is Hitt, Volden and Wiseman (2017), which briefly analyzes the case of a single developer facing a veto player. Our analysis differs substantially; we incorporate multiple developers, and characterize effects on moderation and quality of policies, as well as whether centrists benefit

[2]In Callander (2008) an expert acquires general expertise about the mapping between policies and outcomes, but the complexity of such mappings may vary. On maximally complex issues an expert can reveal which policy achieves his desired outcome while censoring information about the full mapping. It is thus as if he can generate policy-specific quality but *only* for his ideal policy; see also Turner (2017).

from the presence of veto players.

Finally, because the costs of investing in quality are paid up-front, our model relates to contests with an "all-pay" component (Baye, Kovenock and de Vries, 1993; Che and Gale, 2003; Siegel, 2009); particularly those involving policy-motivated competitors (Ashworth and Bueno de Mesquita (2009), Balart, Casas and Troumpounis (2022), Hirsch (2023*a*)) or multiple decisionmakers (Jordan and Meirowitz (2012)). Our model has two primary differences with previous contest models that complicate the equilibrium analysis. The first is that developers are policy-motivated rather than rent seeking, in that the "loser" cares about the exact policy crafted by the "winner." This makes sense for political environments where competing actors care about a collective policy decision. The second is that our model features players who can only block proposals, i.e., veto players. In the presence of veto players, investing in quality is strategically beneficial in two ways; it can make a policy more appealing to the decisionmaker, and also help gain the support of the veto players.

Model

The model takes place in three stages. First, two policy developers simultaneously craft new policies to add to the set of alternatives available for consideration. Second, a decisionmaker proposes a policy; either a new one crafted by a developer, or a preexisting one. Finally, a pair of veto players either approve the policy or block it, in which case a status quo prevails.

Policy has two components: *ideology* $y \in \mathbb{R}$ and *quality* $q \in [0, \infty)$. Players' utility functions are:

$$U_i(b) = q - (x_i - y)^2$$

where x_i is i's ideological ideal point.

Policy development Each of two developers (L and R, with ideal points $x_L < 0$ and $x_R > 0$) may simultaneously invest costly resources to develop a new policy $b_i = (y_i, q_i)$ with ideology y_i and quality $q_i \geq 0$ at cost $\alpha_i q_i$, where $\alpha_i > 2$. A developer thus will not invest in quality simply for its own sake, but rather to improve her policy's prospects of being enacted.

Policy choice In Hirsch and Shotts (2015) and Hirsch (2023*b*), policy is chosen by a single decisionmaker with ideal ideology $x_D = 0$. We augment this with two *veto players* $x_{VL} < 0$ and $x_{VR} > 0$. If either rejects the decisionmaker's proposal an exogenous status quo policy b_0 prevails.

The set of possible policy choices consists of all 0-quality policies, any newly-developed policies, and the status quo $b_0 = (y_0, q_0)$. This assumption reflects the idea that the decisionmaker and the veto players collectively have the power to choose policy, but not to develop it. We further assume that the status quo is low-quality ($q_0 = 0$) and within the "gridlock interval" for 0-quality policies ($x_{VL} < y_0 < x_{VR}$); our analysis is thus restricted to circumstances in which policy is stable absent the development of new alternatives. Finally, we consider developers who are more extreme than the veto players ($x_L \leq x_{VL}$ and $x_R \geq x_{VR}$). This is natural if the set of potential developers consists of all actors motivated to change policy, while veto players must be empowered by formal institutional rules.

Robustness The model is unchanged if more veto players are introduced in $[x_{VL}, x_{VR}]$ because only the two most extreme ones pose binding constraints for policy change. Our equilibria are also robust to including additional moderate developers in $[x_L, x_R]$; if their costs of developing quality are no higher than the same-sided extremist developer's costs, they remain inactive. What matters most for our results is that there is some aspect of quality that each developer, decisionmaker, and opposite-side veto player agree on. This ensures that a developer can improve a proposal in ways that appeal to the opposing veto player, with benefits that spill over to the decisionmaker. It is less important that the developers *themselves* agree what constitutes quality. Indeed, absent veto players the decisionmaker benefits if opposing developers disagree about what constitutes quality (Hirsch and Shotts (2015)).

Application: The U.S. Senate

Before analyzing the model we briefly discuss how to map it to the U.S. Senate. Classic models (Brady and Volden (1997); Krehbiel (1998)) model legislatures as consisting of n members with unidimensional spatial preferences differentiated only by their ideal points. Under these assumptions, whether a proposed bill will pass hinges on the preferences of a few pivotal legislators explicitly or implicitly empowered by the rules. Although our model features "one and a half" dimensions by including quality (Groseclose (2007)), this property is unchanged since all legislators value quality equally (see also Hitt, Volden and Wiseman (2017)). As applied to the Senate, our model represents a reduced-form representation of a 100-member legislature; the decisionmaker is the legislator(s) empowered to set the agenda, the veto players are the outermost pivots implied by the decisionmaking rules, and the developers are the most extreme policy-motivated actors with the ability and resources to craft new policies.

Who are these key actors in the Senate? For the veto players the answer is straightforward – the Senate's 3/5 requirement to invoke cloture has effectively made it a supermajoritarian institution for final passage of major legislation (Binder (2015)). Its pivotal members are approximately the 41st and 60th most liberal legislators required to invoke cloture for policy movements rightward or leftward, respectively. For the decisionmaker (who in our model acts as an agenda setter vis-a-vis the veto players), the answer is less straightforward. Following Krehbiel (1998), we argue that the decisionmaker is best thought of as the Senate median; this is "tantamount to assuming that the legislature decides under an *open rule*; that is, no restrictions are placed on amendments or on who can offer them" (Krehbiel, 1998, 25). Although an open rule simplifies the real-world Senate's complex procedures, it is arguably reasonable given each individual Senator's codified right to delay, the cumbersome nature of cloture (Arenberg and Dove, 2012, 12), and the absence of a germaneness requirement (Oleszek et al., 2015). This assumption also helps sharpen the question at hand by stacking the deck against the possibility that veto players will benefit moderates.

Finally, for the developers we argue that the most extreme Senate actors with the ability and resources to craft new policies are the *parties*, working as a collective to pursue a common ideological objective reflected by the median party member's ideal; while this assumption is a poor fit to the mid-

20th century committee-centered Congress, it more accurately describes the Congress that emerged after the 1980s, with party leaders increasingly originating landmark legislation and coordinating its drafting (Sinclair, 2016).

Preliminary Analysis

In the absence of veto players, the decisionmaker can revise any status quo to a low quality policy at his ideological ideal. It is thus *as if* the status quo ideology is $y_0 = x_D = 0$, with the decisionmaker willing to adopt any newly-crafted policy that he prefers over $(0, 0)$. This is depicted in the top panel of Figure 1.

The presence of veto players creates additional hurdles to policy change, which affects decision-making in two ways. First, it expands the range of potential status quos to include ones that are non-centrist: the status quo may be at any $y_0 \in [x_{VL}, x_{VR}]$. Second, for policy change to occur a new policy must be acceptable to *both* veto players, who are collectively more opposed to policy change than the decisionmaker. This can be seen by observing that the upper envelope of the veto players' indifference curves through the status quo is steeper than the decisionmaker's indifference curve; to avoid a veto the decisionmaker must choose a policy that is weakly within this upper envelope, which we call the *veto-proof set*.

The effect of veto players hinges on how this change in the set of acceptable policies affects developers' incentives to invest in quality. A developer may be *less* willing to invest if she is favorably disposed to the status quo, or unwilling to satisfy veto players' demands. Alternatively, she may be *more* willing to invest if she strongly dislikes the status quo, and is willing to put in extra work to satisfy veto players' demands.

Notation We call the decisionmaker's utility from a policy its *score*, $s(y, q) = U_D(y, q) = q - y^2$; scores are useful because they fully characterize how the decisionmaker evaluates the available veto-proof policies. Absent veto players, it is as if the score of the status quo is $s(0, 0) = 0$; the

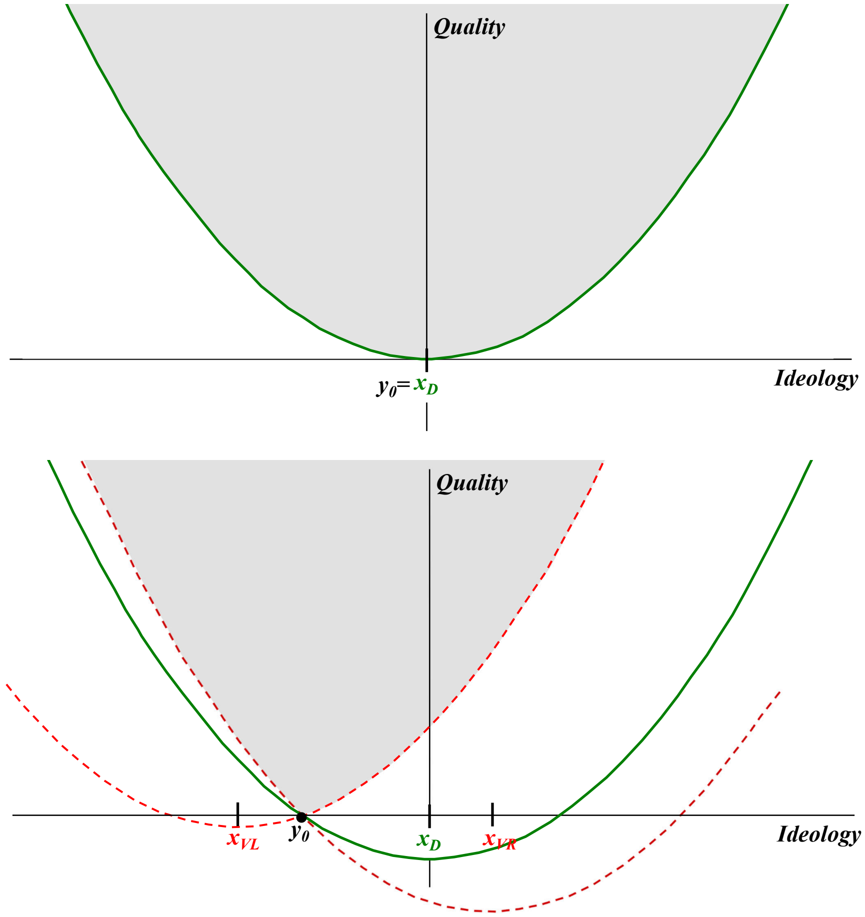

Figure 1: Veto Players' Effect on Decisionmaking. *Green line is decisionmaker's indifference curve. Red dashed lines are veto players' indifference curves.*

decisionmaker chooses the policy with the highest score subject to the constraint that its score is ≥ 0. Veto players increase the range of scores the decisionmaker is willing to accept (to those $\geq U_D(y_0, 0) = -y_0^2$), but restrict the set of acceptable ideologies *given each score*. The following defines the set of veto-proof policies in terms of score s and ideology y, as illustrated in Figure 2.

Definition 1. *A policy* (s, y) *with score* s *and ideology* y *is* veto-proof *if and only if* $y \in [z_L(s), z_R(s)]$, *where* $z_L(s) = y_0 - \frac{s-s_0}{2|x_{VR}|}$, $z_R(s) = y_0 + \frac{s-s_0}{2|x_{VL}|}$, *and* $s_0 = -y_0^2$ *is the score of the status quo.*

After policies have been developed, the decisionmaker chooses the highest-score veto-proof policy

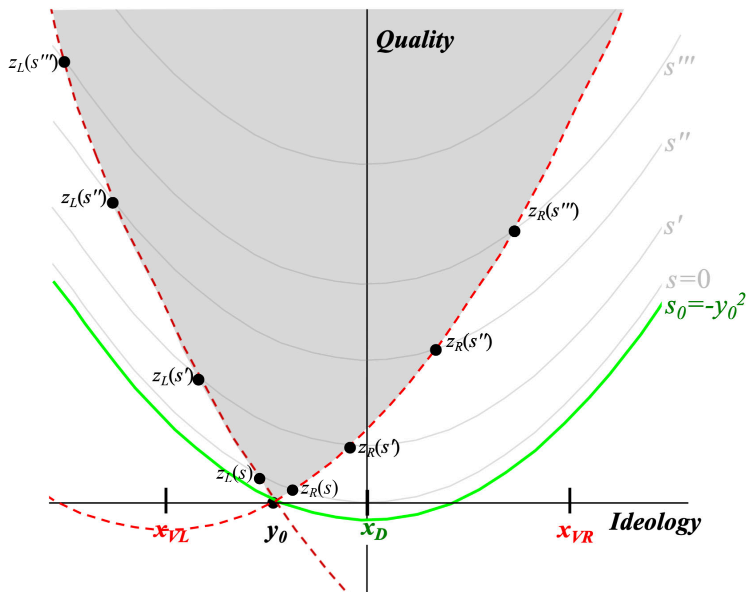

Figure 2: The Veto-Proof Set. *Decisionmaker indifference curves are gray curves. At score s, the range of veto-proof ideologies is $[z_L(s), z_R(s)]$; the right boundary is determined by the left veto player while the left boundary is determined by the right veto player.*

available.

The Monopolist's Problem

To see how veto players influence policy development, we first consider the case of a single developer who is a "monopolist."[3] Because a policy with score s and ideology y must have quality $q = s + y^2$ the up-front cost to developer i of crafting it is $\alpha_i(s + y^2)$, and her policy utility if it is adopted is $V_i(s, y) = U_i(y, s + y^2) = -x_i^2 + s + 2x_i y$. A monopolist's objective is to craft a veto-proof policy (s_i, y_i) that maximizes $-\alpha_i(s_i + y_i^2) + V_i(s_i, y_i)$. It is easily derived that a policy

[3]See also Hirsch and Shotts (2015), Hitt, Volden and Wiseman (2017), Hirsch and Shotts (2018), and Hirsch (2023*b*).

(s_i, y_i) satisfying this objective is an element of the set:

$$\underset{\{(s_i,y_i):s_i\geq s_0,y_i\in[z_L(s_i),z_R(s_i)]\}}{\arg\max}\left\{\underbrace{-(\alpha_i-1)\,s_i}_{\text{score effect}}+\underbrace{2x_iy_i-\alpha_iy_i^2}_{\text{ideology effect}}\right\}. \tag{1}$$

From Equation 1 it is easy to see that without veto players, a monopolist would craft a policy no better for the decisionmaker than $(0, 0)$, because it is as if this is the status quo and there is no constraint on ideology. She sets $s_i = 0$ (minimizing the loss in the first term) and targets the unique ideology that optimally trades off ideological concessions to the decisionmaker against the cost of compensating him with additional quality (maximizing the second term). This optimal ideology is $y_i = \frac{x_i}{\alpha_i}$, a convex combination of the decisionmaker's and monopolist's ideal points, weighted by the cost of quality.

Veto players prevent a monopolist from doing this; they force her to develop a policy within the veto-proof set. What then will a monopolist do? She develops a policy on the closer boundary of the veto-proof set ($z_L(s_L)$ for developer L or $z_R(s_R)$ for developer R), with an ideology that trades off the marginal benefit of moving the outcome closer to her ideal against the marginal cost of producing enough quality to gain the opposite-side veto player's support. Substituting in the optimal ideology $y_i^*(s) = z_i(s_i)$ given a score s into Equation 1, straightforward optimization characterizes the ideology of the optimal policy for a monopolist to develop. As in Hitt, Volden and Wiseman (2017), it is an average of her own ideal point and the ideal point of the opposite-side veto player, weighted by the cost of quality:

$$\hat{y}_i = \begin{cases} \frac{1}{\alpha_L}x_L + \left(1-\frac{1}{\alpha_L}\right)x_{VR} \text{ for } i = L \\ \frac{1}{\alpha_R}x_R + \left(1-\frac{1}{\alpha_R}\right)x_{VL} \text{ for } i = R \end{cases}$$

If the status quo is closer to the developer than $\hat{y}_i$, the marginal cost of moving policy in her direction is too high, so she develops no policy and the status quo is maintained. In sum, we have the following.

Proposition 1. *When developer i is a monopolist, she crafts policy (s_i^{M*}, y_i^{M*}), where*

$$y_i^{M*} = \begin{cases} \min\{y_0, \hat{y}_L\} & \text{for } i = L \\ \max\{y_0, \hat{y}_R\} & \text{for } i = R \end{cases}$$

and $z_i(s_i^{M*}) = y_i^{M*}$. A monopolist invests in policy development if and only if the status quo is farther from her ideal point than her ideal monopoly policy: $s_i^{M*} > s_0 \iff |y_0 - x_i| > |\hat{y}_i - x_i|$.

Whenever a new policy is developed, its ideology $\hat{y}_i$ depends on the tradeoff at the margin between ideological gains and costs of generating quality. With linear costs, this ideology does not depend on the status quo. The level of quality, however, *does* depend on the status quo, because a status quo closer to the opposite-side veto player's ideal forces the developer to generate more quality to gain his support. This yields the following.

Corollary 1. *At any status quo y_0 where policy development occurs ($s_0 < s_i^{M*}$), the monopoly score s_i^{M*} is strictly increasing (decreasing) in y_0 when $i = L\,(R)$.*

The farther is the status quo from the monopolist, the more change she wants, the more quality she must generate to gain the veto player's support, and the more the decisionmaker benefits.

The Competitive Problem

We next describe key properties of equilibria in the main model where two developers compete. We say that developer i is *active* when she develops a veto-proof policy with score $s_i > s_0$ (and strictly positive quality), and *inactive* if she exerts no effort and "develops" the only low-quality veto-proof policy, i.e., the status quo.

Equilibria of the competitive model can be complex; one or both developers may be active or inactive, and when active could mix over a continuum of scores as well combinations of ideology and quality at each score. However, we show in the Appendix that it is without loss of generality to consider strategy profiles of the following form.

Remark 1. *We consider strategy profiles in which each developer*

1. *only crafts veto-proof policies* ($s_i \geq s_0$ *and* $y_i \in [z_L(s_i), z_R(s_i)]$)

2. *chooses the score* s_i *of her policy according to a (potentially degenerate) cumulative distribution function* $F_i(s_i)$

3. *crafts a unique policy* $(s_i, y_i(s_i))$ *at each score* s_i.

Because each developer crafts a unique policy at each score, to describe equilibria we focus on the *probability distributions over scores* that the developers' policies generate. In a pure strategy equilibrium these distributions are degenerate, i.e., each developer places probability 1 on a single score. In a mixed strategy equilibrium they place probability weight on a continuum of scores.

Lemma 1. *In any equilibrium satisfying the conditions in Remark 1, there is a developer* k *and two scores,* $\underline{s}$ *and* $\overline{s}$*, satisfying* $s_0 \leq \underline{s} \leq \overline{s}$ *such that*

- *developer* k*'s score CDF* F_k *has support* $s_0 \cup [\underline{s}, \overline{s}]$ *and exactly one atom at* s_0*,*

- *developer* $-k$*'s score CDF* F_{-k} *has support* $[\underline{s}, \overline{s}]$ *and exactly one atom at* $\underline{s}$*.*

Appendix B.1 fully characterizes fully characterizes necessary and sufficient conditions for a pair of score CDFs to constitute an equilibrium. The relative orientation of $\underline{s}$ and $\overline{s}$ in Lemma 1 determines whether the equilibrium is pure or mixed. When $\underline{s} = \overline{s}$ the equilibrium is in pure strategies, with at most one developer $-k$ being active and crafting a score $\underline{s} = \overline{s}$ policy. Alternatively, when $\underline{s} < \overline{s}$ the equilibrium is in mixed strategies, with both developers crafting policies over a continuum of scores $[\underline{s}, \overline{s}]$ as well as at their respective atoms.

We now describe some key properties of pure and mixed equilibria. Later, in Proposition 2, we give parameter conditions for how the type of equilibrium depends on the status quo and the veto players' extremism in a special case with symmetric actors.

Pure Strategy Equilibria

Whenever a pure strategy equilibrium exists it takes the following form.

Lemma 2. *In a pure strategy equilibrium ($\underline{s} = \overline{s}$), the developer k with the lower monopoly score is inactive (i.e., crafts score s_0), while the other developer $-k$ crafts her monopoly policy $\left(s_{-k}^{M*}, y_{-k}^{M*}\right)$ from Proposition 1 (so that $\underline{s} = \overline{s} = s_{-k}^{M*}$).*

In any pure strategy equilibrium at least one developer must be inactive. If both were active, one would be strictly better off either dropping out or crafting a slightly-higher quality policy to win for sure. The inactive developer must have the lower monopoly score; otherwise, her opponent would strictly prefer crafting her monopoly policy rather than allowing her competitor to act as a monopolist (which results in an even worse policy for her than the status quo). Finally, the active developer must craft her monopoly policy, because absent competition her incentives are the same as a monopolist. (If both developers' monopoly scores are s_0, both remain inactive).

While the preceding explains why pure strategy equilibria take a particular form, it doesn't explain why they exist at all—why doesn't the inactive developer simply craft a policy slightly better for the decisionmaker than her opponent's policy? Indeed, that's what occurs in the model absent veto players, which lacks pure strategy equilibria even when developers differ in extremism and ability (Hirsch (2023*b*)). Our model works differently because veto players sometimes force the active monopolist to craft a policy that is sufficiently high-quality to insulate it from potential competition. Figure 3 depicts a pure strategy equilibrium for particular parameter values.

Mixed Strategy Equilibria

We now give intuition for the form of mixed equilibria ($\underline{s} < \overline{s}$) in Lemma 1, and flesh out some details via an example. One developer k has an atom at s_0, i.e., she is sometimes inactive. The size of k's atom determines the optimal policy (score, ideology, and quality) for the other developer $-k$ to develop when winning with probability $F_k(s_0)$. Developer $-k$ crafts this policy, which has score

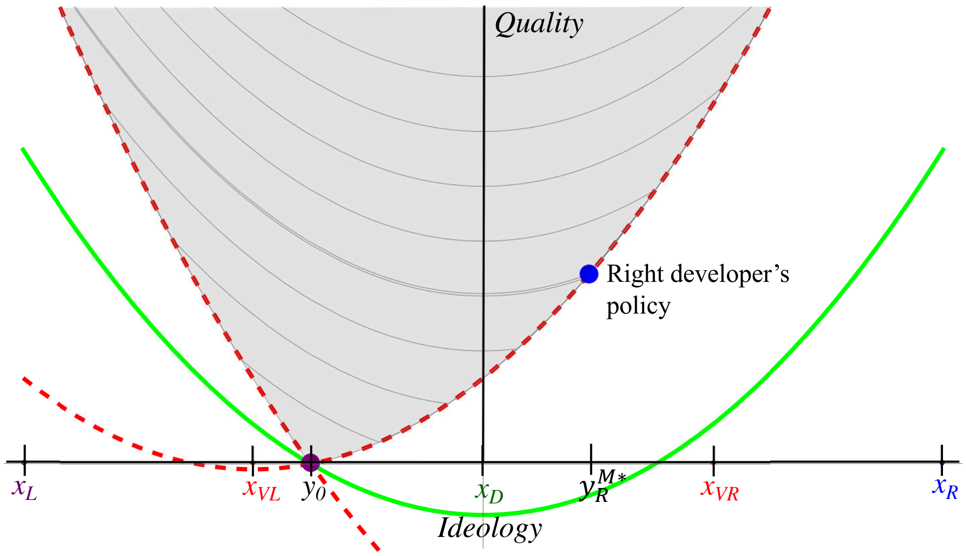

Figure 3: A Pure Strategy Equilibrium. *R develops a policy (blue dot) of sufficient quality to gain the support of the left veto player. L prefers to sit out (purple dot) rather than develop any policy that can defeat it.*

$\underline{s}$, with a probability that makes k indifferent between sitting out versus developing a policy with score $\underline{s}$ that wins with probability $F_{-k}(\underline{s})$. For scores between $\underline{s}$ and $\overline{s}$, the developers mix smoothly, maintaining each other's indifference at the margin over paying additional costs to develop higher-score policies that are more likely to be enacted.[4]

Figure 4 presents an example of a mixed strategy equilibrium. In the example, L is developer k (inactive with probability $F_L(s_0)$) and R is developer $-k$ (always active). This is intuitive because $y_0 < 0$, so R is more dissatisfied with the status quo.

Looking at R's strategy, with probability $F_R(\underline{s})$ she develops a policy at the blue dot in the

[4]As shown in the differential equations in Part 3 of Proposition B.1 in Appendix B, developer $-i$'s strategy has a density at each score that maintains i's indifference given i's expected net marginal cost of producing a higher-score policy, the possible relaxation of constraints imposed by veto players, and i's ideological benefit of winning rather than losing at that score.

right panel; otherwise she mixes smoothly over policies on the blue curve with scores in $(\underline{s}, \bar{s}]$. Her policies are constrained by the left veto player, i.e., they are on the boundary of the veto-proof set. It may seem counterintuitive that R sometimes produces a policy at score $\underline{s}$, because L (when active) never develops a score below $\underline{s}$; R could therefore develop a lower-score policy and still win with the same probability, $F_L(s_0)$. However, R doesn't just care about the decisionmaker's support; she also needs to gain the left veto player's assent. And just as a monopolist is willing to craft a policy at a score strictly greater than s_0 to gain a veto player's assent, so too is a developer whose opponent is sometimes inactive. In this example, R's optimal score-$\underline{s}$ policy trades off the up-front costs to develop a policy that gains the left veto player's assent against the benefits of getting an ideological outcome closer to her ideal point when her opponent chooses to be inactive (which occurs with probability $F_L(s_0)$).

Turning to developer L, with probability $F_L(s_0)$ she is inactive and develops no policy (the purple dot at the status quo). With the remaining probability she mixes over policies on the purple curve with scores in $(\underline{s}, \bar{s}]$. She is willing to invest in developing policies with scores just above $\underline{s}$ because they sometimes win, due to the fact that R has an atom at $\underline{s}$. In this example, L's equilibrium policies are unconstrained by the veto players, i.e., they are not on the boundary of the veto-proof set. Finally, the left panel shows that R's score CDF first order stochastically dominates L's, implying that the decisionmaker is strictly more likely to enact R's policy.

Main Results

We now state our main results. For simplicity we henceforth restrict attention to the case where developers are equally capable ($\alpha_L = \alpha_R = \alpha$), and developers and veto players are equidistant from the decisionmaker ($-x_L = x_R = x_E$, $-x_{VL} = x_{VR} = x_V$).[5] Under these assumptions, any

[5]Details for the symmetric case are in Appendices C (foundational results) and D (how results in the main text follow from these). Propositions C.1 and C.2 provide analytical results for $y_0 = 0$, including the unique mixed equilibrium when a pure equilibrium does not exist. For mixed equilibria

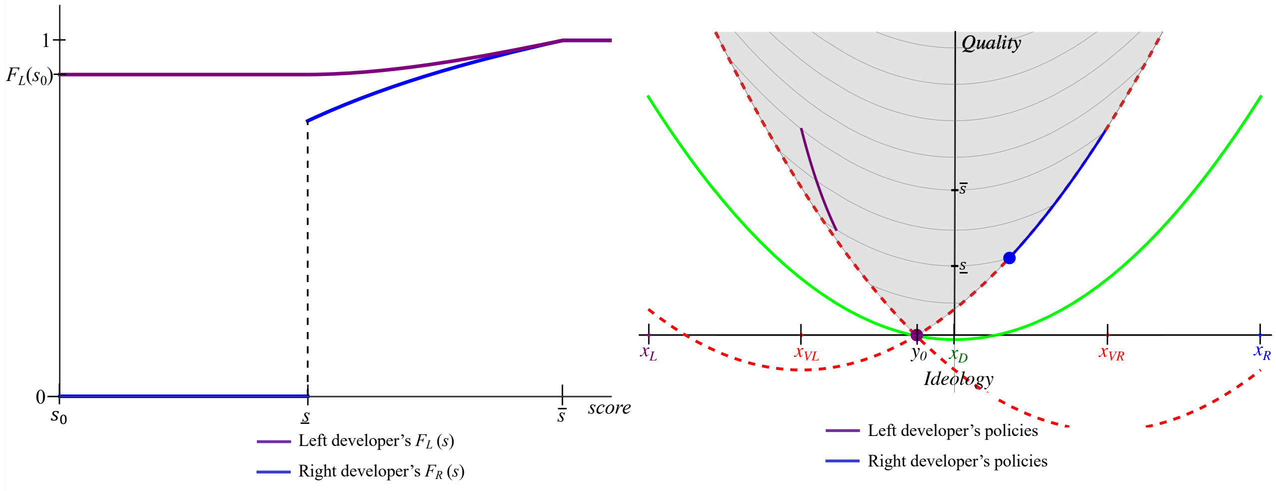

Figure 4: A Mixed Strategy Equilibrium. *Left panel depicts score CDFs. Right panel depicts policies.*

asymmetry in developers' incentives must arise from the location of the status quo.

We refer to the developer farther from the status quo as *more-motivated*, and her opponent as *less-motivated*. The more-motivated developer is more likely to engage in policy development for two reasons. First, she has more to gain: because ideological loss functions are common and convex, she places a greater value on shifts in her ideological direction from the status quo. Second, she has an easier time persuading the opposing veto player to consent to policy changes; e.g., if $y_0 < 0$, it is easier to get the left veto player to agree to a rightward policy shift than it is to get the right veto player to agree to a leftward shift.

Patterns of Activity

Patterns of activity depend on incentives to engage in policy development. What incentivizes a developer to be active? The prospect of shifting policy in her ideological direction; this is more

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with $y_0 \neq 0$, Proposition C.3 analytically proves several key properties, while exact details of developers' strategies are computed numerically using the procedure described at the end of Appendix C, based on analytical results in Proposition B.1. We do not analytically rule out coexistence of pure and mixed equilibria, or multiple mixed equilibria, but find no parameter values exhibiting equilibrium multiplicity in our computational analysis.

attractive when the alternative (either the status quo or her opponent's policy) is far from her ideal point. What deters a developer from being active? The cost of crafting a policy that can gain the support of both veto players and the decisionmaker. This is higher when the opposing veto player is an extremist, and when the opposing developer crafts a high-quality policy that is very appealing to the decisionmaker.

The interplay between these motives generates three possible equilibrium patterns of activity: (i) neither developer is active, (ii) only the more-motivated developer is active, or (iii) the more-motivated developer is always active, while the less-motivated developer is sometimes active (and equilibrium is in mixed strategies). Which pattern arises depends on the extremity of the veto players and the location of the status quo. Figure 5 provides an illustration, varying x_V (on the vertical axis, between 0 and x_E) and y_0 (on the horizontal axis, between $-x_V$ and x_V).[6]

The first possibility (that neither developer is active) occurs in the blue region of Figure 5. Here, the veto players are extreme and the status quo is moderate; each developer chooses not to develop a policy because it is too costly to get the opposing veto player's assent. The condition for this case comes from our monopoly analysis. Recall from Proposition 1 that a monopolist refrains from developing a policy if the status quo is closer to her ideal point than her monopoly policy $\hat{y}_i(x_V)$ (denoting the dependence on x_V explicitly). Thus, there is a pure strategy equilibrium exhibiting *gridlock*, in which neither developer is active, if the status quo is both to the left of L's monopoly policy, and to the right of R's monopoly policy, i.e., if it's sufficiently moderate, $y_0 \in [\hat{y}_R(x_V), \hat{y}_L(x_V)]$. From the definition of the monopoly policies $\hat{y}_i(x_V)$, we can see that the possibility of gridlock requires the veto players to be sufficiently extreme, i.e., $x_V \geq \bar{x}_V = \frac{x_E}{\alpha - 1}$ (so that $\hat{y}_R(x_V) \leq \hat{y}_L(x_V)$).

Outside of the blue region of Figure 5, at least one developer *would* be active as a monopolist. Not surprisingly, the set of active developers always includes the more-motivated one. Whether the

[6]Although the figure and our discussion are for the symmetric model, the same basic logic for which developer(s) will be active in equilibrium also holds with asymmetries in costs and preferences.

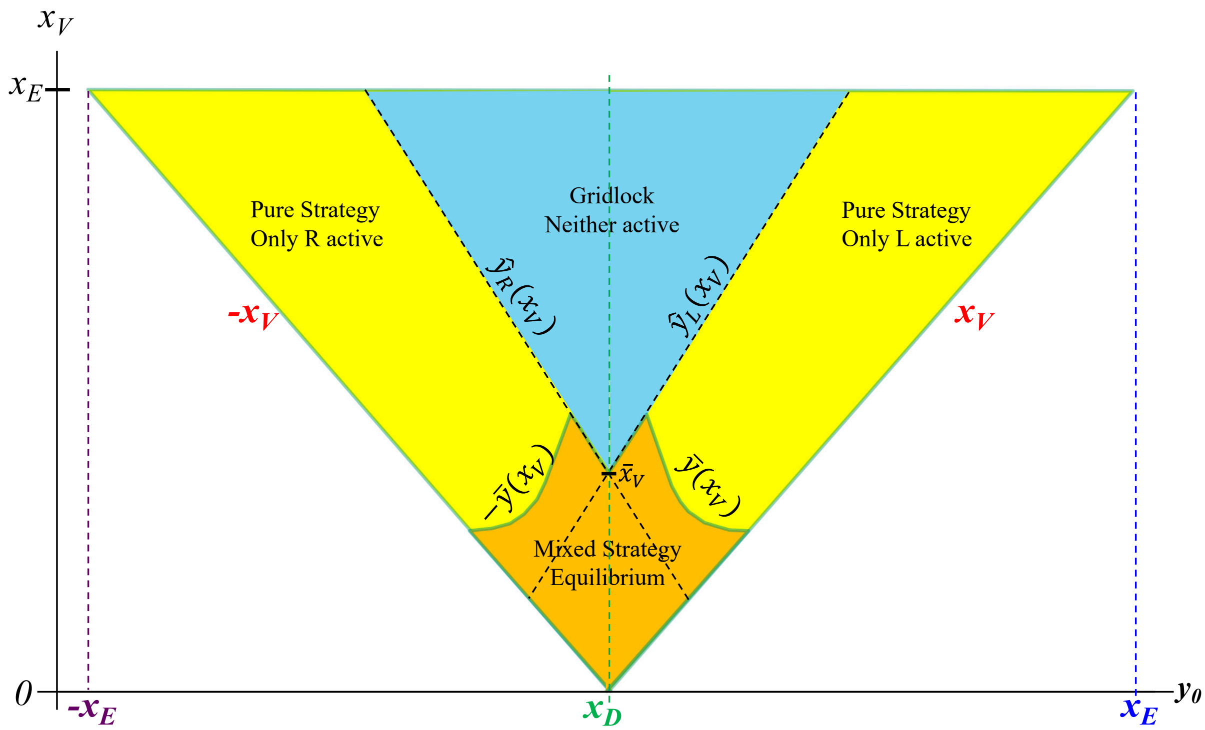

Figure 5: Patterns of Activity. *Displayed as a function of status quo and veto players' extremity*

less-motivated developer is inactive (the yellow regions) or active with strictly positive probability (the orange region) depends on what she would like to do when her more-motivated competitor acts as a monopolist; will she let her policy be enacted, or step in and develop an alternative?

Observe from Proposition 1 that the ideology of a monopolist's policy $\hat{y}_i(x_V)$ is unaffected by the status quo, but has greater quality the more distant is the status quo. Thus, if the more-motivated developer acts like a monopolist, it becomes both more difficult and less intrinsically beneficial for the less-motivated developer to craft a competing policy when the status quo is closer to her. Eventually, there will be a pure-strategy equilibrium in which the more motivated developer acts as a monopolist, and the less-motivated developer chooses to be inactive (the yellow regions in Figure 5). In the proof of Proposition 2 in Appendix D we characterize a cutpoint $\bar{y}(x_V)$ such that R is inactive if $y_0 \geq \bar{y}(x_V)$ and L is inactive if $y_0 \leq -\bar{y}(x_V)$.

Conversely, if the status quo is more moderate than $\bar{y}(x_V)$ (the orange region in Figure 5), equilibrium must sometimes involve active competition. The intuition is as follows. With moderate veto players and a moderate status quo, the more-motivated developer only needs to invest in a small amount of quality to get the opposing veto player to agree to a policy change. But if she did this, her opponent would only need to invest in a small amount of quality to swing policy back in her preferred direction. Thus, in equilibrium both developers are active—the more-motivated developer always, and the less-motivated developer with strictly positive probability—and they compete to craft policies that are appealing to the decisionmaker and acceptable to the veto players. In equilibrium, the more-motivated developer's policy is more appealing to the decisionmaker in a first-order stochastic dominance sense. In sum, we have the following analytically-derived result.

Proposition 2. *Equilibria depend on the extremism of the veto players x_V and the status quo y_0.*

1. *If $x_V \geq \bar{x}_V = \frac{x_E}{\alpha-1}$ and $y_0 \in [\hat{y}_R(x_V), \hat{y}_L(x_V)]$ there is a unique equilibrium and neither developer is active.*

2. *Otherwise, at least one developer is active:*

   (a) *The more-motivated developer is always active.*

   (b) *If $y_0 \notin [-\bar{y}(x_V), \bar{y}(x_V)]$, there is a pure-strategy equilibrium in which the less-motivated developer is inactive.*

   (c) *If $y_0 \in [-\bar{y}(x_V), \bar{y}(x_V)]$, any equilibrium is in mixed strategies and the less-motivated developer is sometimes active.*

3. *The more-motivated developer's policies have first-order stochastically higher scores, and thus are strictly more likely to be enacted than the less-motivated developer's policies.*

At a broad level, the proposition shows that asymmetric activity is a fundamental feature of our model even when the developers are equally extreme and capable, because of their differential willingness and ability to shift policy from a non-centrist status quo.

Effect of veto players' ideological extremism We next examine how the veto players' ideological extremism affects patterns of policy development. As can be seen toward the bottom of Figure 5, if veto players and the status quo are moderate, the more-motivated developer is always active but her opponent only sometimes is. As x_V increases (moving vertically), the probability that the less-motivated developer is active decreases monotonically. For sufficiently high values of x_V, even the more-motivated developer is deterred from developing an enactable policy. Formally, we have the following computationally-derived result.

Proposition 3. *The extremism of the veto players affects policy development activity as follows.*

1. *The probability that the less-motivated developer is active is strictly decreasing in x_V unless the equilibrium is in pure strategies, in which case it is constant at* 0.[7]

2. *The more-motivated developer is active if and only if the veto players are sufficiently moderate,* $x_V < \frac{\alpha|y_0|+x_E}{\alpha-1}$.

Increasingly extreme veto players reduce participation in policy development. At lower levels of extremism they make activity more asymmetric; the less-motivated developer increasingly disengages, while the more motivated developer continues to participate. At higher levels of extremism they also deter the more motivated developer from engaging, resulting in gridlock.

Changes in Senate policymaking Returning to our application, we argue that our predictions are broadly consistent with patterns of policymaking since the 1970s. It is well-established that the Senate has become increasingly polarized. More crucially, the Senate's veto players under the

[7] This property is exhibited by our computational solutions across the entire parameter space whenever the equilibrium is mixed. We also show analytically that whenever there is a pure equilibrium for a particular x_V (so that the less-motivated developer is always inactive) there remains a pure equilibrium for strictly higher values of x_V – see Appendix D.

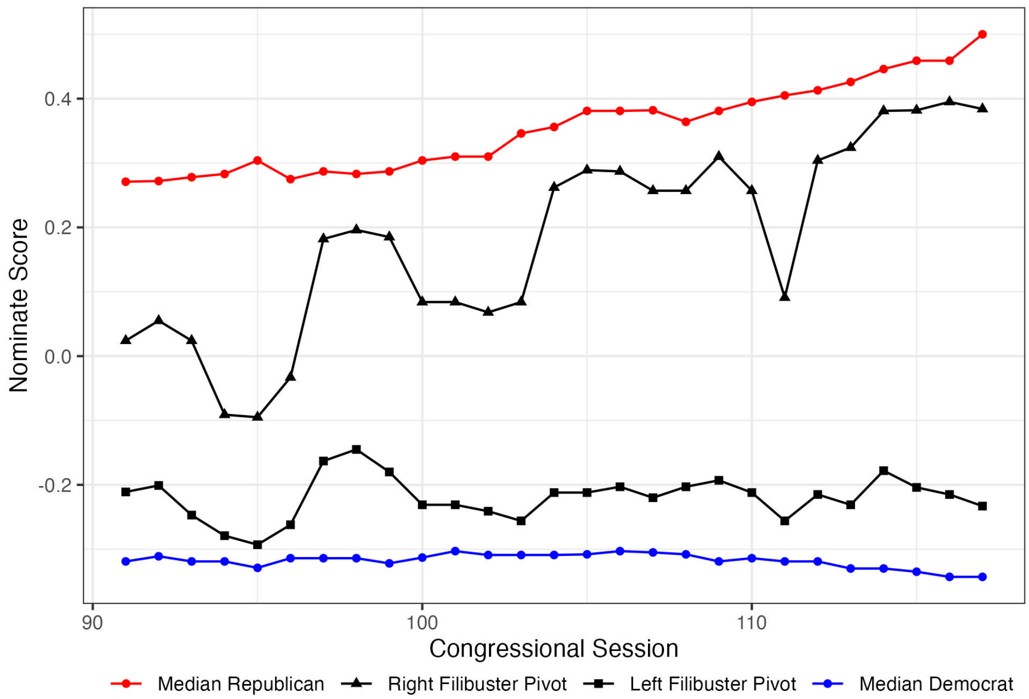

Figure 6: *Senate Filibuster Pivots since 1970s*

filibuster (the 41st and 60th most liberal senators) have diverged, as shown in Figure 6. Indeed, they have diverged more rapidly than the party medians (who we've argued are represented by the developers), so the *ratio* $\frac{x_V}{x_E}$ of veto player-to-developer extremism has increased over time. (Although Proposition 3.1 is stated in terms of x_V, the same results obtain if the ratio $\frac{x_V}{x_E}$ increases: increasingly asymmetric policy development, followed by gridlock.) Both patterns are well-documented in the empirical literature.

The first pattern—asymmetric activity—can be seen by contrasting the current highly-partisan policy development process with the traditional "textbook Congress," in which members of both parties actively worked in committees to develop serious proposals. Over time, majority leaders have played an increasingly central role in "negotiating the details of major bills" (Smith, 2011, p. 135) and "shaping the content of legislation" (Smith and Gamm, 2020, p. 216). For their part, members of the minority have disengaged from creating serious proposals, and instead adopted a strategy of obstructionism (Lee, 2016).

The second pattern—stalemate—is also well-established. It has become increasingly difficult for anyone, including majority party leaders, to get substantial new policies enacted. Nowadays, major policy changes most often occur via budget reconciliation (which doesn't require supermajorities) or during extraordinary crises such as 9/11, the financial meltdown of 2007-8, and Covid-19. For most policy issues, including salient ones, legislative gridlock and stalemate have become common (Binder, 2015).

Thus, both the increasing asymmetry in policy development activity and the overall decline in successful policy development are consistent with our model. Many scholars see these twin developments as hallmarks of the Senate's decline as an effective institution for crafting public policy. As noted by Smith (2014) (p. 14), "an institution that once encouraged creativity, cross-party collaboration, individual expression, and the incubation of new policy ideas has become gridlocked."

Decisionmaker Utility

We next analyze how veto players affect a centrist decisionmaker's utility. We do this not as a normative measure of social welfare, but rather to make predictions about institutional design choices in settings where centrists have decisive power over such choices.

In spatial models without policy development the decisionmaker always benefits from eliminating veto players, because doing so allows him to revise any non-centrist status quo $y_0 \neq 0$ to reflect his own ideal ideology. With policy development, however, veto players don't always induce gridlock – policy change can still occur if a developer crafts a sufficiently-high quality policy to gain the veto players' assent (see also Hitt, Volden and Wiseman (2017)). This opens up the possibility that veto players may benefit the decisionmaker.

Without veto players We first establish a baseline for decisionmaker utility absent veto players. This is *not* the decisionmaker's utility for a zero-quality policy at his ideal point (as it would be in a classic spatial model), but rather his expected utility from competitive policy development without

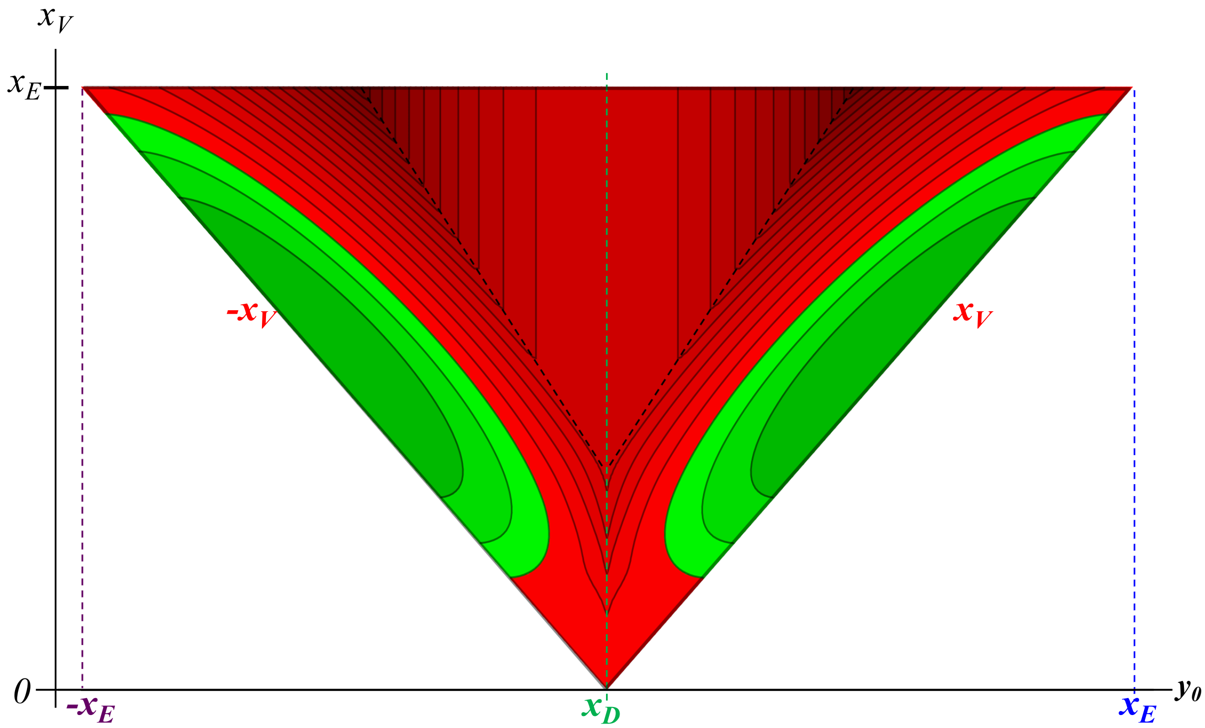

Figure 7: Decisionmaker's Net Utility Gain from Eliminating Veto Players. *Red region shows gains and green shows losses from eliminating veto players. Darkness of shading indicates magnitude of gain or loss.*

veto players. In the Hirsch and Shotts (2015) model, both developers are always active, and mix over policies with strictly positive scores.

Corollary 2. *Absent veto players the decisionmaker's utility is*

$$EU_D^0 = 4x_E^2 \left(\int_0^1 2F \left(\int_0^F \frac{G}{\alpha\,(\alpha - G)} dG \right) dF \right) = 4x_E^2 \left(\alpha + \frac{1}{2} - \frac{2}{3\alpha} - (\alpha^2 - 1) \ln \left(\frac{\alpha}{\alpha - 1} \right) \right).$$

This utility does not depend on the status quo because it is "as if' the status quo is the decisionmaker's ideal point with 0 quality when there is unrestricted competition. Also note that $EU_D^0 > 0$; a unitary decisionmaker strictly benefits from competitive policy development relative to enacting his own ideal with zero quality.

With veto players We next examine the decisionmaker's expected utility with veto players using Proposition 2, which we denote as $EU_D^{VP}(x_V, y_0)$. When there is gridlock, $EU_D^{VP}(x_V, y_0) = -y_0^2$, which is unambiguously worse than his utility from competitive policy development absent veto players. When there is a pure strategy equilibrium with one active developer, $EU_D^{VP}(x_V, y_0)$ is the score of that developer's monopoly policy from Proposition 1. When there is a mixed strategy equilibrium we calculate $EU_D^{VP}(x_V, y_0)$ using numerical integration (see Appendix C.3). Comparing $EU_D^{VP}(x_V, y_0)$ against EU_D^0 yields the following.[8]

Proposition 4. *The decisionmaker prefers to eliminate the veto players if they* or *the status quo are sufficiently moderate. Otherwise he prefers to maintain them.*

Figure 7 illustrates when the decisionmaker would be better off eliminating the veto players as a function of their extremism (on the vertical axis) and the location of the status quo (on the horizontal axis). In the red region the decisionmaker benefits from eliminating them; clearly, this includes the region where veto players induce gridlock (the inner triangle). Conversely, in the green region he benefits from preserving them.

The figure has three important features. First, there *is* a green region—in contrast to a classic spatial model, the decisionmaker can sometimes benefit from veto players. Second, a necessary condition for the decisionmaker to benefit is that the status quo is sufficiently noncentrist—this contrasts with a classic spatial model, in which the worst status quos for a centrist are those gridlocked far from his ideal point. Third, *observable* competition isn't necessary for this benefit to obtain;

[8]This result is derived using a mixture of analytic and numerical analysis; see Appendix D. We analytically derive necessary and sufficient conditions for a pure strategy equilibrium with veto players, as well as decisionmaker utility when these conditions hold. We further prove analytically that the decisionmaker is strictly worse off with veto players whenever $y_0 = 0$. When $y_0 \neq 0$ and the equilibrium with veto players is mixed, we conduct the utility comparison using numerical integration of the computationally-derived mixed equilibria, which are always unique.

indeed, within most of the green region in Figure 7 only the more-motivated developer is active, as can be seen by comparison with Figure 5.

Why can a centrist decisionmaker benefit from veto players when the status quo is very noncentrist? Because one developer is highly motivated to change it, and a somewhat-extreme opposing veto player demands a high quality alternative in order to do so. Why does this coincide with reduced policy development activity? Because these demands force the motivated developer to craft an appealing policy that is difficult to defeat (see Corollary 1), thereby deterring her less-motivated competitor from trying. The surprising empirical implication is that the absence of observable competition—and *apparent* monopoly over development by one side—is not prima facie evidence of dysfunctional politics or agenda control. Rather, in the presence of veto players this occurs when the status quo is extreme, and only one side is highly motivated to change it.

Having discussed when and why the decisionmaker can benefit from veto players, we now discuss what can go wrong, i.e., what happens in the red region of Figure 7. Veto players can have three distinct negative effects: (i) dampening productive competition, (ii) inducing gridlock, and (iii) allowing for new non-centrist policies that are relatively low quality.

The first effect occurs when the veto players and the status quo are very moderate, as in the bottom center of Figure 7. Here, policy change is easy to achieve, but the developers aren't highly motivated to invest in quality because the status quo is also moderate. Although equilibrium involves both developers sometimes being active (see Figure 5), veto players simply dampen the intensity of their productive competition. This is because veto players limit both the upside of engaging in development (by constraining policy change in one's own direction), and the downside of disengaging from development (by constraining policy change in one's opponent's direction).

The second effect occurs when veto players are more extreme but the status quo is still moderate. In this case, veto players demand high quality to consent to change, but a moderate status quo limits the developers' motivation to provide this quality. The result is gridlock, with both developers declining to craft a new policy (the triangular region in the top center of Figure 7, which corresponds to the blue triangle in Figure 5). Veto-player induced gridlock in our model is actually worse for the decisionmaker than in the classic spatial model because it does not just stop the decisionmaker from getting his ideal; it also prevents productive competition.

The third effect occurs when the veto players are more extreme and the status quo is neither sufficiently moderate to induce gridlock, nor sufficiently extreme to adequately motivate the more-distant developer. This effect dominates in the portions of the red region in Figure 7 for which only the more-distant developer is active (i.e., the overlap with the yellow regions of Figure 5). Here the active developer crafts a non-centrist policy of sufficient quality to gain veto players' support over the status quo, but of insufficient quality to surpass the benefit from unconstrained competition.

A final property worth noting is that if policy development costs are sufficiently high ($\alpha > \tilde{\alpha} \approx 3.68$, as in Figure 7 where $\alpha = 3.75$), then it is not only *very moderate* veto players who are harmful regardless of the status quo – it is *very extreme* ones as well. Under these circumstances, no feasible status quo is extreme enough to induce the more-motivated developer to craft a policy better for the decisionmaker than what he would have received under unconstrained competition.

Filibusters We last use our model to reexamine a critical question in legislative studies: why does the Senate allow a submajority of 41 members to block legislation that a majority prefers to the status quo? The Senate is a self-organizing body, and both constitutional scholarship and Senate history support the proposition that a simple majority may eliminate or modify the filibuster (Gold and Gupta, 2004). However, as documented by Binder and Smith (2001), there has there never been a Senate majority in support of eliminating the filibuster by reducing the cloture requirement to 51 votes. Most recently, in 2022 the Senate voted 52-48 against a one-time exception to the filibuster to pass a voting rights bill. At the time, 21 Democrats supported eliminating the filibuster, 27 supported changes such as requiring a "talking filibuster," and two of the most moderate (Manchin and Sinema) opposed any changes (Rieger and Adrian, 2022).

In classic spatial models, supermajority rules harm centrists by preventing them from altering policies to reflect their own ideal point, so centrists' support for the filibuster presents a puzzle. One previously-offered explanation is that centrists use supermajority requirements to counterbalance the power of non-centrist agenda-setters (Peress (2009); Krehbiel and Krehbiel (2023)). As previously described, however, it is unclear whether formal agenda setting power is actually robust in the Senate. Moreover, if it is, then such agenda power can *only* exist with the consent of a Senate majority (Krehbiel, 1992); any theoretical explanation of the filibuster that relies on formal agenda power must therefore also explain why the majority would add an additional procedure (the filibuster) to address its shortcomings, rather than simply revoke it.

In contrast, our model shows that even absent formal agenda power centrists can benefit from supermajority requirements that create de facto veto players, because they can encourage the development of higher quality and more moderate policies. As shown in Proposition 4 and Figure 7, centrists are most likely to benefit from the filibuster when the 41th and 60th most liberal Senators are somewhat non-centrist and the status quo is also non-centrist. A non-centrist status quo could occur in policy areas that are rapidly changing such as financial regulation or health care; given limits on Senators' time and attention, the issues most likely to receive legislative attention are arguably precisely such issues.

Finally, it's important to note that our model does not imply that centrists *always* benefit from the filibuster. Rather, as shown in Proposition 4 and Figure 7, there are several circumstances where veto players are harmful. In the context of recent debates about the filibuster, one is particularly relevant: when veto players are very extreme and policy development is very costly. Does this describe the contemporary Senate? As shown in Figure 6, the filibuster pivots have indeed become increasingly polarized over time. Simultaneously, Congress has disinvested in its own policymaking capacity by reducing the proportion of staffers who work on policy in members' offices (Crosson et al., 2021), limiting personnel funding, and reducing funding for agencies like the CBO, CRS, and GAO

(Reynolds, 2020). Indeed, scholars and commentators have become concerned that it is increasingly difficult for members' offices to craft high-quality policies. Thus, our model suggests that although centrists may have benefitted from the filibuster in the past, calls for reform may become increasingly persuasive if these trends persist.

Conclusion

In this paper we have analyzed how veto players affect competitive policy development. Absent veto players, competing developers always craft policies that benefit a centrist decisionmaker. The effect of veto players depends on the status quo. If veto players are quite moderate, then they dampen productive competition, thereby making the decisionmaker worse off. If they are sufficiently noncentrist, however, a developer dissatisfied with the status quo is willing to work hard to craft a high-quality alternative, and an opposing veto player forces her to do so, thereby benefitting the decisionmaker. By implication, veto players benefit a centrist decisionmaker precisely when standard spatial models predict that they are most harmful, i.e., when the status quo is non-centrist. Under such circumstances, the developer satisfied with the status quo often refrains from crafting a competing policy, reflecting the fact that a veto player has already forced her competitor to craft a reasonably-moderate and high quality policy. Veto players are thus most beneficial to the decisionmaker when they also inhibit observable competition.

Our model yields testable predictions on the number of well-developed policy proposals that are created for a given issue: multiple serious proposals are likely when the status quo is centrist or when veto players are absent. It further yields predictions about the quality of policies that are adopted; centrist policies that are enacted tend to be of lower quality relative to noncentrist ones, because the latter must be more carefully crafted to gain broad approval.

Our model also has surprising implications for the allocation of policymaking capacity in Congress. A natural intuition is that the best way to allocate policymaking capacity in polarized times is to invest in shared resources that are accessible to all members. Our model suggests that this intuition may be off-target; reformers may do better by instead giving resources to non-centrist policy developers. A natural fear is that such developers will exploit their capacity to further extreme objectives, as suggested by the pejorative characterization of "adversarial clientilism" in Drutman and LaPira (2020). However, if the developers are constrained—by potential competition or opposing veto players—then they need to craft policies that are sufficiently high-quality and moderate to actually be enacted.

Our model suggests a number of avenues for future work. First, we do not consider dynamic effects, as in models of pivotal politics where yesterday's policies become today's status quos (Buisseret and Bernhardt (2017), Dziuda and Loeper (2018)). It would be interesting to examine how such dynamic considerations shape legislators' preferences over quality; for example, might a legislator actually prefer that a policy far from her ideological ideal be low-quality so as to make it easier to dislodge in the future? (e.g., Callander and Martin (2017)).

Second, we have focused on the effects of eliminating veto players, because that is the focus of recent debates about the filibuster. However, it also would be possible to shift their location by raising or lowering the supermajority threshold, rather than completely doing away with the filibuster; this is a natural topic for future work.

Finally, our model also could be extended to examine broader questions about institutional design. Suppose the decisionmaker could determine both the location of veto players (either directly or indirectly via supermajority rules) and the resources for each developer (e.g., party or committee staff allocations); would empowering veto players and granting policy development resources be used as complements or substitutes? And how do these tools affect the ideology of the resulting policies? Suppose the decisionmaker could also choose the developers' ideologies, as in a chamber majority choosing the chairs of key legislative committees; would supermajority constraints and "preference outlier" committee chairs be linked, and if so how? We hope to explore these and other avenues in future work.

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Supporting Information for

Veto Players and Policy Development

Contents

A General Equilibrium Analysis

We use notation DM (decisionmaker), DEV (developer), and VP (veto player). The following definitions are easily verified.

Definition A.1.

1. *The implied quality of a policy* (s, y) *is* $q = s + y^2$, *and the score of the status quo is* $s_0 = -y_0^2$.

2. *Player* i*'s utility for policy* (s, y) *is* $V_i(s, y) = U_i\left(y, s + y^2\right) = -x_i^2 + s + 2x_i y$.

3. *DEV* i*'s cost to craft policy* (s, y) *is* $\alpha_i\left(s + y^2\right)$.

4. *All VPs weakly prefer policy* (s, y) *to the status quo if and only if* $s \geq s_0$ *and* $y \in Y_V(s) = [z_L(s), z_R(s)]$, *where* $z_i(s) = y_0 - \frac{s - s_0}{2x_{V-i}}$.

A.1 Proof of Proposition 1 (Equilibrium of Monopoly Variant)

(s_0, y_0) is the unique veto-proof policy already available to DM, and any policy both VPs weakly prefer to (s_0, y_0) is also weakly preferred by DM. So it's optimal for DM to propose whatever the monopolist crafts – any other feasible proposal is vetoed and proposing DEV's policy only results in an outcome different from (s_0, y_0) when it's veto proof and therefore also weakly preferred by DM over (s_0, y_0). We henceforth restrict attention to such profiles.

Thus the game is as if DEV is an agenda setter directly proposing to the VPs, who break indifference in her favor. Wlog we restrict DEV's choice space to veto-proof policies since (s_0, y_0) is a free veto-proof option and crafting any non veto-proof policy results in the same outcome at weakly higher cost. It is easily verified that an optimal (s_i^{M*}, y_i^{M*}) for DEV must satisfy

$$\underset{\{(s_i,y_i):s_i\geq s_0,y_i\in[z_L(s_i),z_R(s_i)]\}}{\arg\max}\left\{\underbrace{-\left(\alpha_i-1\right)s_i}_{\text{score effect}}+\underbrace{2x_iy_i-\alpha_iy_i^2}_{\text{ideology effect}}\right\}. \tag{A.1}$$

Wlog suppose DEV is $i = R$. Our assumptions that $y_0 \in [x_{VL}, x_{VR}]$ and $x_{VR} \leq x_R$ imply $y_0 \leq x_R$, so DEV wishes to move policy rightward. We proceed in three steps. *Step 1.* R never develops (s_R, y_R) with $y_R < y_0$ because she would be better off proposing (s_0, y_0) at zero cost. Because $x_{VL} < 0 < x_{VR}$, any policy developed with $y_R \geq y_0$ is veto proof iff the left VP weakly prefers it to (s_0, y_0). *Step 2.* At any $y_R \geq y_0$, R's optimal policy must satisfy $y_R = z_R(s_R)$; because $\alpha_R > 1$, for any policy not on the boundary of the veto proof set, R is strictly better off developing a lower quality policy at the same ideology on the boundary.

Step 3. We find the optimal (s_R, y_R) for R with $y_R \geq y_0$ and $y_R = z_R(s_R)$. Applying Step 2 and Def. A.1, the optimal s_R at y_R is $s_R = 2x_{VL}(y_0 - y_R) - s_0$. Substituting into Eq. A.1, R maximizes $-(\alpha_R - 1)(2x_{VL}(y_0 - y_R) - s_0) + 2x_R y_R - \alpha_R y_R^2$. Differentiating w.r.t. y_R and setting $= 0$ yields $\hat{y}_R = \frac{1}{\alpha_R} x_R + \left(1 - \frac{1}{\alpha_R}\right) x_{VL}$. For $y_0 \leq \hat{y}_R$, DEV optimally crafts a policy at $\hat{y}_R$. For $\hat{y}_R < y_0$ she is strictly better off sitting out than with any (s_R, y_R) with $y_R > y_0$ and $y_R = z_R(s_R)$.

A.2 Preliminary Analysis of Competitive Model

A DEV pure strategy (s_i, y_i) is a two-dimensional element of the set of scores and ideologies implying weakly positive-quality policies: $\mathbb{B} \equiv \left\{(s, y) \in R^2 \mid \left(s - y_0^2\right) + y^2 \geq 0\right\}$. A mixed strategy

σ_i is a probability measure over the Borel subsets of $\mathbb{B}$.[1] We first justify restricting attention to strategy profiles consisting of only veto-proof policies.

Lemma A.1. *Consider an equilibrium strategy profile in which the DEVs sometimes develop policies that fail veto-proofness; then the modified strategy profile in which each DEV develops the status quo whenever the original profile called for her to develop a policy that failed veto-proofness is also an equilibrium that yields the same distribution over outcomes and payoffs.*

Proof: Because $x_{VL} < 0 < x_{VR}$, (s_0, y_0) is the unique score-minimizing policy among those that are veto proof, and the unique veto-proof policy that is 0-quality. Now consider any $\{(s_i, y_i), (s_{-i}, y_{-i})\}$ s.t. i's policy fails veto-proofness; we argue that the alternative profile $\{(s_i = s_0, y_i = y_0), (s_{-i}, y_{-i})\}$ in which i develops (s_0, y_0) yields the same probability distribution over outcomes. If (s_{-i}, y_{-i}) is both veto-proof and distinct from (s_0, y_0) then $s_{-i} > 0$, in either profile DM is strictly better off proposing (s_{-i}, y_{-i}) than any other feasible policy, and it will be accepted for sure. Otherwise, in either profile any feasible proposal either fails veto-proofness or is equal to (s_0, y_0), so any feasible proposal by DM results in (s_0, y_0).

The preceding observation yields the desired result through a series of observations. First, in any equilibrium strategy profile, each DEV must never develop a strictly positive-quality policy that fails veto proofness; a DEV who did so could profitably deviate to a strategy in which she instead develops (s_0, y_0), because outcomes would be unaffected and she would strictly save on costs. Second, whenever developing a 0-quality policy that fails veto-proofness is a best response for i, so too is developing (s_0, y_0); thus, altering i's equilibrium strategy to have her develop (s_0, y_0) whenever she previously developed a 0-quality policy that failed veto-proofness remains a best response for i. Finally, altering i's equilibrium strategy to have her develop the status quo whenever she previously developed a 0-quality policy that failed veto proofness does not change $-i$'s utility from developing any (s_{-i}, y_{-i}), and therefore the set of strategies that are a best response for her. QED

Having restricted strategy profiles to veto-proof policies Y_V, let $F_i(s)$ denote the CDF over scores induced by i's mixed strategy σ_i; when both DEVs' policies are veto proof, a policy with the strictly highest score is the outcome. We derive necessary and sufficient equilibrium conditions in a series

[1] For technical convenience we restrict attention to strategies generating score CDFs that can be written as the sum of an absolutely continuous and a discrete distribution.

of lemmas. Let $\bar{\Pi}_i(s_i, y_i; \sigma_{-i})$ denote i's expected utility for developing $(s_i, y_i) \in Y_V$ if a score tie is broken in her favor, which is i's utility from developing a policy with $s_i \geq s_0$ where $-i$ has no atom. Regardless of whether $-i$ has an atom at s_i, i can achieve utility arbitrarily close to $\bar{\Pi}_i(s_i, y_i; \sigma_{-i})$ by developing an $\varepsilon-$higher score policy, so

$$\bar{\Pi}_i(s_i, y_i; \sigma_{-i}) = -\alpha_i\left(s_i + y^2\right) + F_{-i}(s_i) \cdot V_i(s_i, y_i) + \int_{s_{-i} > s_i} V_i(s_{-i}, y_{-i})\, d\sigma_{-i}. \tag{A.2}$$

The first term is the up-front cost of quality. With probability $F_{-i}(s_i)$, i's opponent develops a policy with a lower score; in this case, i's policy is enacted, yielding utility $V_i(s_i, y_i)$. With the remaining probability $-i$'s policy is enacted, yielding utility $V_i(s_{-i}, y_{-i})$. Only the first two terms of Eq A.2 are affected by y_i. Differentiating yields $-2\alpha_i y_i + 2F_{-i}(s_i)\, x_i$, which is strictly decreasing in y_i. Given s_i, there's a unique *strictly* optimal value of y_i in the veto-proof interval $[z_L(s_i), z_R(s_i)]$.

Lemma A.2. *At any score $s \geq s_0$ where $F_{-i}(\cdot)$ has no atom or i would win in a tie, $(s, y_i^*(s))$ is i's strictly best score-s policy where $y_i^*(s) = \hat{y}_i(s; F_{-i}(s))$ and*

$$\hat{y}_i(s; F_{-i}) = \min\left\{\max\left\{z_L(s), \frac{x_i}{\alpha_i}F_{-i}\right\}, z_R(s)\right\}.$$

Lemma A.2 states that at $s > s_0$, i's best combination of ideology and quality to generate a veto-proof policy is unique. The optimal ideology is the closest veto-proof ideology to the *unconstrained optimal ideology* $\frac{x_i}{\alpha_i}F_{-i}(s)$, which only depends on the score s *indirectly* through its impact on the probability of winning $F_{-i}(s)$. We say a strategy profile satisfies *ideological optimality* if the DEVs target the strictly best veto proof ideologies (i.e. $y_i = y_i^*(s_i)$) with probability 1.

We next establish the absence of ties at scores $> s_0$, a property we term *no ties.*

Lemma A.3. *In equilibrium there is 0-probability of a tie at scores $s > s_0$.*

Proof: Suppose not, so there's a strictly positive probability of a tie at a score $s > s_0$; we show at least one DEV must have a strictly profitable deviation. Let $p_i^s > 0$ denote the probability developer i crafts a score-s policy; this may involve mixing over distinct policies with the same score. Next let $\bar{y}_i^s$ be the *expected ideology of i's policy* conditional on crafting a score-s policy; the up-front cost to i of crafting $(s, \bar{y}_i^s)$ is weakly lower than the *expected cost* of mixing according to her strategy conditional on crafting a score s policy (since from part 3 of Def A.1 quality costs are convex in ideology holding score fixed). Third, let $y_D^s = \max\{\min\{0, z_R(s)\}, z_L(s)\}$; this is the closest veto-proof ideology to

0, and from part 3 of Def A.1 is the *cheapest* score-s veto-proof ideology to target. Finally, let $\bar{y}^s$ denote the *expected ideology of the final policy outcome* conditional on a tie at score s.

To see at least one DEV has a strictly profitable deviation, first observe each DEV can achieve her equilibrium utility by mixing according to her strategy conditional on developing only score-s policies; it thus suffices to show at least one can do strictly better deviating. Next recall each DEV's policy utility $V_i(s, y)$ is linear in y, so getting (s, y) for sure or a mix of score-s outcomes with expected ideology y yields identical policy utility. There are then three subcases to consider.

Suppose first $\bar{y}^s \neq y_D^s$. Because the DEVs wish to move ideology in strictly opposite directions holding score fixed (part 2 of Def A.1), exactly one DEV k *strictly* prefers policy (s, y_D^s) to policy $(s, \bar{y}^s)$, implying her policy utility from getting the former for sure is strictly higher than her expected policy utility from a tie at score s. If k plays according to her strategy conditional on crafting a score-s policy, she gets policy utility $V_k(s, \bar{y}_k^s)$ with probability $F_{-k}(s) - p_{-k}^s$ (when her opponent crafts a policy with *strictly* lower score than s) and $V_k(s, \bar{y}^s)$ with probability p_{-k}^s (when her opponent crafts a score-s policy). If she were instead to deviate and craft (s, y_D^s) with probability $\frac{p_{-k}}{F_{-k}(s)}$ and $(s, \bar{y}_k^s)$ with probability $1 - \frac{p_{-k}}{F_{-k}(s)}$ *and always win in a tie at score* s, she would get policy utility $V_k(s, \bar{y}_k^s)$ with probability $F_{-k}(s) - p_{-k}^s$ and policy utility $V_k(s, y_D^s) > V_k(s, \bar{y}^s)$ with probability p_{-k}^s. This would be strictly profitable since the latter strategy yields strictly higher policy utility at a weakly lower up-front cost. Finally, while k may not be able to achieve exactly this utility (since she's not assured to win in a tie at score s) she can achieve utility arbitrarily close to it using otherwise-identical policies with scores just above s, and therefore has a strictly profitable deviation.

Suppose next that $y_D^s = \bar{y}^s$. If at least one DEV k crafts a policy other than (s, y_D^s) with strictly positive probability, then the deviation in the preceding paragraph is profitable, because she would strictly save on the up-front cost of policy development when crafting $(s, \bar{y}_k^s)$.

Suppose finally that $y_D^s = \bar{y}^s$ and both DEVs craft (s, y_D^s) with probability 1 conditional on crafting a score-s policy. Then each DEV's utility from crafting (s, y_D^s) is as if she always wins in a tie so her equilibrium utility is exactly $\bar{\Pi}_i(s, y_D^s; \sigma_{-i})$. If at least one DEV k's strictly-best veto proof ideology $y_k^*(s)$ at score s from Lemma A.2 differs from y_D^s, she has a strictly profitable deviation to developing a policy at ideology $y_k^*(s)$ with score just above s. Alternatively, if both DEVs' strictly-best veto proof ideologies at score s are exactly y_D^s, then (using the definition of $y_i^*(s)$ in Lemma A.2 and that $F_i(s) > 0\ \forall i$) y_D^s must be on the boundary of the veto-proof set

($y_D^s = z_j(s)$ for some $j \in \{L, R\}$). If $-j$ were to deviate by crafting (s_0, y_0) her net utility gain would be $\alpha_{-j}\left(s + (z_j(s))^2\right) + \int_{s_0}^{s}(V_{-j}(s_j, y_j) - V_{-j}(s, z_j(s)))\, d\sigma_j$. Finally, since $|x_i| \geq |x_{Vi}|$ $\forall i$, we have that $(s, z_j(s))$ is the *weakly worst veto proof policy with score* $s_{-j} \in [s_0, s]$ for $-j$; hence the utility gain is *strictly* positive and this is a profitable deviation. QED

Lemmas A.2 – A.3 jointly imply that in equilibrium, DEV i can compute her expected utility *as if* her opponent only crafts veto-proof policies of the form $\left(s_{-i}, y_{-i}^*(s_{-i})\right)$. Her expected utility from crafting *any* veto-proof policy (s_i, y_i) with score $s_i \geq s_0$ where $-i$ has no atom (or a tie would be broken in i's favor) is therefore $\bar{\Pi}_i^*(s_i, y_i; \mathbf{F}) = -\alpha_i\left(s_i + y_i^2\right) + F_{-i}(s_i) \cdot V_i(s_i, y_i) + \int_{s_i}^{\infty} V_i\left(s_{-i}, y_{-i}^*(s_{-i})\right) dF_{-i}$, and her utility from crafting the *best* veto-proof policy with score s_i (where $-i$ has no atom or a tie would be broken in her favor) is $\bar{\Pi}_i^*(s_i, y_i^*(s_i); \mathbf{F}) =$

$$\bar{\Pi}_i^*(s_i; \mathbf{F}) = -\alpha_i\left(s_i + [y_i^*(s_i)]^2\right) + F_{-i}(s_i) \cdot V_i(s_i, y_i^*(s_i)) + \int_{s_i}^{\infty} V_i\left(s_{-i}, y_{-i}^*(s_{-i})\right) dF_{-i}. \qquad \text{(A.3)}$$

We now establish several useful properties.

Lemma A.4. *$\bar{\Pi}_i^*(s; \mathbf{F})$ is right-continuous, and $\lim_{s \to \hat{s}^-}\left\{\bar{\Pi}_i^*(s; \mathbf{F})\right\} \leq \bar{\Pi}_i^*(\hat{s}; \mathbf{F}) \forall \hat{s} > s_0$ when the strategy profile satisfies ideological optimality and no ties.*

Proof: $\bar{\Pi}_i^*(s; \mathbf{F})$ inherits right-continuity of $F_{-i}(s)$; its only potential points of discontinuity over $\hat{s} > s_0$ are at scores where $-i$ has an atom. We show that $\lim_{s \to \hat{s}^-}\left\{\bar{\Pi}_i^*(s; \mathbf{F})\right\} \leq \bar{\Pi}_i^*(\hat{s}; \mathbf{F})$ for $\hat{s} > s_0$. The property holds if $\bar{\Pi}_i^*(s; \mathbf{F})$ is continuous at $\hat{s}$, so suppose $-i$ has an atom at $\hat{s}$; then by Lemma A.3 i does not, and at the atom $-i$ develops $\left(\hat{s}, y_{-i}^*(\hat{s})\right)$. Let $y_i^{\hat{s}-} = \lim_{s \to \hat{s}^-}\{y_i^*(s)\}$ denote i's optimal ideology if she were to just lose at score $\hat{s}$. Then $\bar{\Pi}_i^*\left(\hat{s}, y_i^{\hat{s}-}; \mathbf{F}\right) - \lim_{s \to \hat{s}^-}\left\{\bar{\Pi}_i^*(s; \mathbf{F})\right\} = p_{-i}^{\hat{s}}\left(V_i\left(\hat{s}, y_i^{\hat{s}-}\right) - V_i\left(\hat{s}, y_{-i}^*(\hat{s})\right)\right)$. Finally, we have $V_i\left(\hat{s}, y_i^{\hat{s}-}\right) - V_i\left(\hat{s}, y_{-i}^*(\hat{s})\right) \geq 0$ because we have $V_i\left(\hat{s}, y_i^{\hat{s}-}\right) \geq V_i\left(\hat{s}, y_D^{\hat{s}}\right) \geq V_i\left(\hat{s}, y_{-i}^*(\hat{s})\right)$, recalling that $y_D^{\hat{s}}$ is defined in the proof of Lemma A.3 as the ideology closest to 0 that may be attached a score-$\hat{s}$ policy and remain veto-proof. The first inequality comes from Lemma A.2 applied to i; the second inequality comes from Lemma A.2 applied to $-i$. Finally $\bar{\Pi}_i^*(\hat{s}; \mathbf{F}) \geq \bar{\Pi}_i^*\left(\hat{s}, y_i^{\hat{s}-}; \mathbf{F}\right)$ from the definition of $\bar{\Pi}_i^*(\hat{s}; \mathbf{F})$. QED

We next show that in equilibrium any score in the support of DEV i's score CDF $F_i(s)$ must maximize $\bar{\Pi}^*(s_i; \mathbf{F})$, a property we term *score optimality*.

Lemma A.5. *For all i and $\hat{s}$ in the support of $F_i(\cdot)$, $\bar{\Pi}_i^*(\hat{s}; \mathbf{F}) = \max_{s \geq s_0}\left\{\bar{\Pi}_i^*(s; \mathbf{F})\right\}$.*

Proof: First, equilibrium requires $\max_{s\geq s_0}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}$ to be well-defined; otherwise i would not have a best response. Next, for any $\hat{s}\geq s_0$, $\bar{\Pi}_i^*(\hat{s};\mathbf{F})\leq\max_{s\geq s_0}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}$; hence it suffices to show that $\hat{s}$ in the support of F_i implies $\bar{\Pi}_i^*(\hat{s};\mathbf{F})\geq\max_{s\geq s_0}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}$.

Suppose first that $\hat{s}\geq s_0$ is the support of $F_i(\cdot)$ but $\Pr(s_i\in(\hat{s}-\epsilon,\hat{s}))=0$ for sufficiently small ϵ. Then either i has an atom at $\hat{s}$ or $\Pr(s_i\in(\hat{s},\hat{s}+\epsilon))>0\ \forall\epsilon$. If i has an atom at $\hat{s}$ her utility at $\hat{s}$ must be exactly $\bar{\Pi}_i^*(\hat{s};\mathbf{F})$ – either because $\hat{s}>s_0$ and $-i$ has no atom there by Lemma A.3, or because $\hat{s}=s_0$ and there is a unique veto proof ideology y_0 (so the DM's tie breaking rule doesn't matter). Regardless, $\lim\limits_{s\to\hat{s}^+}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}=\bar{\Pi}_i^*(\hat{s};\mathbf{F})\geq\max_{s\geq s_0}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}$; otherwise i would have a strictly profitable deviation taking probability weight from $\hat{s}$ or a neighborhood above and reallocating to scores yielding utility arbitrarily close to $\max_{s\geq s_0}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}$.

Suppose next that $\hat{s}\geq s_0$ is in the support of $F_i(\cdot)$ and $\Pr(s_i\in(\hat{s}-\epsilon,\hat{s}))>0\ \forall\epsilon>0$. Then $\hat{s}>s_0$ (by the restriction to veto-proof policies) and $\lim\limits_{s\to\hat{s}^-}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}\geq\max_{s\geq s_0}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}$, since otherwise i would have a strictly profitable deviation taking probability weight from a neighborhood below $\hat{s}$ and reallocating to scores that yield utility arbitrarily close to $\max_{s\geq s_0}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}$. But by Lemma A.4 $\bar{\Pi}_i^*(\hat{s};\mathbf{F})\geq\lim\limits_{s\to\hat{s}^-}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}$ so $\bar{\Pi}_i^*(\hat{s};\mathbf{F})\geq\max_{s\geq s_0}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}$. QED

Finally, we show the preceding necessary conditions are also sufficient for equilibrium.

Lemma A.6. *When each DEV only crafts veto-proof policies, the properties of ideological optimality, no ties, and score optimality are jointly necessary and sufficient for equilibrium.*

Proof: Necessity is already shown. Note that strategy profile satisfying no ties, ideological optimality, and score optimality yields utility equal to $\max_{s\geq s_0}\left\{\bar{\Pi}_i^*(s;\mathbf{F})\right\}=U_i^*$. Also, i's utility for developing (s_i,y_i) where $-i$ has no atom is $\bar{\Pi}_i^*(s_i,y_i;\mathbf{F})\leq\bar{\Pi}_i^*(s_i;\mathbf{F})\leq U_i^*$, so consider $(\hat{s},\hat{y}_i)$ at $\hat{s}$ where $-i$ has an atom; then i's utility $\Pi_i^*(\hat{s},\hat{y}_i;\mathbf{F})$ from $(\hat{s},\hat{y}_i)$ is $\leq\max\left\{\bar{\Pi}_i^*(\hat{y}_i,\hat{s};\mathbf{F}),\lim\limits_{s\to\hat{s}^-}\left\{\bar{\Pi}_i^*(\hat{y}_i,\hat{s};\mathbf{F})\right\}\right\}$ since either $V_i(\hat{s},\hat{y}_i)\geq V_i(\hat{s},\hat{y}_{-i}(\hat{s}))$ (so i prefers to always win at the atom) or $V_i(\hat{s},\hat{y}_i)<V_i(\hat{s},\hat{y}_{-i}(\hat{s}))$ (so i prefers to always lose at the atom). But both quantities are $\leq\bar{\Pi}_i^*(\hat{s};\mathbf{F})\leq U_i^*$ since we have $\lim\limits_{s\to\hat{s}^-}\left\{\bar{\Pi}_i^*(\hat{y}_i,\hat{s};\mathbf{F})\right\}\leq\lim\limits_{s\to\hat{s}^-}\left\{\bar{\Pi}_i^*(\hat{s};\mathbf{F})\right\}\leq\bar{\Pi}_i^*(\hat{s};\mathbf{F})$ by Lemmas A.2 and A.4. QED

B Characterizing Score-Optimal CDFs

We begin by establishing some preliminary properties. We first rule out the possibility that in equilibrium a DEV crafts a veto-proof policy with score $s_i>s_0$ and ideology $y_i^*(s_i)$ further away from her ideal ideology than y_0. This will also imply that a DEV only crafts policies interior to the veto proof set or on the closer boundary.

Lemma B.1. *If* $s_i > s_0$ *is* $\in$ *supp*$\{F_i(\cdot)\}$ *then* $F_{-i}(s) > \frac{y_0}{x_i/\alpha_i}$ *and* $|x_i - y_i^*(s)| < |x_i - y_0|$.

Proof: We show $|x_i - y_i^*(s_i)| \geq |x_i - y_0|$ implies $\bar{\Pi}_i^*(s_i; \mathbf{F}) - \bar{\Pi}_i^*(s_0; \mathbf{F}) < 0$, which yields our desired property by contrapositive and score optimality. Suppose $|x_i - y_i^*(s_i)| \geq |x_i - y_0|$; from the definition of $y_i^*(s_i)$, $sign(x_i) = sign(y_0)$ and $F_{-i}(s) \leq \frac{y_0}{x_i/\alpha_i}$. Now i's utility difference from developing *any* veto proof policy (s_i, y_i) with score $s_i > s_0$ vs. developing no policy is:

$$\bar{\Pi}_i^*(s_i, y_i; \mathbf{F}) - \bar{\Pi}_i^*(s_0; \mathbf{F}) = -\alpha_i\left(s_i + y_i^2\right) + \int_{s_0}^{s_i}\left(V_i(s_i, y_i) - V_i\left(s_{-i}, y_{-i}^*(s_{-i})\right)\right) dF_{-i}$$

Since each DEV is weakly more extreme than the same-sided VP, $(s_i, z_{-i}(s_i))$ is weakly worst for i among veto-proof policies with $s \in [s_0, s_i]$, so from part 2 of Def A.1 this utility difference is $\leq -\alpha_i\left(s_i + y_i^2\right) + F_{-i}(s_i) \cdot 2x_i\left(y_i - z_{-i}(s_i)\right)$, which is strictly negative when $F_{-i}(s) \leq \frac{y_0}{x_i/\alpha_i}$. Since (s_i, y_i) is veto proof, the preceding is $\leq -\alpha_i\left(s_i + y_i^2\right) + \left(\frac{y_0}{x_i/\alpha_i}\right) \cdot 2x_i\left(y_i - z_{-i}(s_i)\right)$. From Lemma A.2 the veto proof y_i maximizing the preceding is $y_i^* = \min\left\{\max\left\{z_{-i}(s), \hat{y}_i\left(s; \frac{y_0}{x_i/\alpha_i}\right), z_i(s)\right\}\right\} = y_0$; substituting and simplifying yields $-\alpha_i\left(1 - \frac{y_0}{x_{Vi}}\right)\left(s_i + y_0^2\right)$, which is < 0 since $|y_0| \leq |x_{Vi}|$. QED

Next, the form of the equilibrium score CDFs depends on the objective functions $\bar{\Pi}_i^*(s_i; \mathbf{F})$ at continuity points. Using Eq A.3, differentiating[2] and simplifying at points of continuity yields:

$$\begin{aligned}\frac{\partial \bar{\Pi}_i^*(s_i; \mathbf{F})}{\partial s_i} &= -\left(\alpha_i - F_{-i}(s_i)\right) + \max\left\{D_i\left(s_i, F_{-i}(s_i)\right), 0\right\} \\ &\quad + f_{-i}(s_i) \cdot \left(V_i\left(s_i, y_i^*(s_i)\right) - V_i\left(s_i, y_{-i}^*(s_i)\right)\right) \qquad \text{(A.4)}\end{aligned}$$

where $D_i(s, F) \equiv \frac{\alpha_i}{\left|x_{V_{-i}}\right|} \cdot sign(x_i) \cdot \left(F\frac{x_i}{\alpha_i} - z_i(s)\right)$. The 1st and 3rd terms are identical to the model without VPs (Hirsch and Shotts (2015)): they are the net quality cost to a developer of raising her score and the net ideological benefit of doing so. The 2nd term is due to VPs and captures a DEV's benefit from increasing her score when $y_i^*(s)$ is *constrained* by VPs. $D_i(s, F)$ is continuous in s and F, strictly decreasing (increasing) in the former (latter), and $y_i^*(s) = \frac{x_i}{\alpha_i}F_{-i}(s) \rightarrow D_i\left(s, F_{-i}(s)\right) \leq 0$. Also, since $y_i^*(s_i) = \hat{y}_i\left(s_i; F_{-i}(s_i)\right)$, $\bar{\Pi}_i^*(s_i; \mathbf{F})$ may be written as:

$$\begin{aligned}\bar{\Pi}_i^*(s_i; \mathbf{F}) &= F_{-i}(s_i) \cdot V_i(s_0, y_0) + \int_{s_0}^{s_i}\left(-\left(\alpha_i - F_{-i}(s_i)\right) + \max\left\{D_i\left(\tilde{s}, F_{-i}(s_i)\right), 0\right\}\right) d\tilde{s} \\ &\quad + \int_{s_i}^{\infty} V_i\left(s_{-i}, y_{-i}^*(s_{-i})\right) dF_{-i} \qquad \text{(A.5)}\end{aligned}$$

[2] At points of left-discontinuity the expression that follows is actually the right-derivative.

by observing that $-\alpha_i\left(s_i+\left[\hat{y}_i\left(s_i;F_{-i}\left(s_i\right)\right)\right]^2\right)+F_{-i}\left(s_i\right)\cdot V_i\left(s_i,\hat{y}_i\left(s_i;F_{-i}\left(s_i\right)\right)\right)$

$$\begin{aligned} &= -\alpha_i\left(s_0+\left[\hat{y}_i\left(s_0;F_{-i}\left(s_i\right)\right)\right]^2\right)+F_{-i}\left(s_i\right)\cdot V_i\left(s_0,\hat{y}_i\left(s_0;F_{-i}\left(s_i\right)\right)\right) \\ &\quad +\int_{s_0}^{s_i}\frac{\partial}{\partial\tilde{s}}\left(-\alpha_i\left(\tilde{s}+\left[\hat{y}_i\left(\tilde{s};F_{-i}\left(s_i\right)\right)\right]^2\right)+F_{-i}\left(s_i\right)\cdot V_i\left(\tilde{s},\hat{y}_i\left(\tilde{s};F_{-i}\left(s_i\right)\right)\right)\right)d\tilde{s} \\ &= F_{-i}\left(s_i\right)\cdot V_i\left(s_0,y_0\right)+\int_{s_0}^{s_i}\left(-\left(\alpha_i-F_{-i}\left(s_i\right)\right)+\max\left\{D_i\left(\tilde{s},F_{-i}\left(s_i\right)\right),0\right\}\right)d\tilde{s}. \end{aligned}$$

The final equality uses the fact that $\hat{y}_i\left(s_0;F_{-i}\left(s_i\right)\right)=y_0$ and $-\alpha_i\left(s_0+\left[\hat{y}_i\left(s_0;F_{-i}\left(s_i\right)\right)\right]^2\right)=0$.

We next prove our first lemma on the form of equilibrium score CDFs, which states that if a DEV targets a score $\hat{s}_i$ despite there being a veto-proof lower score $s_i\in[s_0,\hat{s}_i)$ winning with equal probability ($F_{-i}\left(s_i\right)=F_{-i}\left(\hat{s}_i\right)$), then $y_i^*\left(\hat{s}_i\right)=z_i\left(\hat{s}_i\right)$ and $\hat{s}_i$ must be the *lowest* score in the support of i's score CDF (which we henceforth denote $\underline{s}_i=min_s\{\text{supp}\{F_i\left(\cdot\right)\}\}$).

Lemma B.2. *If $\hat{s}_i\in supp\{F_i(\cdot)\}$ and there exists some $s_i\in[s_0,\hat{s}_i)$ such that $F_{-i}(s_i)=F_{-i}(\hat{s}_i)$, then $F_{-i}(\hat{s}_i)>0$, $\hat{s}_i=\underline{s}_i$ and $y_i^*(\hat{s}_i)=z_i(\hat{s}_i)$.*

Proof: Suppose $\hat{s}_i\in$supp$\{F_i\left(\cdot\right)\}$ and $\exists s\in[s_0,\hat{s}_i)$ s.t. $F_{-i}\left(s\right)=F_{-i}\left(\hat{s}_i\right)$. Let $\underline{\hat{s}}_i\geq s_0$ be $\min\left\{s_i:F_{-i}\left(s_i\right)=F_{-i}\left(\hat{s}_i\right)\right\}$, which is well-defined by right-continuity of CDFs. We first argue that $\hat{s}_i$ must be i's *only support point* over $[\underline{\hat{s}}_i,\hat{s}_i]$. From Eq A.4, since $F_{-i}\left(s_i\right)$ is constant and $=F_{-i}\left(\hat{s}_i\right)$ over this interval, $\bar{\Pi}_i^*\left(s_i;\mathbf{F}\right)$ is continuous and for almost all $s_i\in(\underline{\hat{s}}_i,\hat{s}_i)$

$$\frac{\partial\bar{\Pi}_i^*\left(s_i;\mathbf{F}\right)}{\partial s_i}=-\left(\alpha_i-F_{-i}\left(\hat{s}_i\right)\right)+\max\left\{D_i\left(s_i,F_{-i}\left(\hat{s}_i\right)\right),0\right\} \tag{A.6}$$

Thus $y_i^*\left(\hat{s}_i\right)\neq F_{-i}\left(\hat{s}_i\right)\frac{x_i}{\alpha_i}$ (otherwise $\hat{s}_i$ could not be in the support by Lemma A.5) so $y_i^*\left(s_i\right)=z_i\left(s_i\right)\ \forall s_i\in[\underline{\hat{s}}_i,\hat{s}_i]$. Then Eq A.6 is linear and strictly decreasing in s_i over $[\underline{\hat{s}}_i,\hat{s}_i]$, so $\bar{\Pi}_i^*\left(s_i;\mathbf{F}\right)$ is strictly concave. So if $\hat{s}_i$ is in i's support, it can be the only maximizer and $-\left(\alpha_i-F_{-i}\left(\hat{s}_i\right)\right)+D_i\left(s_i,F_{-i}\left(\hat{s}_i\right)\right)>0, \forall s_i\in[s_0,\hat{s}_i)$. Finally, $F_{-i}\left(\hat{s}_i\right)>0$, since $F_{-i}\left(\hat{s}_i\right)=0$ implies the above evaluated at $s_i=s_0$ is $-\alpha_i\left(1-\frac{y_0}{x_{V_{-i}}}\right)<0$.

We next show $\hat{s}_i$ must be i's *lowest* support point. Suppose not. Since supports are closed, i has a *next lowest* support point $s_i'\in[s_0,\underline{\hat{s}}_i)$, and $F_{-i}\left(s_i'\right)<F_{-i}\left(\hat{s}_i\right)$, so $-i$ has a strictly positive probability of crafting a score in $(s_i',\underline{\hat{s}}_i]$. We then argue that (a) $-i$ must have an atom at $\underline{\hat{s}}_i\in(s_i',\hat{s}_i)$ with $y_{-i}^*\left(\underline{\hat{s}}_i\right)=z_{-i}\left(\underline{\hat{s}}_i\right)$ and (b) $F_{-i}\left(s_i\right)$ is constant around s_i' (so i too works on the boundary at s_i'). To see this, recall that since i has no support over $(s_i',\hat{s}_i)$, $F_i\left(s_{-i}\right)$ is constant over $[s_i',\underline{\hat{s}}_i]$ (because $\underline{\hat{s}}_i<\hat{s}_i$ and CDFs are right-continuous). By the argument in the preceding paragraph, now applied to $-i$, this implies $-i$ has a unique support point over this closed interval at which she crafts a policy on her boundary. This then yields the desired properties when combined with the definition of $\underline{\hat{s}}_i$ and $F_{-i}(s_i') < F_{-i}(\underline{\hat{s}}_i) = F_{-i}(\hat{s}_i)$.

We last show $\bar{\Pi}_i^*(\hat{s}_i; \mathbf{F}) - \bar{\Pi}_i^*(s_i'; \mathbf{F})$ is strictly positive, implying s_i' cannot be in i's support. We rewrite $\bar{\Pi}_i^*(\hat{s}_i; \mathbf{F}) = -\alpha_i \left(\hat{s}_i + [z_i(\hat{s}_i)]^2\right) + F_{-i}(\hat{s}_i) \cdot V_i(\hat{s}_i, z_i(\hat{s}_i)) + \int_{\hat{s}_i}^{\infty} V_i\left(s_{-i}, y_{-i}^*(s_{-i})\right) dF_{-i}$ as

$$-\alpha_i \left(s_i' + \left[z_i(s_i')\right]^2\right) + F_{-i}(s_i') \cdot V_i\left(s_i', z_i(s_i')\right) + p_{-i}^{\underline{\hat{s}}_i} \cdot V_i\left(s_i', z_i(s_i')\right)$$
$$+ \int_{s_i'}^{\hat{s}_i} \left(-\left(\alpha_i - F_{-i}(\hat{s}_i)\right) + D_i\left(s_i, F_{-i}(\hat{s}_i)\right)\right) ds_i + \int_{\hat{s}_i}^{\infty} V_i\left(s_{-i}, y_{-i}^*(s_{-i})\right) dF_{-i}$$

which then yields (since $y_i^*(s_i') = z_i(s_i')$) that $\bar{\Pi}_i^*(\hat{s}_i; \mathbf{F}) - \bar{\Pi}_i^*(s_i'; \mathbf{F}) =$

$$\int_{s_i'}^{\hat{s}_i} \left(-\left(\alpha_i - F_{-i}(\hat{s}_i)\right) + D_i\left(s_i, F_{-i}(\hat{s}_i)\right)\right) ds_i + p_{-i}^{\underline{\hat{s}}_i} \cdot \left(V_i\left(s_i', z_i(s_i')\right) - V_i\left(\underline{\hat{s}}_i, z_{-i}(\underline{\hat{s}}_i)\right)\right)$$

The second term is positive since $(\underline{\hat{s}}_i, z_{-i}(\underline{\hat{s}}_i))$ is the weakly worst veto proof policy for i with score $\in [s_0, \underline{\hat{s}}_i]$ (recalling that each developer is more extreme than the same sided veto player). The first term has already been shown to be strictly positive. QED

B.1 Equilibrium Conditions on Score CDFs

Using the preceding we now characterize necessary and sufficient conditions for a pair of score CDFs to support an equilibrium. There are four types of potential equilibria, each characterized by the combination of two properties: (a) whether one of the DEVs is *always active* (i.e., crafts a policy with score $s > s_0$ with probability 1), and (b) whether the equilibrium is *pure* or *mixed*. Which type of equilibrium a particular candidate set of score CDFs $(F_L(\cdot), F_R(\cdot))$ must be, and thus the necessary and sufficient conditions for score optimality, may be determined by considering two quantities; (a) the maximum *lowest score* in the support of the two CDFs – which we denote $\underline{s} = \max_i \{\underline{s}_i\}$ – and the maximum *highest score* in the support of the two CDFs – which we denote $\bar{s} = \max_i \{\bar{s}_i\}$, where $\bar{s}_i = \max_s \{\text{supp}\{F_i(\cdot)\}\}$.

Proposition B.1. *A profile of score CDFs* $\mathbf{F}$ *satisfies score optimality i.f.f. the following hold.*

1. *If* $s_0 = \underline{s} = \bar{s}$ *(so that* $F_i(s_0) = 1\ \forall i$*), then* $\alpha_i - 1 \geq D_i(s_0; 1)\ \forall i$.

2. *If* $s_0 < \underline{s}$*, then there exists a* $k \in \{L, R\}$ *such that*

- *DEV k is **sometimes-inactive** (i.e. $s_0 = \underline{s}_k < \underline{s}$), never crafts a policy with score $\in (s_0, \underline{s}]$ (so that $0 < F_k(s_0) = F_k(\underline{s})$), and has a probability of inactivity $F_k(\underline{s})$ satisfying*

$$\alpha_{-k} - F_k(\underline{s}) = D_{-k}(\underline{s}; F_k(\underline{s}))$$

- *DEV $-k$ is **always-active** (i.e. $s_0 < \underline{s}_{-k} = \underline{s}$ so that $F_{-k}(s) = 0\ \forall s \in [s_0, \underline{s})$), and has a probability $F_{-k}(\underline{s})$ of crafting a score-$\underline{s}$ policy satisfying $\bar{\Pi}_k^*(s_0; \mathbf{F}) \geq \bar{\Pi}_k^*(\underline{s}; \mathbf{F})$, which may written in the following two equivalent forms:*

$$\alpha_k \left(\underline{s} + [y_k^*(\underline{s})]^2\right) \geq F_{-k}(\underline{s}) \cdot 2x_k \cdot (y_k^*(\underline{s}) - z_{-k}(\underline{s}))$$

$$\int_{s_0}^{\underline{s}} ((\alpha_k - F_{-k}(\underline{s})) - \max\{D_k(s; F_{-k}(\underline{s})), 0\})\, ds \geq F_{-k}(\underline{s}) \cdot \left(\frac{x_k}{x_{V_k}} - 1\right)(\underline{s} - s_0)$$

3. *If $\underline{s} < \bar{s}$, then $\forall i \in \{L, R\}$ and $s \in [\underline{s}, \overline{s}]$ the score CDF $F_{-i}(s)$ is continuous and satisfies:*

$$(\alpha_i - F_{-i}(s)) - \max\{D_i(s; F_{-i}(s)), 0\} = f_{-i}(s) \cdot 2|x_i|(y_R^*(s) - y_L^*(s))$$

4. *If **both** $s_0 < \underline{s}$ and $\underline{s} < \bar{s}$, then DEV $-k$'s probability $F_{-k}(\underline{s})$ of crafting a score-$\underline{s}$ policy also satisfies $\bar{\Pi}_k^*(s_0; \mathbf{F}) \leq \bar{\Pi}_k^*(\underline{s}; \mathbf{F})$*

Before the proof we explain how Prop. B.1 characterizes both pure and mixed equilibrium conditions.

First observe that any pure strategy equilibrium must satisfy $\underline{s} = \overline{s}$, since Condition (3) dictates that both DEVs' equilibrium score CDFs have full support over $[\underline{s}, \overline{s}]$. There are then only two possible forms of pure strategy equilibria. The first is that both DEVs are inactive ($s_0 = \underline{s} = \overline{s}$). Condition (1) is the necessary and sufficient condition for this to be an equilibrium, and it is easily verified that it is exactly equivalent to both DEVs' monopoly scores s_i^{M*} being equal to s_0 (see Proposition 1). The second is that some DEV $-k$ is always active and develops exactly score $\underline{s} > s_0$ (i.e., $F_{-k}(s) = \mathbf{1}_{s \geq \underline{s}}$) while the other DEV k is always inactive (i.e., $F_k(s) = 1\ \forall s \geq s_0$). The necessary and sufficient conditions for such an equilibrium are those in Condition (2) with these CDFs substituted in. It is easily verified that the first bullet point is equivalent to $\underline{s}$ being DEV $-k$'s monopoly score; thus, such an equilibrium may only exist if an equilibrium with both DEVs inactive does not. The second bullet point states that DEV k weakly prefers to stay out and accept policy $(\underline{s}, y_{-k}^*(\underline{s}))$ rather than enter herself with the policy $(\underline{s}, y_k^*(\underline{s}))$ and win for sure.

When Condition (1) doesn't hold, and Condition (2) doesn't hold with *either* possible assignment of DEV $-k \in \{L, R\}$ (with $\underline{s} = s_{-k}^{M*}$, $F_k(s) = 1$, and $F_{-k}(s) = \mathbf{1}_{s \geq \underline{s}}$) then any equilibrium must necessarily be in mixed strategies with $\underline{s} < \overline{s}$. (Appendix D explicitly derives existence conditions for pure strategy equilibria in terms of underlying parameters using these steps in the symmetric special case). Existence of a mixed equilibrium when a pure one is absent is a straightforward implication of Simon and Zame (1990), observing that the strategy space is compact after eliminating strictly dominated strategies. Condition (3) describes the differential equations that govern any equilibrium score CDFs over the positive-measure mixing interval $[\underline{s}, \overline{s}]$. There are then again two possible equilibrium forms (although the second is non-generic). The first form is that $s_0 < \underline{s}$ (one DEV $-k$ is always active); then the boundary condition on $F_k(\underline{s})$ is determined by the first bullet point in Condition (2) (which states that DEV $-k$ wants to "work up" to score $\underline{s} > s_0$), while the boundary condition on $F_{-k}(\underline{s})$ is determined by the combination of the second bullet point in Condition (2) and Condition (4) (which together imply that DEV k is indifferent between staying out and targeting score $\underline{s}$ if she is assured to win in a tie). The second form is that $s_0 = \underline{s}$ (both DEVs are sometimes inactive); Condition (3) then fully characterizes both the differential equations and boundary conditions (since $y_R^*(\underline{s}) - y_L^*(\underline{s}) = 0$), with Conditions (2) and (4) being either redundant or trivially satisfied.

Proof: ***(Necessity)*** We first show necessity of (1). If $s_0 = \underline{s} = \overline{s}$ then $F_i(s_i) = 1\ \forall i \in \{L, R\}$ and $s_i \geq s_0$ and $\bar{\Pi}_i^*(s_i; \mathbf{F})$ is continuously differentiable; thus score optimality requires that $\left.\frac{\partial \bar{\Pi}_i^*(s_i;\mathbf{F})}{\partial s_i}\right|_{s_i = s_0} \leq 0\ \forall i$, which is exactly the stated condition.

We now show necessity of (2). Suppose $\underline{s} > s_0$; then there must be *exactly one* DEV with $\underline{s}_i = \underline{s}$ (if both had $\underline{s}_i = \underline{s}$ then by Lemma B.2 each has an atom at $\underline{s}$, violating Lemma A.3). Denote this DEV $-k$. By the definition of $\underline{s}$, $\underline{s}_k \in [s_0, \underline{s})$. Because $F_{-k}(s_k) = 0\ \forall s_k \in (s_0, \underline{s})$, no such score can be in the support of $F_{-k}(\cdot)$ by the first property in Lemma B.2, so $\underline{s}_k = s_0$. Moreover, k cannot have an atom at $\underline{s}$ (because then $-k$ could not have an atom at $\underline{s}$ by Lemma A.3, which would in turn imply a contradiction via the first part of Lemma B.2). Thus k must have an atom at s_0 equal to $F_k(\underline{s})$, with $F_k(s_k)$ constant over $s_k \in [s_0, \underline{s}]$. And applying Lemma B.2, the fact that $\underline{s} \in$supp$\{F_{-k}\}$ and $F_k(s_k)$ is constant over $s_k \in [s_0, \underline{s}]$ implies $F_{-k}(\underline{s}) > 0$.

Having established the form of the DEVs' strategies, we now characterize $F_{-k}(\underline{s})$. The fact that $\bar{\Pi}_k^*(s_0; \mathbf{F}) \geq \bar{\Pi}_k^*(\underline{s}; \mathbf{F})$, in the second bullet point of (2), follows from Lemma A.5. For the two equivalent conditions on $F_{-k}(\underline{s})$, first recall that, as shown above, k doesn't have an atom at $\underline{s}$, which implies, via Lemma B.2, that $y^*_{-k}(\underline{s}) = z_{-k}(\underline{s})$. The first equivalence in the second bullet point of (2) comes from Eq A.3 and the second equivalence comes from Eq A.5.

We last turn to to the characterization of $F_k(\underline{s})$. Since CDFs are right continuous, $\bar{\Pi}^*_{-k}(s_{-k};\mathbf{F})$ is constant over $[s_0,\underline{s})$ and continuous over $[s_0,\underline{s}+\epsilon)$ for sufficiently small ϵ, so $-(\alpha_{-k} - F_k(\underline{s})) + D_{-k}(\underline{s}, F_k(\underline{s})) + f_k(\underline{s}) \cdot \left(V_{-k}\left(\underline{s}, y^*_{-k}(\underline{s})\right) - V_{-k}(\underline{s}, y^*_k(\underline{s}))\right)$ is the right derivative of $\bar{\Pi}^*_{-k}(\underline{s};\mathbf{F})$; the first two terms are the left derivative of $\bar{\Pi}^*_{-k}(\underline{s};\mathbf{F})$, and the third term is ≥ 0. So the first two terms must $= 0$; if they were strictly negative (positive) a score s_{-k} a little bit below (above) $\underline{s}$ would yield a strictly higher value of $\bar{\Pi}^*_{-k}(\underline{s};\mathbf{F})$, violating score optimality.

We next show necessity of (3). Consider $\underline{s} < \bar{s}$. For any $s_i > \underline{s} \geq s_0$ in the support of $F_i(\cdot)$, $F_{-i}(s) < F_{-i}(s_i)\ \forall s \in [s_0, s_i)$; if not then by Lemma B.2 $s_i = \underline{s}_i$, contradicting the definition of $\underline{s}$. Next we argue that any support points strictly above $\underline{s}$ must be common; if not then $\exists\, \hat{s}_i \in$supp$\{F_i(\cdot)\}$ s.t. $\hat{s}_i > \underline{s}$ and $\hat{s}_i \notin$supp$\{F_{-i}(\cdot)\}$ and therefore $F_{-i}(\hat{s}_i - \epsilon) = F_{-i}(\hat{s}_i)$ for sufficiently small ϵ, which by Lemma B.2 implies $\underline{s}_i = \hat{s}_i > \underline{s}$, a contradiction. Next we argue that the set of (common) support points strictly above $\underline{s}$ must be convex. If not, there would exist $\hat{s} > \underline{s} \geq s_0$ in the common support s.t. neither developer has support in a neighborhood immediately below, so $F_i(s) < F_i(\hat{s})\ \forall s \in [s_0, \hat{s})$ would require both developers to have atoms at $\hat{s}$, contradicting the no ties from Lemma A.3. Since supports are closed, the interval $[\underline{s}, \bar{s}]$ must therefore be in the support of both developer's CDFs. Thus by the score optimality property in Lemma A.5, $\bar{\Pi}^*_i(s;\mathbf{F}) = U^*_i\ \forall s \in [\underline{s}, \bar{s}]$, further implying that the score CDFs are absolutely continuous over $(\underline{s}, \bar{s})$, and therefore that $\frac{\partial}{\partial s}\left(\bar{\Pi}^*_i(s;\mathbf{F})\right) = 0$ for almost all $s \in [\underline{s}, \bar{s}]$. This straightforwardly yields the stated differential equation.

We last show necessity of (4). If $\underline{s}_0 < \underline{s} < \bar{s}$ then by (3), $\underline{s}$ is also in the support of $F_k(\cdot)$; score optimality thus requires $\bar{\Pi}^*_k(s_0;\mathbf{F}) \leq \bar{\Pi}^*_k(\underline{s};\mathbf{F})$.

This concludes the argument that (1) $-$ (4) are necessary for score optimality.

(Sufficiency) For all possibilities, $\bar{\Pi}^*_i(\underline{s};\mathbf{F}) = \bar{\Pi}^*_i(\bar{s};\mathbf{F})\ \forall i$ (since $\underline{s} = \bar{s}$ or $\underline{s} < \bar{s}$ and both are in the support of both DEVs' CDFs); so to show score optimality for i we need only show that scores $s_i > \bar{s}$ and scores $s_i \in [s_0, \underline{s}]$ outside of i's support cannot deliver a strictly higher value of $\bar{\Pi}^*_i(\cdot;\mathbf{F})$.

To argue that $\bar{\Pi}^*_i(s_i;\mathbf{F}) \leq \bar{\Pi}^*_i(\bar{s};\mathbf{F})\ \forall i$ and $s_i > \bar{s}$, observe that from Eq A.6, $\frac{\partial}{\partial s_i}\left(\bar{\Pi}^*_i(s;\mathbf{F})\right) = -(\alpha_i - 1) + \max\{D_i(s_i, 1), 0\}$ for $s_i > \bar{s}$ and is weakly decreasing in s_i; thus it suffices to show that at $\bar{s}$, $-(\alpha_i - 1) + \max\{D_i(\bar{s}, 1), 0\} \leq 0$. If (1) holds this is immediate. If (3) holds, it follows from the differential equation since $y^*_R(\bar{s}) > y^*_L(\bar{s})$.

If neither (1) nor (3) hold then $s_0 < \underline{s} = \overline{s}$ so $F_i(\underline{s}) = F_i(\bar{s}) = 1$. Then (2) implies there is an always-active DEV $-k$, and the condition holds for $-k$ from the first bullet point of (2) since $F_k(\underline{s}) = 1$. For k, the second bullet point in (2) combined with $s_0 < \underline{s} = \overline{s}$ and $F_i(\underline{s}) = F_i(\bar{s}) = 1$ yields $\int_{s_0}^{\underline{s}} ((\alpha_k - 1) - \max\{D_k(s;1), 0\})\, ds \geq \left(\frac{x_k}{x_{V_k}} - 1\right)(\bar{s} - s_0)$. Thus $-(\alpha_k - 1) + \max\{D_k(\bar{s};1), 0\} > 0$, combined with the fact that $D_k(s;F)$ is strictly decreasing in s, would imply the left hand side is strictly negative, contradicting the inequality since $\left(\frac{x_k}{x_{V_k}} - 1\right)(\bar{s} - s_0) \geq 0$.

Finally, the property that $\bar{\Pi}_i^*(s_i;\mathbf{F}) \leq \bar{\Pi}_i^*(\underline{s};\mathbf{F})$ $\forall i$ and $s_i \leq \underline{s}$ that are also *not* in supp$\{F_i(\cdot)\}$ is true by construction of the necessary conditions. Observe that the potential existence of such a score requires $s_0 < \underline{s}$ (property 2), a sometimes inactive DEV k with $\underline{s}_k = 0$, and DEV $-k$ always active with $\underline{s}_{-k} = \underline{s}$. The condition in the first bullet point of (2) implies $\bar{\Pi}_{-k}^*(\underline{s};\mathbf{F}) > \bar{\Pi}_{-k}^*(s_{-k};\mathbf{F})$ $\forall s_{-k} \in [s_0, \underline{s})$. Also, $F_{-k}(s_k) = 0$ $\forall s_k \in [s_0, \underline{s})$ implies $\bar{\Pi}_k^*(s_0;\mathbf{F}) > \bar{\Pi}_k^*(s_k;\mathbf{F})$, and the condition in the second bullet point of (2) is that $\bar{\Pi}_k^*(s_0;\mathbf{F}) \geq \bar{\Pi}_k^*(\underline{s};\mathbf{F})$, completing the argument. QED

We also note that since $F_{-i}(s)$ is assumed to be a mixture of discrete and absolutely continuous distributions, it's absolutely continuous over $[\underline{s}, \overline{s}]$. Also, $D_i(s;F_{-i}(s))$ and $y_i^*(s) = \hat{y}_i(s;F_{-i}(s))$ are continuous in $F_{-i}(s)$; thus, part (3) of Prop B.1 implies $f_i(s)$ is continuous over $[\underline{s}, \bar{s}]$.

C Symmetric Special Case

We now impose $|x_{Vl}| = |x_{Vr}| = x_V \leq |x_L| = |x_R| = x_E$ and $\alpha_L = \alpha_R = \alpha_R$ and prove properties of equilibrium for this case. Under symmetry, $z_i(s) = 2y_0 - z_{-i}(s)$ at any s, DEV i is strictly constrained by VPs iff $\frac{x_E}{\alpha} F_{-i}(s) > sign(x_i) \cdot z_i(s) = sign(x_i) \cdot y_0 + \frac{s - s_0}{2x_V}$, so that $D_i(s,F) = \frac{\alpha}{x_V}\left(F\frac{x_E}{\alpha} - \left(sign(x_i) \cdot y_0 + \frac{s-s_0}{2x_V}\right)\right)$, and $D_i(s,F) - D_{-i}(s,F) = -sign(x_i) \cdot \frac{\alpha}{x_V} 2y_0$. Thus $y_0 = 0 \to D_L(s,F) = D_R(s,F)$ and $y_0 < (>)\, 0 \to D_R(s,F) > (<)\, D_L(s,F)$; this determines which DEV is more active.

C.1 Equilibrium with $y_0 = 0$

Proposition C.1. *Suppose $y_0 = 0$ so that $s_0 = 0$, $D_L(s,F) = D_R(s,F) = D(s,F) = \frac{\alpha}{x_V}\left(F\frac{x_E}{\alpha} - \frac{s}{2x_V}\right)$, and $-z_L(s) = z_R(s) = z(s) = \frac{s - s_0}{2x_V}$).*

- *If $\alpha - 1 \geq D_R(s_0, 1) \iff \alpha \geq \frac{x_E}{x_V} + 1$, then in equilibrium both DEVs are inactive with probability $F(0) = 1$ (so $0 = \underline{s} = \bar{s}$)*
- *If $\alpha \in \left(2, 1 + \frac{x_E}{x_V}\right)$ then in equilibrium both DEVs are inactive with probability $F(0) = \frac{\alpha}{1 + \frac{x_E}{x_V}} < 1$ so $0 = \underline{s} < \bar{s}$. Now let $s(F)$ denote the inverse of $F(s)$ (so $s(F(0)) = 0$), let $\check{F} =$*

$\min\left\{F(0)\cdot\left(\frac{1+3\frac{x_E}{x_V}}{1+2\frac{x_E}{x_V}}\right),1\right\}$, *and let* $\breve{s}$ *solve* $\frac{x_E}{\alpha}\breve{F}=z(\breve{s}) \iff \breve{s}=\frac{2x_Ex_V}{\alpha}\breve{F}$.

– *for* $F\in\left[F(0),\breve{F}\right]$ *we have* $|y_i^*(s(F))|=z(s)$ *and* $s(F)$ *equal to*

$$\hat{s}(F)=\frac{2x_V^2}{\alpha}\left(3\frac{x_E}{x_V}+1\right)\cdot(F-F(0))$$

– *for* $F\in(\breve{F},1]$ *we have* $|y_i^*(s(F))|=\frac{x_E}{\alpha}F$ *and* $s(F)$ *equal to*

$$\tilde{s}(F)=\breve{s}+\int_{\breve{F}}^{F}4x_E^2\frac{G}{\alpha(\alpha-G)}dG=\breve{s}+4x_E^2\left(\ln\left(\frac{\alpha-\breve{F}}{\alpha-F}\right)-\frac{F-\breve{F}}{\alpha}\right)$$

Finally, the DM's equilibrium utility is $\int_{F(0)}^{\breve{F}}2F\cdot\hat{s}(F)\,dF+\int_{\breve{F}}^{1}2F\cdot\tilde{s}(F)\,dF=$

$$\frac{4x_V^2}{\alpha}\left(3\frac{x_E}{x_V}+1\right)\left(\frac{[F(0)]^3-\breve{F}^2\left(3F(0)-2\breve{F}\right)}{6}\right)$$
$$+\breve{s}\cdot\left(1-\breve{F}^2\right)+4x_E^2\left(\left(1-\breve{F}\right)\left(\alpha+\frac{1+\breve{F}}{2}-\frac{2-\breve{F}\left(1-\breve{F}\right)}{3\alpha}\right)-\left(\alpha^2-1\right)\log\left(\frac{\alpha-\breve{F}}{\alpha-1}\right)\right)$$

Proof: We show by construction that there exists a solution to score optimality satisfying $\underline{s}=s_0=0$, and that it is the unique solution with this property.

For the first bullet point, note that if $D(s_0,1)=\frac{x_E}{x_V}\leq\alpha-1 \iff \frac{\alpha}{1+\frac{x_E}{x_V}}\geq 1$, inactivity ($s_0=\underline{s}=\bar{s}$) is the unique equilibrium of the form $s_0=\underline{s}$.

For the second bullet point, note that if $D(s_0,1)=\frac{x_E}{x_V}>\alpha-1$, inactivity ($0=s_0=\underline{s}=\bar{s}$) is *not* an equilibrium. We solve for a solution of the form $0=s_0=\underline{s}<\bar{s}$ which is unique by construction. Since $y_i^*(s_0)=y_0=0\ \forall i$, $\alpha-F_{-i}(0)-D(0;F_{-i}(0))=0 \iff F_{-i}(0)=\frac{\alpha}{1+\frac{x_E}{x_V}}\ \forall i$. In a neighborhood above 0, $y_i^*(s)=z_i(s)$ and $D(s_i,F_i(s))\geq 0$; substituting into the differential equations from part 3 of Prop B.1, simplifying, and rearranging yields $\alpha-\left(1+\frac{x_E}{x_V}\right)F_{-i}(s)+\frac{\alpha}{2x_V^2}s=f_{-i}(s)\cdot 2\frac{x_E}{x_V}\cdot s$, $\forall i$. Since the differential equation and boundary condition at $s_0=0$ is the same $\forall i$ the solution in this neighborhood is a common CDF $\hat{F}(s)$ satisfying $\alpha-\left(1+\frac{x_E}{x_V}\right)\hat{F}(s)+\frac{\alpha}{2x_V^2}s=\hat{f}(s)\cdot 2\frac{x_E}{x_V}\cdot s$ and the boundary condition is $\alpha-\left(1+\frac{x_E}{x_V}\right)\hat{F}(0)=0$. The system has the following linear solution, whose inverse is the function $\hat{s}(F)$ in the proposition: $\hat{F}(s)=\left(\frac{\alpha}{2x_V^2}\right)\left(\frac{1}{3\frac{x_E}{x_V}+1}\right)s+F(0)$. By linearity of $\hat{F}(\cdot)$ and $z(s)$ there is a unique $\breve{s}$ s.t. $\frac{x_E}{\alpha}\hat{F}(\breve{s})=\frac{\breve{s}}{2x_V}$, which is $\breve{s}=2x_V^2\left(\frac{1+3\frac{x_E}{x_V}}{1+\frac{x_E}{x_V}}\right)\left(\frac{\frac{x_E}{x_V}}{1+2\frac{x_E}{x_V}}\right)$ so $\hat{F}(\breve{s})=$

$\alpha\left(\frac{\left(1+3\frac{x_E}{x_V}\right)}{\left(1+\frac{x_E}{x_V}\right)\left(1+2\frac{x_E}{x_V}\right)}\right)$, and a unique $\hat{s}$ s.t. $\hat{F}(\hat{s})=1$, which is $\hat{s}=2x_V^2\left(\frac{1+3\frac{x_E}{x_V}}{1+\frac{x_E}{x_V}}\right)\left(\frac{1+\frac{x_E}{x_V}}{\alpha}-1\right)$. Then we have two possibilities for the unique solution.

First, we may have $\breve{s}\geq\hat{s}$, which occurs when $\alpha\geq\frac{\left(1+\frac{x_E}{x_V}\right)\left(1+2\frac{x_E}{x_V}\right)}{\left(1+3\frac{x_E}{x_V}\right)}$. In this case $\frac{x_E}{\alpha}\hat{F}(s)>\frac{s}{2x_V}$ $\forall s\in[0,\hat{s})$, so $0=\underline{s}<\bar{s}=\hat{s}$ with $F_i(s)=\hat{F}(s)$ $\forall i$ is the unique solution of the form $s_0=\underline{s}<\bar{s}$ so the inverse is $\hat{s}(F)$ for $F\in[F(0),1]$.

Second, we may have $\breve{s}<\hat{s}$. Then $\hat{F}(\breve{s})=\breve{F}=\alpha\left(\frac{\left(1+3\frac{x_E}{x_V}\right)}{\left(1+\frac{x_E}{x_V}\right)\left(1+2\frac{x_E}{x_V}\right)}\right)<1$ and the differential equation in a neighborhood above $\breve{s}$ is $\alpha-F_{-i}(s)=f_{-i}(s)\cdot\frac{x_E^2}{\alpha}\left(F_{-i}(s)+F_i(s)\right)$, $\forall i$, with boundary condition $F_{-i}(\breve{s})=F_i(\breve{s})=\breve{F}$. The solution is a common CDF $F_{-i}(s)=F_i(s)=\tilde{F}(s)$ satisfying $\tilde{F}(\breve{s})=\breve{F}$ and $\tilde{f}(s)=\frac{\alpha}{4x_E^2}\frac{\alpha-\tilde{F}(s)}{\tilde{F}(s)}$ $\forall s\in[\breve{s},\bar{s}]$. This is strictly concave; since $z(s)$ is linear we then have $\frac{x_E}{\alpha}\tilde{F}(s)<z(s)$ for $s\in(\breve{s},\bar{s}]$ as required. To derive an analytic expression observe that the inverse $\tilde{s}(F)$ of $\tilde{F}(s)$ satisfies $\tilde{s}'(F)=\frac{4x_E^2}{\alpha}\left(\frac{F}{\alpha-F}\right)$ and $\tilde{s}\left(\breve{F}\right)=\breve{s}$, so $\tilde{s}(F)=\breve{s}+\int_{\breve{F}}^{F}4x_E^2\frac{G}{\alpha(\alpha-G)}dG$.

Lastly, in any symmetric mixed strategy equilibrium with $\underline{s}=s_0=0\leq\bar{s}$ the DM's payoff is the maximum of the two scores offered. So DM utility is $\int_0^{\bar{s}}s\cdot\frac{\partial}{\partial s}\left([F(s)]^2\right)ds=\int_0^{\bar{s}}s\cdot 2F(s)f(s)\,ds=\int_0^{\bar{s}}2F(s)\cdot s(F(s))\cdot f(s)\,ds=\int_{F(0)}^{1}2F\cdot s(F)\,dF$, where the last equality follows from a change of variables and $F(\bar{s})=1$. Thus in the model with VPs, DM utility is $\int_{F(0)}^{\breve{F}}2F\cdot\hat{s}(F)\,dF+\int_{\breve{F}}^{1}2F\cdot\tilde{s}(F)\,dF=\int_{F(0)}^{\breve{F}}2F\cdot\left(\frac{2x_V^2}{\alpha}\left(3\frac{x_E}{x_V}+1\right)\cdot(F-F(0))\right)dF+\int_{\breve{F}}^{1}2F\cdot\left(\breve{s}+\int_{\breve{F}}^{F}4x_E^2\frac{G}{\alpha(\alpha-G)}dG\right)dF=\frac{4x_V^2}{\alpha}\left(3\frac{x_E}{x_V}+1\right)\int_{F(0)}^{\breve{F}}F\cdot(F-F(0))\,dF+\left(1-\breve{F}^2\right)\breve{s}+4x_E^2\int_{\breve{F}}^{1}2F\left(\int_{\breve{F}}^{F}\frac{G}{\alpha(\alpha-G)}dG\right)dF$, which evaluates to the expression in the proposition. QED

Proposition C.2. *If $y_0=0$ the DM's equilibrium payoff in the model without VPs FOSD her payoff in the symmetric equilibrium that we characterize in in Prop C.1 with VPs.*

Proof: Let $F_V(s)$ denote the DEVs' common score CDF in Prop C.1, and let $F_C(s)$ denote their common score CDF in the unique symmetric equilibrium of the model without VPS in Hirsch and Shotts (2015). In both models the DM chooses the policy with the maximum score, so to show FOSD of DM utility without veto players it suffices to show FOSD of the equilibrium score CDF.

Let $r=\frac{x_E}{x_V}$ and note that $r\geq 1$ by assumption. From Hirsch and Shotts (2015) $F_C(s)$ is a continuous strictly increasing function over $[0,\bar{s}_C]$ satisfying $F_C(0)=0$ and $F_C(\bar{s}_C)=1$ where $\bar{s}_C=4x_E^2\left(\log\left(\frac{\alpha}{\alpha-1}\right)-\frac{1}{\alpha}\right)>0$; the CDF has a (well-defined) inverse over $F\in[0,1]$, $\tilde{s}_C(F)=4x_E^2\int_0^F\frac{G}{\alpha(\alpha-G)}dG$.

Using Prop C.1, FOSD holds immediately when $\alpha \geq 1+r$, because DEVs are completely inactive with VPs. Consider next when the equilibrium with VPs has activity and is mixed, i.e. $\alpha \in (2, 1+r)$ so $F_V(0) = \frac{\alpha}{1+r} \in (0,1)$. We wish to show $F_C(s) < F_V(s)\ \forall s \in [0, \bar{s}_V]$ (where $F_V(\bar{s}_V) = 1$), further implying that $F_C(s) < F_V(s) = 1\ \forall s \in (\bar{s}_V, \bar{s}_C)$ and $F_C(s) = F_V(s) = 1\ \forall s \geq \bar{s}_C$. To do so we work with the inverse score CDFs and show $s_C(F) > s_V(F)\ \forall F \in [F_V(0), 1]$.

Recall from Prop C.1 that $s_V(F) = \hat{s}(F) = \frac{2x_V^2}{\alpha}(3r+1)(F - F_V(0))$ for $F \in \left[F(0), \breve{F}\right]$, where $\breve{F} = \min\left\{\frac{\alpha}{1+r}\left(\frac{1+3r}{1+2r}\right), 1\right\} > F_V(0)$. Also, $\hat{s}(F)$ is linear and note that $\tilde{s}_C(F)$ is strictly convex. Straightforwardly, $\hat{s}'\left(\frac{\alpha}{1+r}\left(\frac{1+3r}{1+2r}\right)\right) = \tilde{s}'_C\left(\frac{\alpha}{1+r}\left(\frac{1+3r}{1+2r}\right)\right) = \frac{2x_V^2}{\alpha}(3r+1)$. It thus suffices to show $\tilde{s}_C\left(\breve{F}\right) > \hat{s}\left(\breve{F}\right)$ since (using Prop C.1) either (a) $\breve{F} = 1 \iff \frac{\alpha}{1+r}\left(\frac{1+3r}{1+2r}\right) \geq 1$, $s_V(F) = \hat{s}(F)$ and $\tilde{s}'_C(F) < \hat{s}'(F)\ \forall F \in [F(0), 1]$, or (b) $\breve{F} < 1 \iff \frac{\alpha}{1+r}\left(\frac{1+3r}{1+2r}\right) < 1$, $s_V(F) = \hat{s}(F)$ and $\tilde{s}'_C(F) < \hat{s}'(F)\ \forall F \leq \left[\breve{F}(0), \breve{F}\right]$, and $s_V(F) = \tilde{s}(F)$ and $\tilde{s}'(F) = \tilde{s}'_C(F) = 4x_E\frac{G}{\alpha(\alpha-G)}\ \forall F \in (\breve{F}, 1]$. So we need only show $4x_E^2\int_0^{\breve{F}}\frac{G}{\alpha(\alpha-G)}dG > \frac{2x_V^2}{\alpha}(3r+1)\left(\breve{F} - \left(\frac{\alpha}{1+r}\right)\right) \Leftrightarrow 2r^2\int_0^{\breve{F}}\frac{G}{\alpha(\alpha-G)}dG > \frac{1}{\alpha}(3r+1)\left(\breve{F} - \left(\frac{\alpha}{1+r}\right)\right)$. Either $\breve{F} = \frac{\alpha}{1+r}\left(\frac{1+3r}{1+2r}\right) < 1$ and the r.h.s. reduces to $\frac{1}{\alpha}\breve{F}r$, or $\breve{F} = 1 \iff \frac{\alpha}{1+r}\left(\frac{1+3r}{1+2r}\right) \geq 1 \iff \frac{\alpha}{1+r} \geq \frac{1+2r}{1+3r}$ and the r.h.s is $\leq (3r+1)\left(1 - \frac{1+2r}{1+3r}\right) = \frac{1}{\alpha}r = \frac{1}{\alpha}\breve{F}r$. In either case, the inequality holds when $2r^2\int_0^{\breve{F}}\frac{G}{\alpha(\alpha-G)}dG > \frac{1}{\alpha}\breve{F}r$, which reduces to $2r\int_0^{\breve{F}}\frac{G}{\alpha-G}dG > \breve{F}$. Since $\alpha - G \geq \alpha - 1$ the preceding holds if $\frac{2r}{\alpha-1}\int_0^{\breve{F}}G \cdot dG > \breve{F}$, which reduces to $\frac{r}{\alpha-1}\breve{F} > 1$. If $\breve{F} = 1$ this holds, because $F_V(0) < 1$ implies $\alpha < 1+r$. If $\breve{F} = \frac{\alpha}{1+r}\left(\frac{1+3r}{1+2r}\right) < 1$ then because $\frac{1+3r}{1+2r} > 1$ it suffices to show that $\frac{r}{\alpha-1}\frac{\alpha}{1+r} > 1$ which holds because $\alpha < 1+r$. QED

C.2 Equilibrium with $y_0 \neq 0$

We next partially characterize equilibrium when $y_0 \neq 0$. Wlog we consider $y_0 < 0$.

Proposition C.3. *Suppose $y_0 < 0$ so that $s_0 = -y_0^2 < 0$, and*

$$\begin{aligned} D_L(s,F) &= \frac{\alpha}{x_V}\left(F\frac{x_E}{\alpha} + z_L(s)\right) = \frac{\alpha}{x_V}\left(F\frac{x_E}{\alpha} - z_R(s) + 2y_0\right) \\ &< \frac{\alpha}{x_V}\left(F\frac{x_E}{\alpha} - z_R(s)\right) = D_R(s,F)\ \ \forall (s,F) \end{aligned}$$

Then in any equilibrium with activity ($s_0 < \bar{s}$), participation is asymmetric (so $s_0 < \underline{s} \leq \bar{s}$)

- *if a DEV is unconstrained at a particular score ($F_{-i}(\hat{s})\frac{x_E}{\alpha} \leq sign(x_i) \cdot z_i(\hat{s})$) they are unconstrained at all higher scores ($F_{-i}(s)\frac{x_E}{\alpha} < sign(x_i) \cdot z_i(s)\ \forall s > \hat{s}$)*
- *L is sometimes inactive ($L = k$); R is always active ($R = -k$) and therefore strictly constrained by the VPs at the lowest score $\underline{s}$*

- *whenever R is unconstrained* ($\frac{x_E}{\alpha}F_L(s) \leq z_R(s)$) *so is L* ($\frac{x_E}{\alpha}F_R(s) > z_L(s)$)

- *R's score CDF is first-order stochastically dominant, i.e.* $F_R(s) \leq F_L(s)$, *and* $\exists \breve{s} \in (\underline{s}, \bar{s}]$ *such that* $F_R(s) < F_L(s)$ *for* $s < \breve{s}$ *and* $F_R(s) = F_L(s)$ *for* $s \geq \breve{s}$.

Proof: We first consider pure strategy equilibria with activity ($s_0 < \underline{s} = \bar{s}$). From Prop B.1 and symmetry for all parameters except y_0, any such equilibrium with activity must be asymmetric, have $\alpha - 1 = D_{-k}(\underline{s}; 1)$ and

$$\int_{s_0}^{\underline{s}} ((\alpha - 1) - \max\{D_k(s; 1), 0\})\, ds \geq \left(\frac{x_E}{x_V} - 1\right)(\underline{s} - s_0) \geq 0, \tag{A.7}$$

Suppose $-k = L$ and $k = R$; then $D_L(\underline{s}; 1) = \alpha - 1$ (from the first condition). But since $D_L(\underline{s}; 1) < D_R(s; 1)$ and $D_i(s; 1)$ is strictly decreasing in s, the left hand side of Eq A.7 would be strictly negative, a contradiction. Thus any pure strategy equilibrium with activity must have $k = L$ and $-k = R$, and it is easily verified that only one pure strategy equilibrium can exist and satisfies the remaining properties in Prop C.3.

We next consider mixed strategy equilibria ($s_0 \leq \underline{s} < \bar{s}$). Now, at any $s \in (\underline{s}, \bar{s}]$ where $D_i(s; F_i(s)) < 0 \iff \frac{x_E}{\alpha}F_{-i}(s) < sign(x_i) \cdot z_i(s)$ we must have $F_{-i}(s)$ strictly concave, because from the differential equations in part (3) of Prop B.1, $f_{-i}(s) = \frac{\alpha - F_{-i}(s)}{2x_E \cdot \left(y_R^*(s) - y_L^*(s)\right)}$, which is strictly decreasing in s.

Next, we show that at any $\breve{s} \in (\underline{s}, \bar{s}]$ where $D_i(\breve{s}; F_{-i}(\breve{s})) = 0 \iff \frac{x_E}{\alpha}F_{-i}(\breve{s}) = z_i(\breve{s})$ and also $D_i(s; F_{-i}(s)) > 0 \iff \frac{x_E}{\alpha}F_{-i}(s) > sign(x_i) \cdot z_i(s)$ in a neighborhood immediately below, $F_{-i}(s)$ is also strictly concave at $\breve{s}$ and in a neighborhood below, and $\frac{x_E}{\alpha}f_{-i}(\breve{s}) < \frac{\partial(sign(x_i) \cdot z_i(s))}{\partial s} = \frac{1}{2x_V}$. To see this first observe that $\frac{x_E}{\alpha}f_{-i}(\breve{s}) \leq \frac{\partial(sign(x_i) \cdot z_i(s))}{\partial s} = \frac{1}{2x_V}$; otherwise $\frac{x_E}{\alpha}F_{-i}(\breve{s}) = z_i(\breve{s})$ would imply $\frac{x_E}{\alpha}F_{-i}(s) < sign(x_i) \cdot z_i(s)$ in a neighborhood below $\breve{s}$, contradicting our premise. Next, our premises imply that in a neighborhood below $\breve{s}$, $(\alpha - F_{-i}(s)) - D_i(s; F_{-i}(s)) = f_{-i}(s) \cdot 2x_E \cdot (y_R^*(s) - y_L^*(s))$ $\iff f_{-i}(s) = \frac{(\alpha - F_{-i}(s)) + \frac{\alpha}{x_V}\left(F_{-i}(s)\frac{x_E}{\alpha} - sign(x_i) \cdot z_i(s)\right)}{2x_E \cdot \left(y_R^*(s) - y_L^*(s)\right)}$. Since $y_R^*(s) - y_L^*(s)$ is strictly increasing in s, to show $f_{-i}(s)$ strictly decreasing (i.e., $F_{-i}(s)$ strictly concave) in a neighborhood below $\breve{s}$ it suffices to show the numerator is strictly decreasing. The derivative of the numerator is $-f_{-i}(s) + \frac{\alpha}{x_V}\left(f_{-i}(s)\frac{x_E}{\alpha} - \frac{1}{2x_V}\right)$, which is < 0 at $\breve{s}$ and in a neighborhood below $\breve{s}$ since $f_{-i}(\breve{s})\frac{x_E}{\alpha} - \frac{1}{2x_V} \leq 0$. Lastly, since $F_{-i}(s)$ is strictly concave at $\breve{s}$ and in a neighborhood below $\breve{s}$ and $sign(x_i) \cdot z_i(s)$ is linear, we cannot have $f_{-i}(\breve{s})\frac{x_E}{\alpha} = \frac{1}{2x_V}$ since if so we would have $\frac{x_E}{\alpha}F_L(s) < sign(x_i) \cdot z_i(s)$ in a neighborhood below $\breve{s}$, again contradicting our premise.

Finally, note that the preceding arguments jointly imply that whenever $\frac{x_E}{\alpha}F_{-i}(s)$ "reaches" $sign\left(x_i\right)\cdot z_i\left(s\right)$ from above, it crosses at exactly a single point $\breve{s}$ and stays strictly below thereafter (since $z_i(s)$ is linear, $F_{-i}(s)$ is strictly concave in a neighborhood around $\breve{s}$, and once $\frac{x_E}{\alpha}F_{-i}(s)$ is strictly below $sign\left(x_i\right)\cdot z_i\left(s\right)$ the CDF $F_{-i}(s)$ remains strictly concave).

Having established basic properties of score CDFs in neighborhoods around "crossings" between a developer's unbounded optimum and her boundary of the veto proof set, we make some statements about $F_L\left(s\right)-F_R\left(s\right)$ in neighborhoods below scores where it crosses 0 (i.e., scores $\breve{s}\in(\underline{s},\overline{s}]$ where $F_L\left(\breve{s}\right)-F_R\left(\breve{s}\right)=0$). Specifically, if $F_L\left(\breve{s}\right)-F_R\left(\breve{s}\right)=0$ and $\frac{\partial}{\partial s}\left(F_L\left(s\right)-F_R\left(s\right)\right)=f_L\left(s\right)-f_R\left(s\right)<0$ in a neighborhood below $\breve{s}$ then $F_L\left(\breve{s}\right)-F_R\left(\breve{s}\right)$ is strictly decreasing in a neighborhood below $\breve{s}$, implying $F_L-F_R\left(s\right)>0$ in a neighborhood below $\breve{s}$. To sign $F_L\left(s\right)-F_R\left(s\right)$ in a neighborhood below crossings of $F_L(s)-F_R(s)$ with 0 it thus suffices to sign $f_L\left(s\right)-f_R\left(s\right)$. From part (3) of Prop B.1, $\forall i\in\{L,R\}$ and $s\in(\underline{s},\overline{s}]$ the CDFs are continuous and satisfy:

$$\begin{aligned}\left(f_R\left(s\right)-f_L\left(s\right)\right)\cdot 2x_E\left(y_R^*\left(s\right)-y_L^*\left(s\right)\right)&=F_L\left(s\right)-F_R\left(s\right)\\&+\max\left\{D_R\left(s;F_L\left(s\right)\right),0\right\}-\max\left\{D_L\left(s;F_R\left(s\right)\right),0\right\}\end{aligned}\tag{A.8}$$

with $y_R^*\left(s\right)-y_L^*\left(s\right)>0$, which we use to sign $f_R\left(s\right)-f_L\left(s\right)$ around crossings $F_L\left(\breve{s}\right)-F_R\left(\breve{s}\right)=0$.

(Case 1) Suppose $D_R\left(\breve{s};F_L\left(\breve{s}\right)\right)>0$. Then $f_R\left(\breve{s}\right)>f_L\left(\breve{s}\right)$ (using that $D_R\left(s;F\right)>D_L\left(s;F\right)$ $\forall\left(s,F\right)$), so $F_L\left(s\right)-F_R\left(s\right)>0$ and strictly decreasing in a neighborhood below $\breve{s}$ (using that score CDFs $\mathbf{F}$ are absolutely continuous over $(\underline{s},\overline{s})$).

(Case 2) Suppose $D_R\left(\breve{s};F_L\left(\breve{s}\right)\right)\leq 0$. Then $D_R\left(s;F\right)-D_L\left(s;F\right)=\frac{\alpha}{x_V}2\left|y_0\right|>0$ and $F_L\left(\breve{s}\right)-F_R\left(\breve{s}\right)=0$ jointly imply that $D_L\left(\breve{s};F_R\left(\breve{s}\right)\right)<0$. We consider two subcases.

(Subcase 2.i) Suppose $D_R\left(s;F_L\left(s\right)\right)\leq 0$ in a neighborhood below $\breve{s}$. Then over this region the differential equations are $\alpha-F_{-i}\left(s\right)=f_{-i}\left(s\right)\cdot 2x_E\left(y_R^*\left(s\right)-y_L^*\left(s\right)\right)$ $\forall i$. Then $\frac{f_i(s)}{\alpha-F_i(s)}=\frac{f_{-i}(s)}{\alpha-F_{-i}(s)}\rightarrow\int_s^{\breve{s}}\frac{f_i(t)}{\alpha-F_i(t)}dt=\int_s^{\breve{s}}\frac{f_{-i}(t)}{\alpha-F_{-i}(t)}dt\rightarrow\log\left(\frac{\alpha-F_i(s)}{\alpha-F_i(\breve{s})}\right)=\log\left(\frac{\alpha-F_{-i}(s)}{\alpha-F_{-i}(\breve{s})}\right)\rightarrow F_i\left(s\right)-F_{-i}\left(s\right)=0$ when combined with the boundary condition $F_i\left(\breve{s}\right)=F_{-i}\left(\breve{s}\right)$.

(Subcase 2.ii) Suppose $D_R\left(s;F_L\left(s\right)\right)>0$ in a neighborhood below $\breve{s}$, further implying $D_R\left(\breve{s};F_L\left(\breve{s}\right)\right)=0\iff\frac{x_E}{\alpha}F_L\left(\breve{s}\right)=z_R\left(\breve{s}\right)$ when combined with $D_R\left(\breve{s};F_L\left(\breve{s}\right)\right)\leq 0$ by continuity. By our initial arguments, $F_L\left(s\right)$ is strictly concave in a neighborhood below $\breve{s}$ and $\frac{x_E}{\alpha}f_L\left(\breve{s}\right)<z_R'\left(\breve{s}\right)=\frac{1}{2x_V}$. We show this implies $F_L\left(s\right)-F_R\left(s\right)>0$ and strictly decreasing in a neighborhood below $\breve{s}$. Under our premises, in a neighborhood below $\breve{s}$ Eq (A.8) reduces to $\left(f_R\left(s\right)-f_L\left(s\right)\right)\cdot 2x_E\left(y_R^*\left(s\right)-y_L^*\left(s\right)\right)=\left(F_L\left(s\right)-F_R\left(s\right)\right)+D_R\left(s;F_L\left(s\right)\right)=\left(F_L\left(s\right)-F_R\left(s\right)\right)+\frac{\alpha}{x_V}\left(F_L\left(s\right)\frac{x_E}{\alpha}-z_R\left(s\right)\right)$. Since $F_L\left(\breve{s}\right)=F_R\left(\breve{s}\right)$

and $F_L(\breve{s})\frac{x_E}{\alpha} = z_R(\breve{s})$ we rewrite the r.h.s. as $\int_s^{\breve{s}}(f_R(t) - f_L(t))\,dt + \frac{\alpha}{x_V}\int_s^{\breve{s}}\left(z_R'(t) - f_L(t)\frac{x_E}{\alpha}\right)dt =$ $\int_s^{\breve{s}}\left((f_R(t) - f_L(t)) + \frac{\alpha}{x_V}\left(\frac{1}{2x_V} - f_L(t)\frac{x_E}{\alpha}\right)\right)dt$. Since $\lim_{s\to\breve{s}}\left((f_R(s) - f_L(s)) + \frac{\alpha}{x_V}\left(\frac{1}{2x_V} - f_L(s)\frac{x_E}{\alpha}\right)\right)$ $= \frac{\alpha}{x_V}\left(\frac{1}{2x_V} - f_L(\breve{s})\frac{x_E}{\alpha}\right) > 0$, the r.h.s. is > 0 for s in a neighborhood below $\breve{s}$, so the l.h.s. is also strictly positive in a neighborhood below $\breve{s}$, implying $f_R(s) - f_L(s) > 0$ in a neighborhood below $\breve{s}$, which yields the desired property.

We now prove the main results. We first show weak FOSD ($F_R(s) \le F_L(s)\ \forall s \in [\underline{s}, \bar{s}]$). Suppose not, so $\exists s \in [\underline{s}, s)$ s.t. $F_L(s) - F_R(s) < 0$. Since $F_L(\bar{s}) - F_R(\bar{s}) = 0$, $\exists \breve{s} \in (s, \bar{s}]$ s.t. $F_L(\breve{s}) - F_R(\breve{s}) = 0$ and $F_L(s) - F_R(s) < 0$ in a neighborhood below; but the preceding arguments already imply $F_L(s) - F_R(s) \ge 0$ in a neighborhood below any $\breve{s}$ where $F_L(\breve{s}) - F_R(\breve{s}) = 0$.

We next argue $s_0 < \underline{s} < \bar{s}$ (any mixed equilibrium is asymmetric). Suppose $s_0 = \underline{s}$; since $y_R^*(s_0) = y_L^*(s_0) = 0$ by part (3) of Prop B.1, $\alpha - F_{-i}(s) = D_i(s_0; F_{-i}(s_0))\ \forall i$; but since $D_R(s;F) > D_L(s;F)\ \forall (s,F)$ and $D_i(s,F)$ is strictly increasing in F, satisfying both equalities would require that $F_L(s_0) < F_R(s_0)$, contradicting FOSD.

We next argue that in any asymmetric equilibrium $k = L$ and $-k = R$. Part (2) of Prop B.1 requires $\alpha - F_k(\underline{s}) = D_{-k}(\underline{s}; F_k(\underline{s}))$ and $\int_{s_0}^{\underline{s}}((\alpha - F_{-k}(\underline{s})) - \max\{D_k(s; F_{-k}(\underline{s})), 0\})\,ds \ge F_{-k}(\underline{s}) \cdot \left(\frac{x_E}{x_V} - 1\right)(\underline{s} - s_0) \ge 0$. Suppose instead $k = R$ and $-k = L$; then $\alpha - F_R(\underline{s}) = D_L(\underline{s}; F_R(\underline{s}))$ implying $\alpha - F_R(\underline{s}) < D_R(\underline{s}; F_R(\underline{s}))$. Then $F_L(\underline{s}) \ge F_R(\underline{s})$ would imply that $\int_{s_0}^{\underline{s}}(((\alpha - F_L(\underline{s}))$ $- \max\{D_R(s; F_L(\underline{s})), 0\})ds < 0$, which would violate the inequality, so instead $F_L(\underline{s}) < F_R(\underline{s})$, but this would violate FOSD.

We next show $F_R(s) < F_L(s)\ \forall s \in [s_0, \underline{s}]$, which is equivalent to $F_R(\underline{s}) < F_L(\underline{s})$; by FOSD it suffices to rule out $F_R(\underline{s}) = F_L(\underline{s})$. Suppose so. By the first bullet point of part (2) of Prop B.1, $D_R(\underline{s}; F_L(\underline{s})) = \alpha - F_L(\underline{s}) > 0$; letting $f_i^+(s) = \lim_{s\to\underline{s}^+}(f_i(s))$, Eq A.8 would then imply $\left(f_R^+(\underline{s}) - f_L^+(\underline{s})\right) \cdot 2x_E\left(y_R^*(\underline{s}) - y_L^*(\underline{s})\right) = D_R(s; F_L(\underline{s})) - \max\{D_L(s; F_R(\underline{s})), 0\} = D_R(s; F_L(\underline{s})) -$ $\max\{D_L(s; F_L(\underline{s})), 0\} > 0$, implying $f_R^+(\underline{s}) > f_L^+(\underline{s})$; but then $F_R(s) - F_L(s) > 0$ in a neighborhood above $\underline{s}$, violating FOSD.

We last show $\exists \breve{s} \in (\underline{s}, \bar{s}]$ s.t. $F_R(s) < F_L(s)$ for $s < \breve{s}$ and $F_R(s) = F_L(s)$ for $s \ge \breve{s}$. Recall that (i) $F_L(s)$ is strictly concave at any $s \in (\underline{s}, \bar{s}]$ where $D_R(s; F_L(s)) < 0$, and (ii) at any $\breve{s} \in (\underline{s}, \bar{s}]$ where $D_R(\breve{s}; F_L(\breve{s})) = 0 \iff \frac{x_E}{\alpha}F_L(\breve{s}) = z_R(\breve{s})$ and also $D_R(s; F_L(s)) > 0 \iff \frac{x_E}{\alpha}F_L(s) > z_R(s)$ in a neighborhood immediately below, $\frac{x_E}{\alpha}f_L(\breve{s}) < z_R'(\breve{s}) = \frac{1}{2x_V}$. When combined with the property that $D_R(\underline{s}; F_L(\underline{s})) = \alpha - F_L(\underline{s})$ these imply that $D_R(s; F_L(s))$ crosses 0 at most once at a $\breve{s}$ (since

$D_R(\breve{s}; F_L(\breve{s})) = 0$ and $\frac{x_E}{\alpha} f_L(\breve{s}) < z_R'(\breve{s})$ imply $D_L(s; F_L(s)) < 0$ in a neighborhood above $\breve{s}$ and therefore $F_L(s)$ is strictly concave and remains strictly concave thereafter by the first argument in this section of the proof on mixed strategy equilibria). Thus either (i) $D_R(s; F_L(s)) > 0\ \forall s \in [\underline{s}, \bar{s})$ implying $F_R(s) < F_L(s)\ \forall s \in [\underline{s}, \bar{s})$, or (ii) there $\exists \breve{s} \in (\underline{s}, \bar{s})$ s.t. $D_R(s; F_L(s)) > (<)(=) 0 \iff s < (>)(=) \breve{s}$. Since $F_R(s) \leq F_L(s)\ \forall s$ and $D_R(s; F) \geq D_L(s; F)$, $D_R(s; F_L(s)) \leq 0 \rightarrow D_L(s; F_R(s)) < 0$, so over $[\breve{s}, \bar{s}]$ the differential equations are $\alpha - F_{-i}(s) = f_{-i}(s) \cdot 2x_E(y_R^*(s) - y_L^*(s))\ \forall i$ which as shown in the proof of *Subcase 2.i* requires $F_i(s) - F_{-i}(s) = 0$ when combined with the boundary condition that $F_i(\bar{s}) = F_{-i}(\bar{s}) = 1$. QED

C.3 Computational Procedure

The preceding analytical results justify the following computational procedure to numerically calculate asymmetric equilibria for $y_0 \neq 0$. We describe the process for the case $y_0 < 0$. First, check whether no activity ($\bar{s} = s_0$) is a pure strategy equilibrium – it is easy to verify from Proposition B.1 that a no-activity pure-strategy equilibrium must be the unique equilibrium. Next, when inactivity fails to be an equilibrium, check whether the unique required strategy profile for a pure strategy equilibrium with activity ($s_0 < \underline{s} = \bar{s}$) with $k = L$ is indeed an equilibrium. If not, search for an asymmetric mixed equilibrium with $s_0 < \underline{s} < \bar{s}$ with $k = L$, which we identify for all parameters (Equilibrium existence can be proved using Simon and Zame (1990)).

Our analytical results handle some, but not all, potential issues of equilibrium multiplicity. When inactivity is an equilibrium it is unique. When inactivity fails to be an equilibrium, *any* equilibrium exhibits asymmetric participation ($s_0 < \underline{s}$) with the more-motivated DEV always active. And whenever an asymmetric pure strategy equilibrium exists, it is the unique pure strategy equilibrium. Our analytical results do not rule out coexistence of a pure and mixed asymmetric equilibrium both with the more motivated DEV always active, nor coexistence of two distinct mixed asymmetric equilibria both with the more motivated DEV always active. We conjecture (and our computational analysis supports) that equilibrium with symmetric DEVs and VPs is unique.

D Competitive Model Results

We now prove the results in the main paper.

Lemma 1. Prop B.1 implies that in a pure strategy equilibrium at most one DEV is active. The case where neither is active is covered in part (1) of the proposition, noting that in this case each DEV's optimal policy to develop is the status quo (s_0, y_0). When exactly one DEV is active, she develops her monopoly policy. To see that the active DEV must be the one with the higher monopoly score, suppose instead $s_{-k}^{M*} > s_k^{M*} > s_0$ and only k is active, developing $\left(s_k^{M*}, y_k^{M*}\right)$. But $-k$ strictly prefers to develop $\left(s_{-k}^{M*}, y_{-k}^{M*}\right)$ and have it enacted rather than (s_0, y_0) and because $|x_{-k}| \geq |x_{V-k}|$, $-k$ strictly prefers (s_0, y_0) over $\left(s_k^{M*}, y_k^{M*}\right)$. Thus $-k$ strictly prefers to enter, a contradiction. QED

Remark 1. (1) follows from Lemma A.1. (3) follows from Lemmas A.2 and A.3. For (2), given (3) the only choice for the DEV is the CDF she uses when choosing score. QED

Lemma 1. Follows from Prop B.1. QED

Proposition 2. For **(1)**, a necessary and sufficient condition for L to develop a policy if R sits out is $y_0 > \hat{y}_L(x_V) = -\frac{1}{\alpha}x_E + \left(1 - \frac{1}{\alpha}\right)x_V$ and a necessary and sufficient condition for R to develop a policy if L sits out is $y_0 < \hat{y}_R(x_V) = \frac{1}{\alpha}x_E + \left(1 - \frac{1}{\alpha}\right)(-x_V)$. Combining yields a necessary and sufficient condition for existence of a pure strategy equilibrium in which both sit out: $\hat{y}_R(x_V) \leq y_0 \leq \hat{y}_L(x_V)$. This requires $\hat{y}_R(x_V) \leq \hat{y}_L(x_V)$, i.e., $x_V \geq \frac{x_E}{\alpha - 1} = \bar{x}_V$.

For **(2)**, from above, in equilibrium at least one DEV must be active when $y_0 \notin [\hat{y}_R(x_V), \hat{y}_L(x_V)]$. From Prop C.3, the more-motivated one is always active. The question is whether the less-motivated developer is active too, in a mixed strategy equilibrium as characterized in Prop C.3. We characterize a cutpoint $\bar{y}(x_V) \geq 0$ s.t. there is a pure strategy equilibrium in which the less-motivated DEV is inactive iff $y_0 \notin [-\bar{y}(x_V), \bar{y}(x_V)]$. Wlog let $y_0 > 0$ so L is more-motivated. We determine whether R's best response is to enter or to sit out when L develops her monopoly policy from Lemma 2. Recall that L's monopoly policy is $\left(s_L^{M*}, y_L^{M*}\right)$ where $y_L^{M*} = -\frac{1}{\alpha}x_E + \left(1 - \frac{1}{\alpha}\right)x_V = z_L\left(s_L^{M*}\right)$. Note that from Def A.1, $z_R(s) - z_L(s) = 2(y_0 - z_L(s))$ and thus $z_R(s) = 2y_0 - z_L(s)$.

It is never optimal for R to develop a policy with a score $> s_L^{M*}$; since $y_0 > 0$, $|z_R(s)| > |z_L(s)|$ so $D_R(s,1) < D_L(s,1)$. By Lemma A.2, if R's best response is to beat L's monopoly policy, it will be at score s_L^{M*} and ideology $\min\left\{\max\left\{z_L\left(s_L^{M*}\right), \frac{x_E}{\alpha}\right\}, z_R\left(s_L^{M*}\right)\right\}$. If $z_L\left(s_L^{M*}\right) = \min\left\{\max\left\{z_L\left(s_L^{M*}\right), \frac{x_E}{\alpha}\right\}, z_R\left(s_L^{M*}\right)\right\}$, R won't enter because doing so means paying costs to develop the same policy L develops. So the best ideology for R with score s_L^{M*} is $\min\left\{\frac{x_E}{\alpha}, z_R\left(s_L^{M*}\right)\right\}$.

Note that $z_R\left(s_L^{M*}\right) = 2y_0 - z_L\left(s_L^{M*}\right) = 2y_0 + \frac{1}{\alpha}x_E - \left(1 - \frac{1}{\alpha}\right)x_V$ so $z_R\left(s_L^{M*}\right) \leq \frac{x_E}{\alpha}$ iff $y_0 \leq \frac{\alpha - 1}{2\alpha}x_V$.

We first consider the case $y_0 \leq \frac{\alpha-1}{2\alpha}x_V$, for which R's optimal score-s_L^{M*} policy is on the boundary and R's net benefit from entering with a policy at $\left(s_L^{M*}, z_R\left(s_L^{M*}\right)\right)$ is $2x_E\left(z_R\left(s_L^{M*}\right) - z_L\left(s_L^{M*}\right)\right)$ $-\alpha\left(s_L^{M*} + \left[z_R\left(s_L^{M*}\right)\right]^2\right) = 4x_E\left(y_0 - z_L\left(s_L^{M*}\right)\right) - \alpha\left(s_L^{M*} + \left(2y_0 - z_L\left(s_L^{M*}\right)\right)^2\right) = 4x_E\left(y_0 - y_L^{M*}\right)$ $-\alpha\left(s_L\left(y_L^{M*}; y_0\right) + \left(2y_0 - y_L^{M*}\right)^2\right) = \left(y_0 - y_L^{M*}\right) \cdot \left(4x_E - \alpha\left(2x_V + 3y_0 - y_L^{M*}\right)\right)$. This is a concave quadratic function of y_0 with zeroes at $y_0 = y_L^{M*}$ and at $\check{y}(x_V)$ that solves

$$\begin{aligned} 0 &= 4x_E - \alpha\left(2x_V + 3y_0 - y_L^{M*}\right), \text{ so} \\ \check{y}(x_V) &= \frac{x_E}{\alpha} - \frac{x_V(\alpha+1)}{3\alpha} \end{aligned} \quad \text{(A.9)}$$

For L to enter as a monopolist requires $y_0 > y_L^{M*}$, so for R to have a profitable deviation to enter and win with policy $\left(s_L^{M*}, z_R\left(s_L^{M*}\right)\right)$ also requires $y_0 < \check{y}(x_V)$.

Next consider the case $y_0 > \frac{\alpha-1}{2\alpha}x_V$, for which R's optimal score-s_L^{M*} policy is off the boundary and R's net benefit from entering at $\left(s_L^{M*}, \frac{x_E}{\alpha}\right)$ is:

$$\begin{aligned} \tilde{G}(y_0; x_V, x_E) &= 2x_E\left(\frac{x_E}{\alpha} - z_L\left(s_L^{M*}\right)\right) - \alpha\left(s_L^{M*} + \left[\frac{x_E}{\alpha}\right]^2\right) \\ &= 2x_E\left(\frac{x_E}{\alpha} - y_L^{M*}\right) - \alpha\left(\left(-y_0^2 + 2x_V\left(y_0 - y_L^{M*}\right)\right) + \left[\frac{x_E}{\alpha}\right]^2\right) \\ &= \alpha y_0^2 - 2\alpha x_V y_0 + 3\frac{x_E^2}{\alpha} - 4x_V x_E + 2x_E x_V\frac{1}{\alpha} + 2x_V^2\alpha\left(1 - \frac{1}{\alpha}\right) \end{aligned} \quad \text{(A.10)}$$

This has zeroes at $y_0 = x_V \pm \frac{1}{\alpha}\sqrt{\alpha^2 x_V^2 - \alpha\left(3\frac{x_E^2}{\alpha} - 4x_V x_E + 2x_E x_V\frac{1}{\alpha} + 2x_V^2\alpha\left(1-\frac{1}{\alpha}\right)\right)}$. If the determinant is negative the benefit of entering at $\left(s_L^{M*}, \frac{x_E}{\alpha}\right)$ is strictly positive $\forall y_0 \in \left[\frac{\alpha-1}{2\alpha}x_V, x_V\right]$. Otherwise, R strictly gains from entering iff $y_0 \in \left[\frac{\alpha-1}{2\alpha}x_V, \tilde{y}(x_V)\right)$ where $\tilde{y}(x_V)$ $= x_V - \frac{1}{\alpha}\sqrt{\alpha^2 x_V^2 - \alpha\left(3\frac{x_E^2}{\alpha} - 4x_V x_E + 2x_E x_V\frac{1}{\alpha} + 2x_V^2\alpha\left(1-\frac{1}{\alpha}\right)\right)}$. Thus, letting

$$\bar{y}(x_V) = \begin{cases} \check{y}(x_V) \text{ if } y_0 \in \left(0, \frac{\alpha-1}{2\alpha}x_V\right] \\ \tilde{y}(x_V) \text{ if } y_0 \in \left[\frac{\alpha-1}{2\alpha}x_V, x_V\right] \end{cases} \quad \text{(A.11)}$$

we have shown that for $y_0 > 0$ there is an equilibrium with only L active iff $y_0 \in (0, \bar{y}(x_V)]$.

Although not covered in part 2 of Prop 2, we note what happens for $y_0 = 0$; Prop C.1 characterizes a symmetric equilibrium. Also, if $y_0 = 0$ there cannot be an equilibrium with exactly one DEV active because $-k$'s monopoly policy $\left(0, y_{-k}^{M*}(0)\right)$ is k's strictly-worst zero-score policy, so if $-k$ develops $\left(0, y_{-k}^{M*}(0)\right)$ then k is strictly better off entering.

(3) follows from Prop. C.3. QED

Proposition 3. For **(1)**, first note that within a pure strategy equilibrium, the less-motivated DEV's probability of being active is constant at 0.

We next show that if there is a pure strategy equilibrium at $\tilde{x}_V$, there is a pure strategy equilibrium $\forall x_V > \tilde{x}_V$. Wlog we show this for $y_0 > 0$.

First note that because a mixed strategy equilibrium requires that the more-motivated DEV L be active, there is a pure strategy equilibrium with neither active iff $y_L^{M*} > y_0 \Leftrightarrow x_V > \frac{\alpha y_0 + x_E}{\alpha - 1}$.

Otherwise, any pure strategy equilibrium must have L active. From Prop 2, this requires $y_0 > \bar{y}(x_V)$, where $\bar{y}(x_V) = \begin{cases} \check{y}(x_V) & \text{if } y_0 \in \left(0, \frac{\alpha-1}{2\alpha} x_V\right] \\ \tilde{y}(x_V) & \text{if } y_0 \in \left[\frac{\alpha-1}{2\alpha} x_V, x_V\right] \end{cases}$ from Eq A.11.

For $y_0 \leq \frac{\alpha-1}{2\alpha} x_V$, from Eq A.9 a pure strategy equilibrium requires $y_0 > \check{y}(x_V) = \frac{x_E}{\alpha} - \frac{x_V(\alpha+1)}{3\alpha} \Leftrightarrow x_V > \frac{3\alpha}{\alpha+1}\left(\frac{x_E}{\alpha} - y_0\right)$.

For $y_0 \geq \frac{\alpha-1}{2\alpha} x_V$, a pure strategy equilibrium requires that R's net benefit from entering at $\left(s_L^{M*}, \frac{x_E}{\alpha}\right)$ if L develops her monopoly policy must be negative, i.e., from Eq A.10, $\tilde{G}(y_0; x_V, x_E) \leq 0$. We show that if $\tilde{G}(y_0; \tilde{x}_V, x_E) \leq 0$ for some $\tilde{x}_V \in (0, x_E)$ then $\tilde{G}(y_0; x_V, x_E) \leq 0\ \forall x_V \in [\tilde{x}_V, x_E]$, i.e., there is still a pure strategy equilibrium if veto players are more extreme. Note that $\tilde{G}(y_0; x_V, x_E)$ is a strictly convex quadratic function of x_V. Thus if $\tilde{G}(y_0; \breve{x}_V, x_E) > 0$ for some $\breve{x}_V \in (\tilde{x}_V, x_E)$, then $\tilde{G}(y_0; x_V, x_E) > 0\ \forall x_V > [\breve{x}_V, x_E]$, and $\tilde{G}(y_0; x_E, x_E) > 0$. But this cannot be the case, because if $x_V = x_E$ then $y_L^{M*} > 0$ and $y_R^{R*} < 0$ and hence R strictly prefers not to pay the marginal cost of moving policy rightward from L's monopoly policy.

We also note that at $y_0 = \frac{\alpha-1}{2\alpha} x_V$, $\check{y}(x_V) = \tilde{y}(x_V)$ (via algebra). Thus if x_V increases from a value $< \frac{\alpha-1}{2\alpha} x_V$ to a value $> \frac{\alpha-1}{2\alpha} x_V$, the arguments above imply that if there is a pure strategy equilibrium at the lower value of x_V there is a also pure strategy equilibrium at the higher value.

The final component of the result is that within a mixed strategy equilibrium region, the probability that the less-motivated DEV is active is strictly decreasing in x_V. This follows from computational analysis of the equilibrium in Prop C.3, holding fixed all parameters except for x_V.

Part 2. Follows directly from Prop 1's condition for a monopolist to invest. QED

Corollary 2. Follows from Eq 3 and Footnote 4 of Hirsch and Shotts (2015). QED

Proposition 4. This follows from a combination of analytical and computational results. The analytical results are the following. First, absent VPs, EU_D^0 is characterized in Corollary 2. Second, if $y_0 = 0$, DM utility with VPs is characterized in Prop C.1, and Prop C.2 shows it is $< EU_D^0$. Third, for parameters where neither DEV is active, DM utility is $s_0 = -y_0^2 < 0 < EU_D^0$. Fourth, for parameters where exactly one DEV is active, DM utility is the monopoly score, s_i^{M*}, which from Corollary 1 is strictly increasing in $|y_0|$.

The final piece of the results is for parameters where both DEVs are active and $y_0 \neq 0$. In this case, we compute DM utility by numerical evaluation of the equilibrium in Prop C.3. QED

Extremist VPs harm DM when $\alpha > \tilde{\alpha}$. In our discussion of DM utility, we note that if $\alpha > \tilde{\alpha} \approx 3.68$, and VPs are extreme then for any y_0 DM utility is higher without VPs. We show this for $x_V = x_E$, noting that by continuity it holds for x_V in a neighborhood below x_E.

We first argue that for $x_V = x_E$ and any y_0 any equilibrium is in pure strategies. Suppose not, i.e., there is an equilibrium in mixed strategies. Note that the less-motivated DEV k must weakly prefer the status quo over any policy in the support of $-k$'s strategy, because one VP is at k's ideal ideology. Also, $sign\,(y_0) = sign\,(x_k)$ because $\alpha > 2$, so there is no policy that k weakly prefers to develop and enact over the status quo, which is a contradiction.

Also note that for any y_0 at which neither DEV is active, DM utility is ≤ 0 and so $< EU_D^0 = 4x_E^2 \int_0^1 2F \left(\int_0^F \frac{G}{\alpha(\alpha - G)} dG \right) dF$ from Corollary 2.

Thus we only need to consider equilibria with exactly one active DEV. From Corollary 1, when policy development occurs, the monopoly score is increasing in $|y_0|$, so to characterize a bound on α we set $y_0 = -x_V = -x_E < 0$, calculate DM utility with R as a monopolist s_R^{M*}, and compare it with EU_D^0. From Prop 2, $y_R^{M*} = \frac{1}{\alpha} x_E + \left(1 - \frac{1}{\alpha}\right)(-x_V) = \frac{2}{\alpha} x_E - x_E$. Also, because the left veto player is indifferent, $q_R^* = \left(y_R^{M*} - (-x_V)\right)^2 - (y_0 - (-x_V))^2 = \frac{4}{\alpha^2} x_E^2$ so DM utility is $s_R^{M*} = x_E^2 \left(\frac{4}{\alpha} - 1\right)$.

Without VPs, from Corollary 2 $EU_D^0 = 4x_E^2 \int_0^1 2F \left(\int_0^F \frac{G}{\alpha(\alpha-G)} dG \right) dF$, which evaluates to $EU_D^0 = 4x_E^2 \left(\left(\alpha + \frac{1}{2} - \frac{2}{3\alpha} - \left(\alpha^2 - 1\right) \ln \left(\frac{\alpha}{\alpha - 1} \right) \right) \right)$. Both $s_R^{M*}$and EU_D^0 are strictly decreasing in α. Evaluating numerically, $EU_D^0 > s_R^{M*}, \forall \alpha > \tilde{\alpha} \approx 3.68$. QED

E Data Notes

Figure 6 uses Nominate scores (from voteview.com, 2/26/2023). If a state had more than 2 Senators during a session, we use scores for the 2 who served longest within that session. We calculate the left filibuster pivot as the 41st most liberal Senator and the right filibuster pivot as the 60th. When calculating party medians, Senators who were independent or members of minor parties but caucused with a major party are treated as members of that party.