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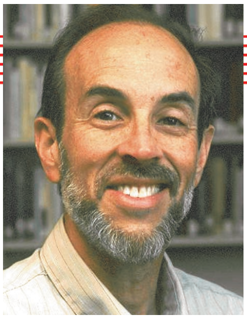

Continuum Mechanics in Physics Education

Jerry Gollub

One of the oddities of contemporary physics education is the nearly complete absence of continuum mechanics in the typical undergraduate or graduate curriculum. Continuum mechanics refers to field descriptions of mechanical phenomena, which are usually modeled by partial differential equations. The Navier–Stokes equations for the velocity and pressure fields of Newtonian fluids provide an important example, but continuum modeling is of course also well developed for elastic and plastic solids, plasmas, complex fluids, and other systems.

Students' main experience with continua, at both the undergraduate and graduate level, occurs in the standard courses in electromagnetism. Fields associated with simple charge distributions are encountered, as is the propagation of electromagnetic waves in various media. On the other hand, parallel experience does not typically occur in students' studies of mechanics, in which continuum phenomena are arguably just as important. A clear understanding of stresses (not just forces) is essential for understanding how materials stretch, bend, and break, and how fluids flow.

It is easy to see why engineers are interested in fluid and solid mechanics, but the subject is also increasingly valuable for physicists and students. Continuum modeling is widely used in astrophysics at many scales, including both stellar interiors and larger-scale phenomena. The subject plays a significant role in the growing field of biological physics, in which structures such as membranes and the cytoskeleton are of great interest. With exposure to the continuum way of thinking about mechanical phenomena, students would have access to many physical aspects of nature: laboratory and geophysical fluid dynamics, the dynamics of deformable materials, part of the growing fields of soft condensed matter physics and complex fluids, and much more.

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Given these facts, how can we account for the nearly total absence of continuum mechanics in typical physics curricula at both the undergraduate and graduate levels? The omission may be in part a consequence of the historical relegation of some of these topics—for example, fluid mechanics—to engineering. However, other factors contribute as well.

- ▶ Our curricula at both the undergraduate and graduate levels are already crowded with other topics. We know from educational research that crowded curricula interfere with attaining deep understanding, so it is difficult to make additions without deleting other topics.
- ▶ The most popular textbooks intended for courses in classical mechanics do not generally treat fluid or solid mechanics. It is hard to teach courses that are not adequately supported by textbooks, especially at the undergraduate level. (Although there are a few fluid dynamics textbooks suitable for physics students, they are mainly intended for courses that are entirely devoted to that topic.)
- ▶ We physicists often do not feel prepared to teach relatively unfamiliar topics. (Co-teaching with an engineering colleague might be an intriguing solution.)
- ▶ We worry about appearing to infringe on subjects that are taught elsewhere in our universities.
- ▶ We are not yet collectively convinced that the need is compelling, despite the wide applicability of fluid and solid mechanics. Perhaps this column will help.

The integration of continuum mechanics into the physics curriculum could yield many benefits. I do not presume that this integration would have to occur as a single separate course. Greater attention to this field could be distributed in various parts of the undergraduate and graduate curricula, including courses in classical mechanics, condensed matter physics, and so on.

Some suggestions

At Haverford College, I generally include an introduction to fluid dynamics in our undergraduate mechanics course. Time for this can be created in several ways, for example by treating oscillations in an earlier "waves and optics" course or by abbreviating the treatment of rigid bodies. I also plan to teach a general interest course called Fluids in Nature for non-majors in 2004.

I like to get students interested in hydrodynamic phenomena by showing them some of the images in the Gallery of Fluid Motion, a competitive feature of the annual American Physical Society's division of fluid dynamics meetings. The entries are available online at http://www.aps.org/units/dfd and will appear soon in book form.[1] Several recent examples are given in figures 1 and 2, but many more are available online. (Click on the American Institute of Physics "AIP Gallery of Fluid Motion" and then either "2003 Gallery" or "Archives.") Readers may also be interested in Steven Vogel's *Life in Moving Fluids*,[2] a fascinating exposition on biological fluid mechanics.

For graduate and upper-level undergraduate students, a modern course in fluid mechanics could easily cover some of the applications to fields I have mentioned in this column and thus provide an opportunity to showcase the diversity of physics and its connections to neighboring disciplines. It is also desirable to explain the limits of traditional hydrodynamics, to show how it is connected to atomic-scale thinking, and to indicate that it can be extended to non-Newtonian fluids.

Potential benefits

What are some of the potential benefits of including fluid and solid mechanics in courses? First, many students have difficulty developing competence in using partial differential equations in physical theories. By applying them to a wide range of mechanical phenomena that can be directly visualized, students might significantly improve their knowledge in this area of applied mathematics that is central to physical modeling. (It would not be a bad experience for most instructors, either!) Concepts such as scaling, dimensional analysis, linear

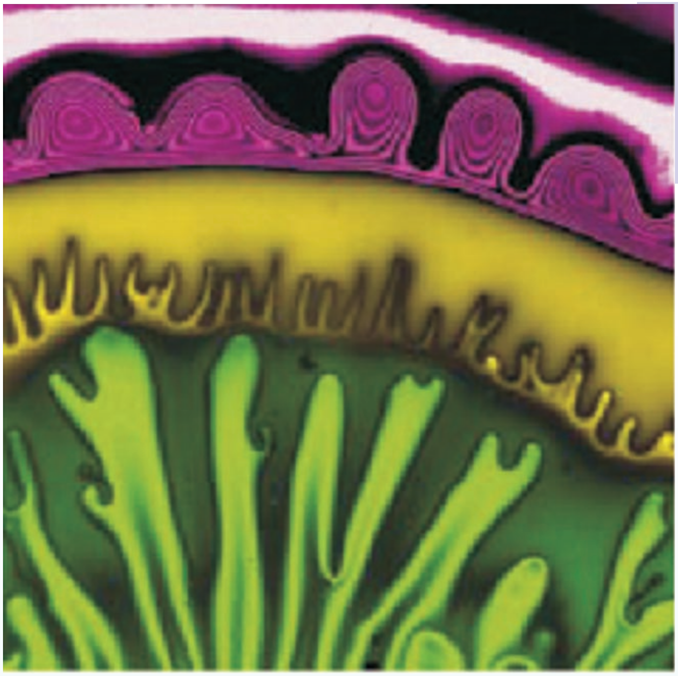

Figure 1. Unstable spreading of a surfactant solution on a thin viscous film. The solution is driven by surface tension gradients. (A. A. Darhuber, S. M. Troian, from the AIP Gallery of Fluid Motion Web site, http://www.aps.org/units/dfd, item S9, 2003.)

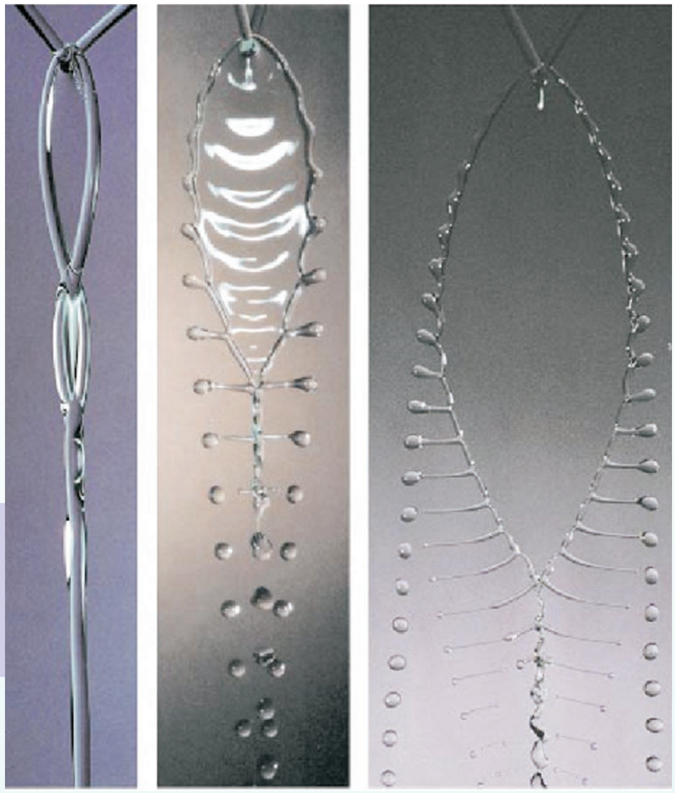

Figure 2. "Fluid fishbones" created by the collision of two jets at flow rates increasing from left to right. (A. E. Hasha, J. W. M. Bush, from the AIP Gallery of Fluid Motion Web site, http://www.aps.org/units/dfd, item S8, 2002.)

and nonlinear stability theory, asymptotic analysis, Fourier methods, and so forth can be effectively taught in the context of continuum mechanics.

Second, continuum mechanics is one way to introduce physics students to nonlinear dynamics, a subject that has wide applications. Perhaps because linear phenomena appear to be so straightforward to model, physics courses still suffer from a preoccupation with them. Among the nonlinear phenomena that can be introduced would be instabilities, chaotic dynamics, and complexity. Although some of these topics can also be treated at the level of individual particles as discussed in a previous column (PHYSICS TODAY, January 2003, page 10), a continuum treatment offers special opportunities because fluid dynamics is inherently nonlinear.

An example of how fluid mechanics can contribute to an understanding of nonlinear dynamics occurs through consideration of mixing in fluids. Although most students learn about Hamiltonian mechanics as undergraduates, a smaller number encounter the fundamental concepts of hyperbolic and elliptic fixed points that characterize the phase space of almost any nonlinear conservative system such as the pendulum. Even fewer students (and faculty) have any recall of these topics, as I know from having asked audiences at various colloquia. Yet these important mathematical structures can be visualized concretely and in real space (rather than phase space) by considering how an impurity is mixed into a fluid whose velocity field is time periodic.[3]

A third advantage is that acquiring the tools of continuum mechanics gives students the potential to understand phenomena that are amazingly diverse and also important—the fracture and failure of solids, instability and pattern formation in flowing fluids, the dynamics of the atmospheric circulation, and the behavior of soft materials such as membranes, emulsions, and biological materials. It is plausible to imagine that the inclusion of topics from these fields in our educational programs would highly motivate some students whom we currently lose to other fields. It would certainly increase students' confidence that their physics knowledge is widely applicable and would contribute to their preparation for a variety of research and employment opportunities. Perhaps over time, our thinking about what students really need to know will evolve to include this multidisciplinary field.

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References

1. M. Samimy, K. S. Breuer, L. G. Leal, P. H. Steen, eds. *A Gallery of Fluid Motion*, Cambridge U. Press, New York (2003).
2. S. Vogel, *Life in Moving Fluids: The Physical Biology of Flow*, Princeton U. Press, Princeton, N.J. (1994).
3. For an elementary discussion, see R. C. Hilborn, *Chaos and Nonlinear Dynamics: An Introduction for Scientists and Engineers*, 2nd ed. Oxford U. Press, New York (2000), p. 436. At a more advanced level, see J. M. Ottino, *The Kinematics of Mixing: Stretching Chaos and Transport,* Cambridge U. Press, New York (1989). ■