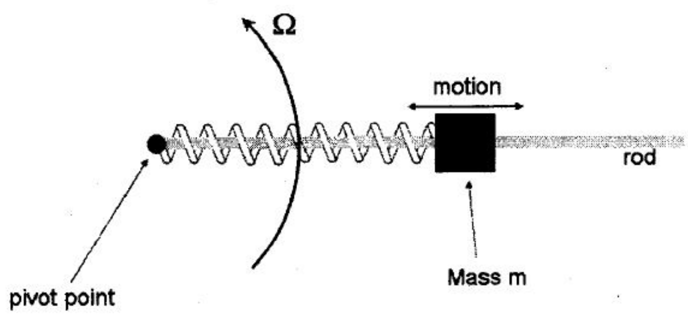
QP53

A long massless rod is attached at one end to a motor which rotates the rod at constant angular speed Ω. A massless spring of force constant k and equilibrium length r_0 is wound around the rod. One end of the spring is attached to the pivot point. At the other end of the spring a mass m is attached. This mass slides along the rod as the spring expands and contracts. There is no friction and no gravity and the motor is powerful enough to keep the angular speed Ω constant. In case you are worrying about the Coriolois force: don't. It is not relevant here.

The natural oscillation frequency of the spring and mass combination would be $\omega_0 = (k/m)^{1/2}$ were it not for the rotation of the rod. Let the radial position of the mass be r, measured from the pivot point.



a) (2 points) Write down Newton's second law for the mass m in the reference frame rotating with the rod. Show that simple harmonic oscillations are still expected, at least for sufficiently small Ω.

b) (1 point) Assuming no oscillations, what is the equilibrium radial position of the mass?

c) (2 points) Find a formula for the natural oscillation frequency ω of the mass/spring system. Show that your formula reduces to the right value when the rod is not rotating (i.e. when $\Omega = 0$).

d) (2 points) You'll see that your formula does something strange when Ω is too large. Describe what happens when this is the case. Are there still nice harmonic oscillations? What is the value of Ω when this strange behavior takes over? If you could not work part (c), try to answer this question based on your physical intuition.

e) (2 points) Sketch the potential energy $U(r)$ of the mass/spring system for Ω less than the critical value and for Ω greater than the critical value.

f) (1 point) Is angular momentum conserved in this problem? Explain your answer.