

QP32

A uniform disk of mass  $m$  and radius  $r$  begins to slide down an inclined plane with an initial velocity  $v_0$  at its center of mass at time  $t = 0$ . The inclined plane has a surface frictional coefficient  $\mu$  and forms an angle  $\theta$  relative to the ground, as shown below. At time  $t = t_1$ , the disk begins to roll down the plane without slipping. The local gravitational acceleration is  $g$ , pointing vertically down.

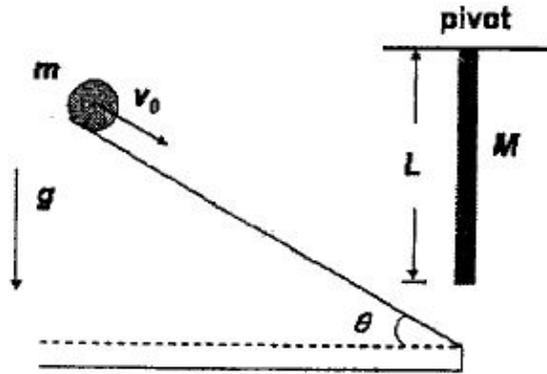


Figure 1A

- (2 points) Express  $t_1$  in terms of  $v_0$ ,  $g$ ,  $\mu$  and  $\theta$ .
- (1 point) Find the minimal frictional coefficient  $\mu$  (in terms of  $g$  and  $\theta$ ) required for the disk to achieve pure rolling motion?

At  $t > t_1$  the disk reaches the end of the inclined plane with a final speed  $v_f$  at its center of mass, and it becomes stuck instantaneously upon impact to the end of a uniform thin rod of length  $L$  and mass  $M$  hanging vertically from the ceiling. The rod-disk assembly swings to the right, as shown below.

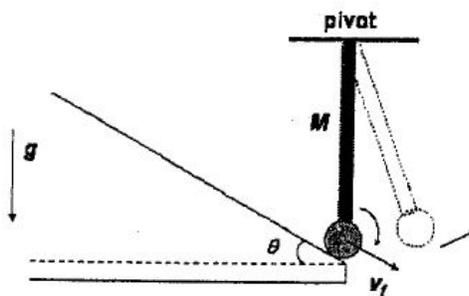


Figure 1B

- (1 point) Find the moment of inertia  $I$  of the rod-disk assembly about the axis through the pivot.
- (2 points) Find the angular momentum (both the magnitude and direction) of the rod-disk assembly about the axis through the pivot after the impact. Express your answer in terms of  $v_f$ ,  $m$ ,  $M$ ,  $r$  and  $L$ . Discuss the condition required for the rod-disk assembly to swing to the right.