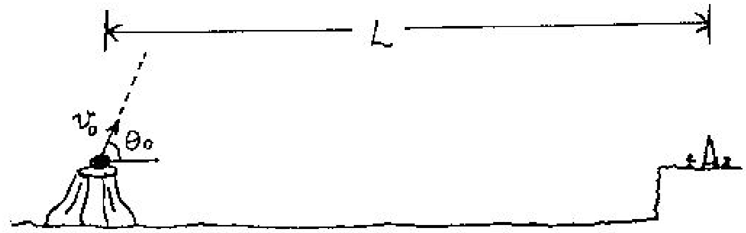
QP2



As shown in the sketch above, a spaceship on a distant planet has landed on a ledge above a vast flat horizontal plain, a distance $L = 100$ km from a volcano. Just as the astronaut team emerges to survey the area, a boulder is ejected at $t = 0$ from the cone of the volcano, at the same height as the ledge, with an initial speed $v_0 = 1$ km/sec and an inital angle θ_0 with the horizontal. Measurements by the team show that the boulder is following a parabolic trajectory, that it reaches its maximum height at $t = 200$ sec, and that it is headed directly for the base of the ship! The astronauts clamber back into the ship and fire their boosters at time t_0 when the boulder is falling at a vertical height of 10 km above the ledge.
Note that numerical answers are required for parts a, b, c and e below.

a) (2 points) Find the angle θ_0 and the acceleration of gravity g_0 on the planet.

b) (2 points) Find the maximum height above the ledge reached by the boulder.

c) (2 points) How much time t_2 does the team have left to escape when they fire their boosters at time t_0?
The astronauts use their main and emergency boosters, which together (for times $t > t_0$) give the ship a total vertical acceleration (in m/sec^2) of $a(t') = A(1 = e^{(t'/\tau)})$, where $\tau = 5$ sec and $t' = t - t_0$. Note that A already includes the planet's gravitational acceleration g_0.

d) (2 points) Find an expression for the speed of the ship $U(t')$ for $t_2 > t' > 0$. You may leave your answer in terms of A, τ and t'.

e) If the base of the ship must reach a height of 1 km above the point where the boulder strikes the ledge to avoid being destroyed by flying debris, find the minimum value of A that will allow the team to escape.