QP1

You board a stationary elevator on the ground floor. A uniform vertical gravitational field $g = -9.8m/s^2$ is present. Before the elevator starts moving, you drop a "super" ball from a height of 2 m above the floor. The "super" ball bounces perfectly elastically from the floor (meaning that the velocity changes direction in a negligible amount of time, without changing its magnitude).

a) (1 point) How long does it take for the ball to reach the floor of the elevator?

b) (2 points) How long does it take for the ball to travel from the floor to its maximum height above the floor (at which point you catch it again)?

Now the elevator starts to accelerate until it reaches a velocity of $v_0 = 0.5m/s$ just before it reaches the second floor. At this point the velocity remains constant at 0.5 m/s. As it passes the second floor you again drop the ball from a height of 2 m above the floor.

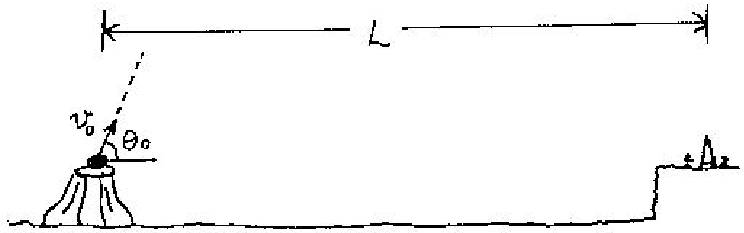
c) (3 points) How long does it take the ball to hit the floor of the elevator? Hint: you might want to express the height of the elevator floor and the ball, with respect to the second floor, as a function of time.

After you've caught the ball, and just as the elevator passes the fourth floor, it starts to accelerate again, with an acceleration give by $a = +kt^2$ with $k = 4m/s^4$. At this point you again drop the ball from a height of 2 m above the floor.

d) (2 points) How long does it take the ball to hit the floor of the elevator?

e) (2 points) After you get off the elevator you decide to throw the ball straight up in the air. This is real air with air friction (note that air friction introduces an additional acceleration that is proportional to the velocity by opposite in direction). Does it take longer for the ball to go up or come down, or are they equal? Explain your reasoning. Hint: you might want to consider the total acceleration on the ball.

QP2



As shown in the sketch above, a spaceship on a distant planet has landed on a ledge above a vast flat horizontal plain, a distance $L = 100$ km from a volcano. Just as the astronaut team emerges to survey the area, a boulder is ejected at $t = 0$ from the cone of the volcano, at the same height as the ledge, with an initial speed $v_0 = 1$ km/sec and an inital angle θ_0 with the horizontal. Measurements by the team show that the boulder is following a parabolic trajectory, that it reaches its maximum height at $t = 200$ sec, and that it is headed directly for the base of the ship! The astronauts clamber back into the ship and fire their boosters at time t_0 when the boulder is falling at a vertical height of 10 km above the ledge.
Note that numerical answers are required for parts a, b, c and e below.

a) (2 points) Find the angle θ_0 and the acceleration of gravity g_0 on the planet.

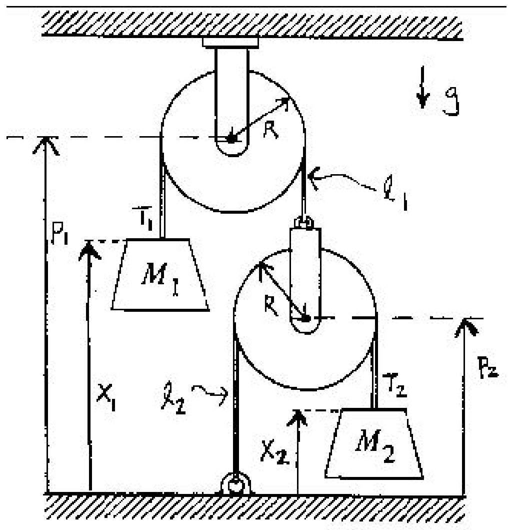
b) (2 points) Find the maximum height above the ledge reached by the boulder.

c) (2 points) How much time t_2 does the team have left to escape when they fire their boosters at time t_0?
The astronauts use their main and emergency boosters, which together (for times $t > t_0$) give the ship a total vertical acceleration (in m/sec^2) of $a(t') = A(1 = e^{(t'/\tau)})$, where $\tau = 5$ sec and $t' = t - t_0$. Note that A already includes the planet's gravitational acceleration g_0.

d) (2 points) Find an expression for the speed of the ship $U(t')$ for $t_2 > t' > 0$. You may leave your answer in terms of A, τ and t'.

e) If the base of the ship must reach a height of 1 km above the point where the boulder strikes the ledge to avoid being destroyed by flying debris, find the minimum value of A that will allow the team to escape.

QP3



Consider the system of masses, strings and pulleys connected as shown.The strings are massless and inextensible, and the pulleys are massless and frictionless. There is a uniform gravitational field g. Note that only the lower pulley is free to move up and down. Treat the first string of length l_1 as beginning at the upper edge of m_1, going around the upper pulley, and ending at the center of the lower pulley. The second string of length l_2 is attached to the floor, goes around the lower pulley and ends at the upper edge of m_2.

a) (1 point) Write down expressions for the lengths l_1 and l_2 of the strings in terms of x_l, x_2, p_1, p_2 and the pulley radius R (consider the hook on the floor to be part of string l_2).

b) (1 point) Sketch the free-body diagram for each mass, and write down the force equations from Newton's second law. Use a_1 and a_2 for accelerations of masses m_1 and m_2, respectively.

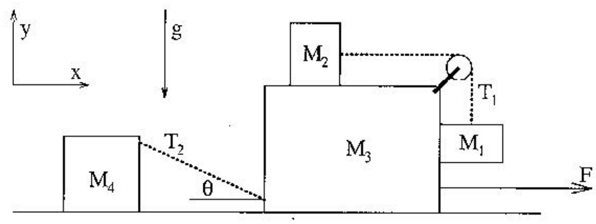
c) (1 point) Using the results from part (a), find the relationship between a_1 and a_2. In other words, $a_1 = ka_2$, where k is a numerical constant.

d) (1 point) Write down the relationship between the tensions in the strings T_1 and T_2. In other words, $T_1 = k_2T_2$, where k_2 is a numerical constant.

e) (1 point) Solve for a_1 and T_2 in terms for m_1, m_2 and g. If you are unable to solve parts (c) and (d), use $a_1 = -a_2$ and $T_1 = 1/3\,T_2$. (Note: there are not the correct relations).

f) (1 point) What is a_2 in the case of $m_1 = m_2$? What is the condition for zero acceleration of the masses?

QP4



The system of blocks and pulleys shown above is being pulled to the right with a force F. This force is adjusted so that block M_1 neither falls nor rises, i.e. the y component of its acceleration vanishes. All surfaces are frictionless, the pulley is massless and frictionless, and the strings (dotted lines) are massless and inextensible. Strings are marked with their tensions and blocks with their masses. All surfaces shown to be in contact remain in contact at all times; in particular block M_4 does not "tip over". The string connecting blocks M_3 and M_4 makes an angle θ with the horizontal.

a) (3 points) Draw free body diagrams for all four masses, with all forces clearly shown and labeled.

b) (2 points) Write Newton's second law for blocks M_1 and M_2 in the x and y directions (i.e. 4 equations).

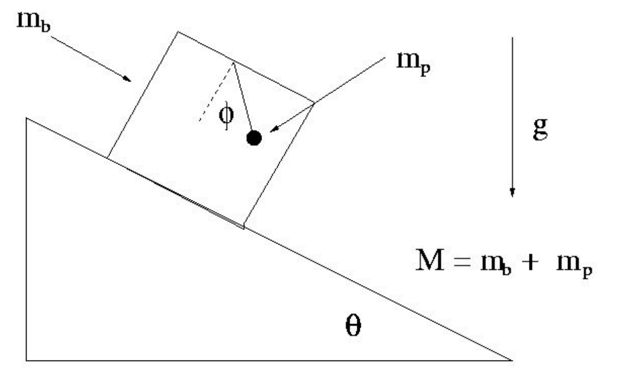
c) (2 points) What acceleration a_3 must block M_3 have? (Recall that block M_1 neither falls nor rises). Give your answer in terms of the masses M_1, M_2, M_3, M_4 and gravitational acceleration g.

d) (1 point) What is the magnitude of the force F?

e) (1 point) What is the normal force N_4 between the floor and block M_4? You may leave your answer in terms of the masses and a_3.

f) (1 point) What is the force F_p (magnitude and direction) of the pulley on the string? You may leave your answer in terms of masses and a_3.

QP5



A hollow box of mass m_b sliding down an inclined plane has a pendulum of mass m_p attached by a massless, inextensible string as shown. Let μ be the coefficient of kinetic friction between the inclined plane and the box.

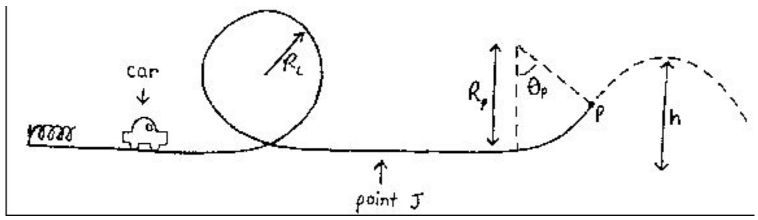
a) (1 point) Sketch the free body diagram and write down the equations from Newton's second law for the box and pendulum as a single unit, where the total mass is $M = m_b + m_p$.

b) (1 point) Solve for the acceleration a down the plane in terms of g, θ and μ.

c) (1 point) Now move into the accelerating frame of the box. Sketch the free body diagram for the pendulum including any "fictious" forces.

d) (1 point) Find the equilibrium angle ϕ the pendulum makes with the line perpendicular to the top of the box for both the case of no friction ($\mu = 0$) and for the case of friction ($\mu \neq 0$) between the box and the inclined plane.

QP6



A child's toy consists of a car on a frictionless track with a circular loop of radius R_L, as shown in the figure. The car is being pushed against the spring (so that the spring is compressed by distance x) and then released. The car is linked to the track so that it slides along the track but cannot fall off. The track ends at point P, and the section of track just before this point is curved with radius R_P as shown. The car is free to fly through the air after leaving P. The car has mass M and the spring has spring constant k.

For parts a - d, assume that the car starts at rest against the spring with the spring compressed by distance x, where x is large enough so that the car has more than enough velocity to go around the loop. Your answers should be in terms of g, k, M, R_L, R_P, x, and θ_P.

a) (2 points) What is the normal force of the track on the car at the top of the loop?

b) (2 points) After the car has let the track at point P, what is the maximum height h reached by the flying car?

Suppose now the child spills jam on the track just after the loop, at point J as shown in the diagram. The jam has coefficient of friction μ and the spill has length d. Your answers should be in terms of d, g, k, M, x and μ.

c) (1 point) What is the velocity of the car after it leaves the spill?

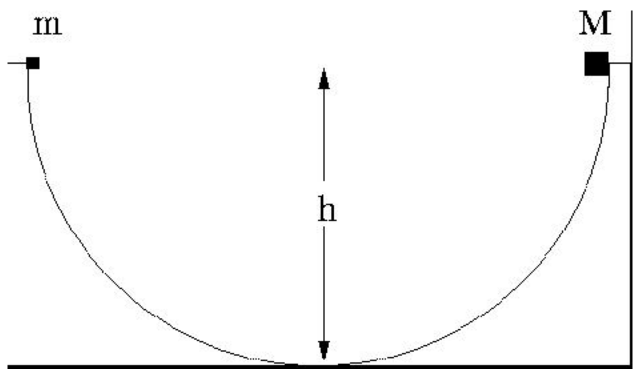
d) (1 point) For what minimum value of d will the car get stuck?

For parts e and f, you are to determine under what conditions the car can make it around the loop. Your answers should be in terms of g, k, M and R_L.

e) (2 points) What is the minimum compression of the spring, x_{min} for the car to just barely reach the top of the loop?

f) (2 point) If the spring were given this minimum compression and the car wasn't linked to the track, at what height would the car fall off?

QP7



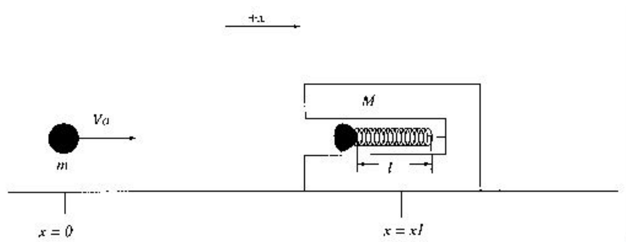
Two masses (M, m) slide without friction (under the influence of a uniform gravitational field g) down the sides of a hemispherical bowl. They each start with zero velocity at the lip of the bowl, which is a height h above the bottom.

a) (1 point) What is the total kinetic energy when the masses first touch (assume the size of the masses is much less than h).

b) (2 points) How high will the masses move if the collision is completely inelastic (i.e. they stick together)?

c) (2 points) Assuming that the collision between the masses is elastic, what is the maximum height that mass m will achieve?

QP8



A spring gun of mass M sits at rest on a frictionless surface. A suction cup is attached to the end of the spring, and a rubber ball of mass m is incident on the spring with velocity V_0. At time $t = 0$, the gun is at $x = xl$ and the ball is at $x = 0$ as shown above. The spring and suction cup have negligible mass, the initial length of the spring is l, and the spring constant is k. When the ball hits the suction cup it sticks and remains attached to the end of the spring. Assume that the spring does not sag (i.e. ignore the effects of gravity). Express your answer in terms of m, M, V_0, xl, l and k.

a) (1 point) For times before the ball hits the suction cup, when the gun sits at $x = xl$, write the location of the center of mass of the gun/ball system as a function of time.

b) (1 point) Suppose a latch is located at the point of maximum compression of the spring, so that the latch stops the ball at this point. What is the final velocity of the ball/gun system after the ball hits the cup and the latch is tripped?

c) (2 points) What fraction of the inital kinetic energy of the ball is stored in the spring?

d) (1 point) When the spring has its maximum compression, by what amount Δl, is the length of the spring decreased?

e) (2 points) At time $t = t2$, when the center of the mass of the gun/ball system is at position $x = x2$, the latch pops loose (but the ball remains attached to the end of the spring). On a graph of x versus time, plot the position of the center of the mass of the system (you may make the origin of your plot ($t2$, $x2$)). Also, on a similar plot, sketch qualitatively the position of the gun (mass M) relative to the center of mass, assuming that $M > m$.

f) **Extra Credit Point** After the latch pops loose, what is the frequency of the osciallation for the system?

QP9

A tetrahedron has its vertices located at four points A, B, C and D where the coordinates of these points are given by:

$$\begin{aligned} A &= (0, 0, 0) \\ B &= (1\,cm, 0, 0) \\ C &= (0, 2\,cm, 0) \qquad (1) \\ D &= (0, 0, 3\,cm) \qquad (2) \\ & \qquad (3) \end{aligned}$$

a) (1 point) It takes a fly 5 seconds to walk with constant speed along the edge from B to C. What are the velocity and speed of the fly?

b) (1 point) Find the area of the side with vertices B, C and D.

c) (2 points) If the bug then flies from point C with a constant speed of 3 cm/sec for 7 seconds along the direction that is perpendicular to face BCD, what is its final position?

QP10

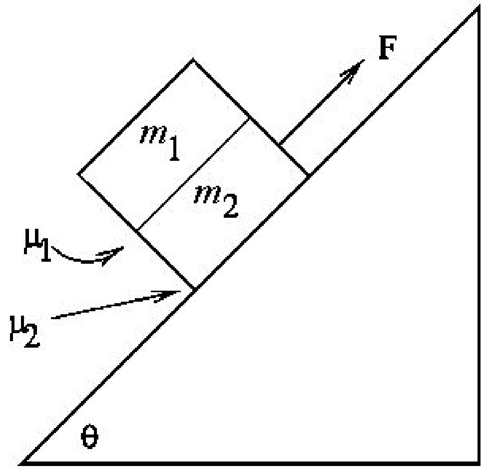
Two long horizontal test tracks at Edwards Air Force base, running parallel and next to each other, were used to compare the performance of a rocket motor and a jet motor. The rocket motor started from rest and accelerated constantly along the first track until it reached exactly half the measured test distance $L/2$ at t_1. At this point the rocket ran out of fuel and then continued at constant speed to the end of the track, over a further distance of $L/2$. A jet motor was started at the same instant as the rocket, at the same starting coordinates $s = 0$ along the second track and ran along the track with constant acceleration for the whole length L. It was observed that both the rocket and the jet motors covered the test distance in exactly the same time T.

a) (2 points) Find t_1/T for the rocket.

b) (2 points) Find the ratio of the acceleration of the jet motor a_2 to the rocket motor a_1.

c) (2 points) Make a plot of the rocket's position as a function of time $s_1(t)$ and the jet's position $s_2(t)$, showing all main features of the motion.

QP11

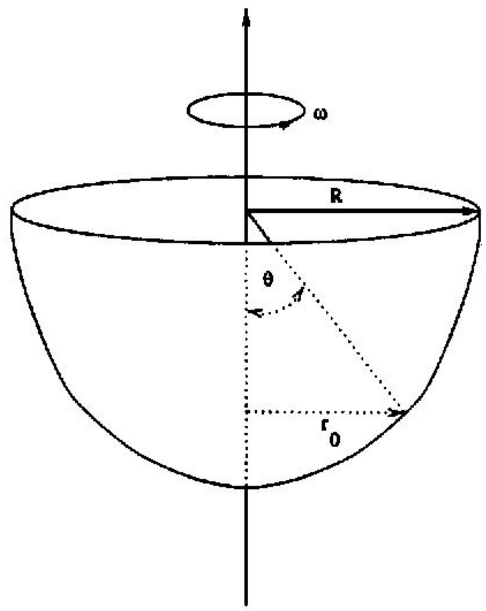


Upon an inclined plane of angle θ is placed a block of mass m_2. Upon m_2 is placed another block of mass m_1. The coefficient of static friction between m_2 and the inclined plane is μ_{2s} and the coefficient of sliding friction is μ_{2k}. Likewise, the coefficient of static friction between m_1 and m_2 is μ_{1s} and the coefficient of sliding friction is μ_{1k}. A force F upward and parallel to the plane is applied to m_2.

a) (2 points) What is the acceleration of m_2 when m_1 just starts to slip on it?

b) (2 points) What is the maximum value of F before this slipping takes place?

QP12



A small marble of mass M is placed on a hemispherical bowl of radius R as shown in the figure. If the bowl is spun (around its vertical axis) with the constant speed ω, the marble eventually settles at a distance r_0 from the axis.

a) (3 points) Find the magnitude and the angle with respect to the vertical axis of the force exerted by the bowl on the marble.

b) (2 points) Find r_0 as a function of ω.

c) (2 points) Note that if ω is too small, the answer to part (b) does not make sense. What happens of the angular velocity ω is too small?

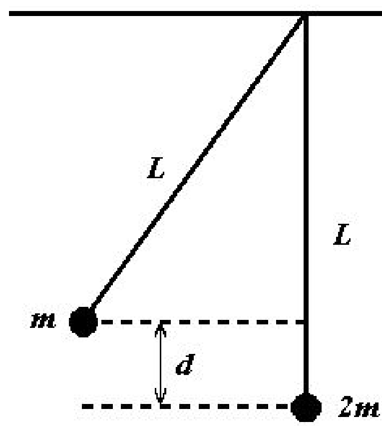
QP13

A spigot pours rice onto a large platform, which is the platform of a scale. At a time $t = 0.0$ sec, the spigot is opened and rice begins to pour out (with initial velocity 0) at a rate of 1.00 kg/sec onto the platform from a height of 10.0 m above.

a) (1 point) At $t = 10$ sec what is the weight of the rice on the platform?

b) (2 points) What does the scale read at this time? Hint: consider the change in vertical momentum per unit time.

QP14



A pendulum of mass $m_1 = m$ is raised a distance d and dropped so that it collides elastically with a second pendulum of mass $m_2 = 2m$.

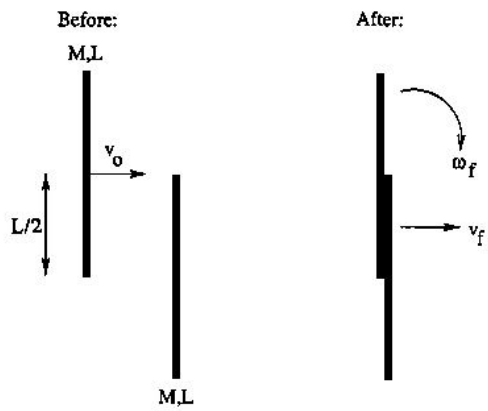
a) (1 point) Find the initial height of the center of mass relative to the lower mass.

b) (1 point) Find the velocities before (v_{1i}, v_{2i}) and after (v_{1f}, v_{2f}) the collision. Express your answers in terms of $v_0 = \sqrt{2gd}$.

c) (3 points) How high does each mass rise? Assuming that the masses reach their maximum heights at the same time, how high does the center of mass rise? Express your answer in terms of d. Use $v_{1f} = -v_0/4$ and $v_{2f} = 3v_0/4$ if you were unable to solve the previous part.

d) (2 points) How high does the system rise if the collision is completely inelastic (i.e. if the balls stick together)? What fraction of the initial energy is lost in this case?

QP15



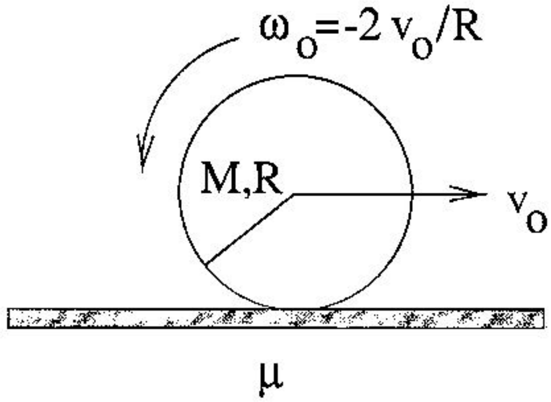
Two equal, uniform, thin rigid rods of length L amd mass M collide and stick together, making a composite rod of length $3L/2$. Initially, one rod is at rest while the other approaches with speed v_0 perpendicular to both rods.

a) (1 point) What is the velocity of the center of mass after the collision?

b) (2 points) Find the moment of interia I_c for the composite rod about its center of mass.

c) (2 points) After the collision, what is the angular velocity ω_f of the composite rod about its center of mass?

QP16



At $t = 0$, a solid sphere of mass M and radius R is spinning backwards while moving forwards on a table top of frictional coefficient μ. The initial velocity of the sphere is v_0 and the initial angular velocity is $\omega_0 = -2v_0/R$ as shown below.

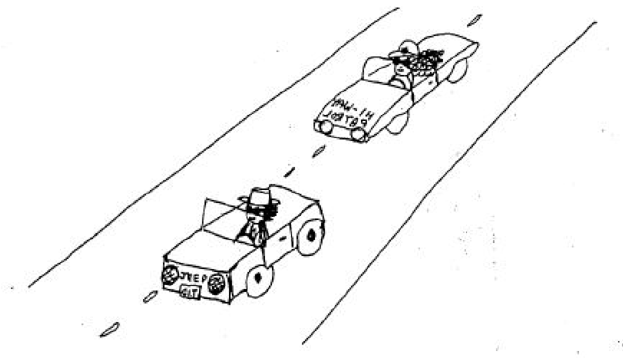
a) (2 points) At what time t_0 does the sphere first roll without slipping?

b) (2 points) For times $t >> t_0$ draw and label a similar picture of the sphere, as above. Indicate the magnitude and direction of both the velocity and angular velocity.

c) (1 point) What is the magnitude and direction of the frictional torque τ_f at $t = 0$? What is it for $t >> t_0$?

QP17

Indiana Jones, on his way home to Caltech after an adventure in the Mojave desert, is doing 80 mph on the 210 Freeway. A California Highway Patrol Officer is parked in her Porsche at a turnout. She starts accelerating at 8 mph/sec as Indiana passes her (1 mile = 1760 yards).



a) (2 points) When will the officer catch up to Indiana?

b) (1 point) How fast would the officer be going when she catches up to Indiana?

c) (1 point) Sketch graphs of $(x)t$ and $v(t)$, labelling both Indiana and the officer.

Actually, when the officer is 100 yards behind Indiana, Indiana slams on his brakes. Assume at this position, the officer's speed is that obtained in part (b). The officer brakes immediately as soon as she sees Indiana's brake lights (zero reaction time). Both cars brake with the same deceleration, d.

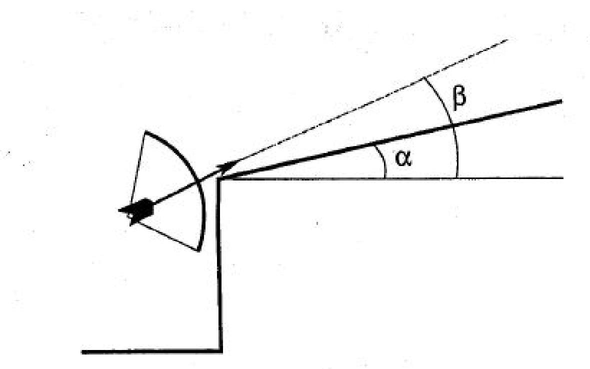
d) (2 points) What time after the brakes are applied do they collide?

e) (1 point) What is their relative speed at the time of the collision?

QP18

(3 points) What is the angle between two intersecting body diagonals of a cube? Give a numerical answer in degrees. A body diagonal connects two corners and passes through the center of the cube.

QP19



Robin Hood is standing at the foot of a hill which makes an angle α with the horizontal. For practicing his recently learnt Phys 1a formulae, he shoots an arrow from a point on the hill, with initial velocity v_0 and under an angle $\beta > \alpha$ with the horizontal. Neglect both the size of the arrow and air friction.

a) (2 points) Express the time needed for the arrow to land in terms of α, β, v_0 and the gravitational acceleration g.

b) (1 point) Show that the distance between the origin and the place of landing is given by

$$l = \frac{2v_0^2}{g cos^2(\alpha)} sin(\beta - \alpha)\, cos(\beta)$$

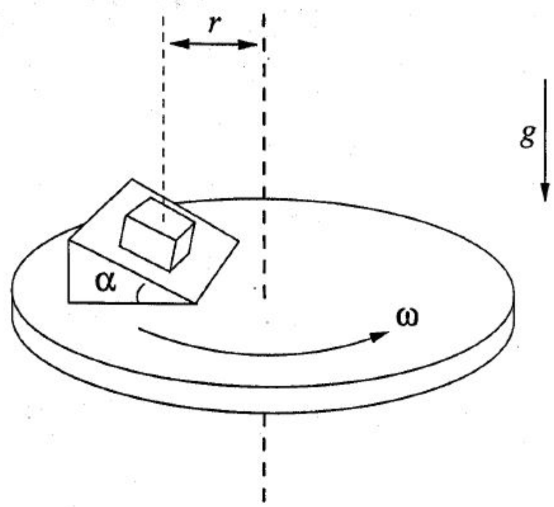
QP20

A rope of length $2l$ and uniform density (here: mass per unit length) ρ is hanging over a nail in a wall with a piece of length l on both sides. Friction, the thickness of the nail, and the thickness of the rope are negligible. When one creates a slight length difference between the two sides, a net force will start to act on the rope, and it will slide off the nail, faster and faster.

a) (1 point) When the length on one side is $l + x (0 < x < l)$, what is the net force on the rope?

b) (3 points) Find the velocity as a function of x. Hint: make use of $\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt}$. What is the velocity when the rope completely comes off the nail?

QP21



An inclined plane of angle α is glued onto a horizontal turntable, as shown in the figure. A block is placed on the included plane a distance r from the axis of rotation of the turntable, and the coefficient of friction between the block and the plane is μ. The turntable spins about its axis with constant angular frequency ω.

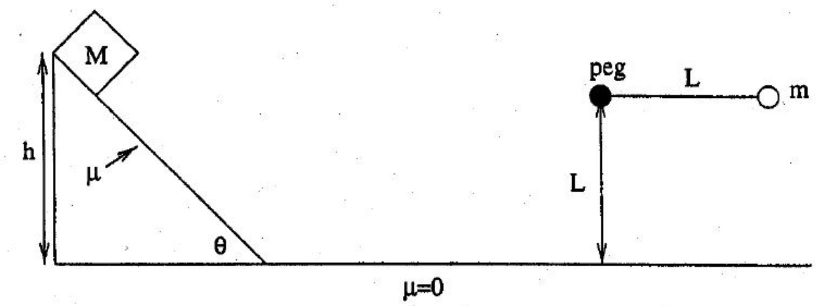
a) (1 point) Draw a free body diagram for the block, showing the forces that act on it.

b) (2 points) Find an expression for the minimum angular velocity, ω_c, to keep the block from sliding down the plane, in terms of g, r, μ and the angle of the plane α.

QP22

A large mass M is released from rest at the top of an inclined plane of angle θ and frictionless coefficient μ. At the same time, a pendulum consisting of a small mass m and massless string of length L is released at height L (horizontally).

The large mass slides down the incline and smoothly onto the table, continues across the frictionless table, and collides with the pendulum's mass just as the pendulum has reached the bottom of its swing and is approaching the large mass. The two masses stick together after the collision.



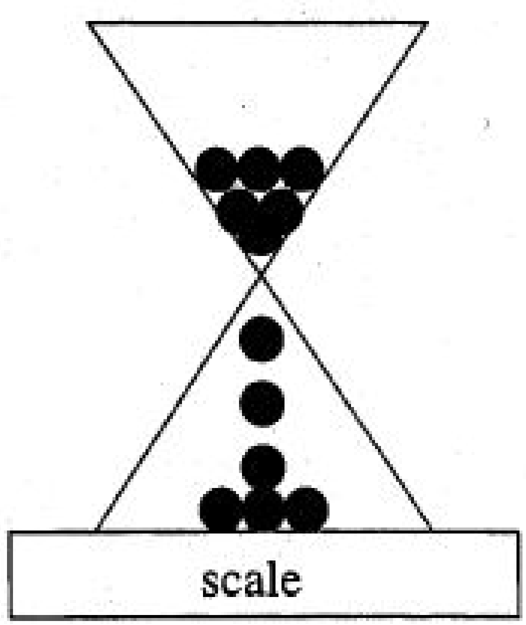
a) (1 point) Assuming the mass M is released at a height h, what is its velocity v_M when it reaches the flat part of the table?

Assume for the rest of the problem that $v_M = \sqrt{gh}$.

b) (2 points) What is the velocity v_C of the combined object immediately after the collision? For what initial height h of the mass M will the combined mass come to rest immediately after the collision?

c) (2 points) Consider the case $M = 2m$. What must the initial height h have been in order for the pendulum of the combined mass to go "over the top" in the counterclockwise direction?

QP23



An hour glass of sand sits on a scale. Initially all the sand (of total mass m) in the glass (of mass M) is held in the upper reservoir. At $t = 0$, the sand is released and it falls at a constant rate $dm/dt = \lambda$ to the bottom of the lower reservoir, as shown. Find the reading of the scale as a function of time.

a) (1 point) From the time $t = 0$ at which the sand is released, until the time $t = t_1$ at which it starts to arrive at the bottom of the reservoir.

b) (1 point) From $t = t_1$ until the time $t = t_2$ at which all of the sand has left the upper reservoir.

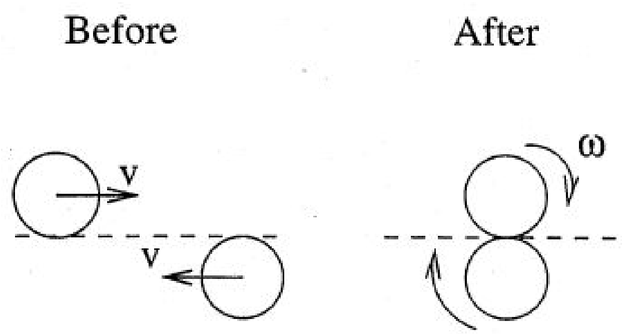
c) (1 point) From $t = t_2$ until the time $t = t_3$ at which all the sand has reached the bottom.

d) (1 point) For all times $t > t_3$.

e) (1 point) Sketch the reading of the scale as a function of time, assuming that $m < M$.

QP24

Two cylindrical pucks, each of mass M and radius R slide towards each other on a smooth frictionless surface. Initially, each has speed v. They undergo a grazing collision and stick together at their edge.



a) (1 point) What is the combined angular momentum of the two pucks about their mutual center of mass before the collision?

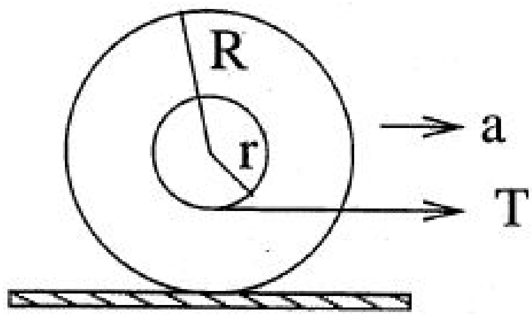
b) (1 point) What is the combined moment of interia of the two pucks about their mutual center of mass after the collision?

c) (2 points) What is ω, the angular speed of the two pucks about their mutual center of mass after the collision?

d) (1 point) What fraction of the original energy is lost to heat duing the collision?

QP25

A yo-yo has mass m, inner radius r, and outer radius R. Its moment of interia is I about its center. The yo-yo rolls without slipping on a horizontal table and is pulled along by a horizontal string wound around its inner radius. The pulling by the string gives rise to an acceleration a.



a) (3 points) Find the tension T in the string, and the force of friction, f.

b) (1 point) Find the minimum coefficient of friction μ_{min} so that the yo-yo rolls without slipping. If $I = kmR^2$, find μ_{min} in terms of k, a and g.

c) (1 point) In the pictures, in which direction does the yo-yo roll? Explain.

QP26

A football quarterback throws a ball directly at a receiver who is at rest 25 meters downfield. The ball is thrown at an angle of 30 degrees to the horizontal and with a speed of 20 meters/sec. Immediately sensing that the ball is going to over-shoot him, the receiver begins to run, with constant acceleration, downfield in the same direction the ball is travelling. Amazingly, he catches the ball. You may assume that the ball is thrown from and caught at the same height.

a) (1 point) How far did the receiver have to run?

b) (1 point) How long after the throw is the catch made?

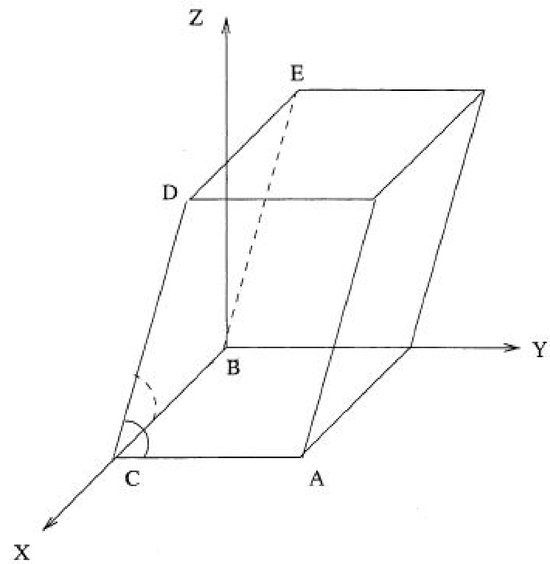
c) (2 points) What was the acceleration of the receiver?

d) (1 point) At the time of the catch, which downfield velocity is larger, the ball's or the receiver's? By how much?

e) (2 points) Next season the receiver decides to quit football and take up the 100 meter dash track event instead. In his first attempt, he manages a time of 10.0 seconds. Assuming that he was able to accelerate at the same rate as found above for a portion of the 100 meter race and then continue at constant speed for the remainder, determine how long the acceleration phase of his race was.

QP27

A prism has its square base of side 4 cm in the XY plane. Its top is 10 cm higher than the base, but offset 2 cm in the $\hat{y}$ direction.

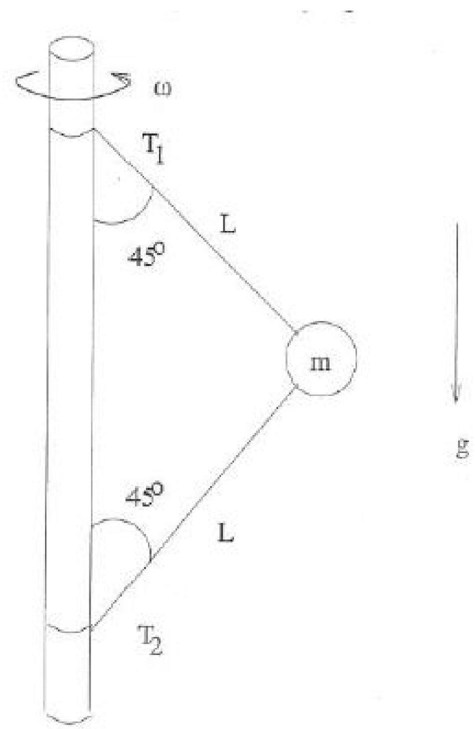


a) (2 points) What are the two angles ACD and BCD between the edges of the prism?

b) (1 point) Find the outward unit normal vector to the side $BCDE$.

QP28

A mass m is connected to a vertical revolving axle by two massless strings of length l, each making an angle of 45° with the axle as shown. Both the axle and mass are revolving with large angular velocity ω. You may neglect the radius of the axle.



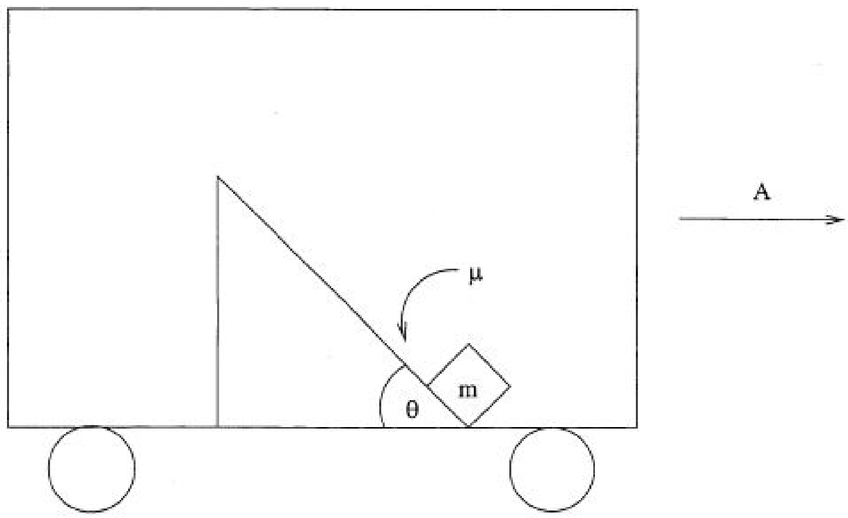
a) (1 point) Draw a free body diagram for the forces acting on the mass m.

b) (2 points) Find the tensions T_1 and T_2 in the two strings. Give your answer in terms of ω, m, L, and g.

c) (1 point) What is the minimum angular velocity ω_{min} such that both strings remain taut? Give your answer in terms of m, L, and g.

QP29

A block of mass m sits on the bottom of an inclined plane of angle θ and friction μ. The whole assembly is inside a car, and the incline is fixed to the car's floor. Throughout the problem, assume the block remains in contact with the incline.



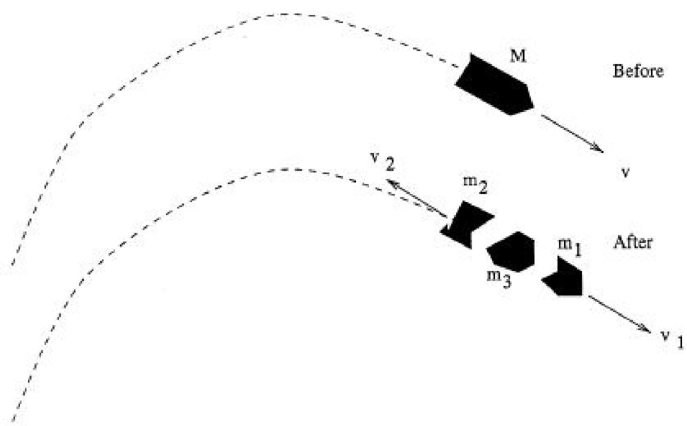
a) (1 point) The car takes off with acceleration A to the right. Describe the ficticious force needed in the rest frame of the car. What is its magnitude and direction on the block?

b) (2 points) Assuming the block begins to move up the incline, draw a force diagram and write down Newton's laws.

c) (3 points) Solve for the acceleration of the block up the incline, in terms of A, θ, μ and g. If $A = 2g$ and the coefficient of friction $\mu = 0.5$, what is the condition on θ so that the block will indeed move up the incline?

QP30

A projectile of mass M initially travelling with speed v explodes in flight into three fragments (see below). An energy E equal to 5 times the initial kinetic energy of the projectile is released in the explosion, and is transformed into additional kinetic energy of two of the projectiles. One fragment of mass $m_1 = M/2$ travels with speed $v_1 = k_1 v$ in the original direction of the projectile, while the second fragment of mass $m_2 = M/6$ travels in the opposite direction with speed $v_2 = -k_2 v$ and the third fragment of mass $m_3 = M/3$ is at rest the instant after the explosion.

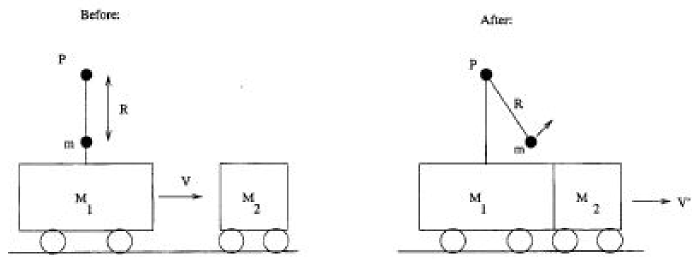


a) (2 points) Write down equations expressing the conservation of momentum and energy in terms of M, k_1, k_2, v and E immediately after the explosion.

b) Find the values of k_1 and k_2.

QP31

A cart of mass M_1 has a pole on it from which a ball of mass $m << M_1, M_2$ hangs from a thin string of negligible mass and length R attached at a point P, as shown in the figure. The cart and ball have initial velocity V (the ball is initially at rest with respect to the cart). The cart crashes into another cart of mass M_2 and sticks to it. All surfaces are frictionless, and you may ignore the mass of the wheels.



a) (2 points) Find the velocity of the two carts V' after the collision.

b) (2 points) Find the smallest initial velocity V so that the ball will complete a circle around the point P after the collision.

c) (2 points) Now, instead of the two carts sticking together, assume an elastic collision and find the smallest initial velocity V so that the ball will complete a circle around the point P after the collision.

QP32

A uniform disk of mass m and radius r begins to slide down an inclined plane with an initial velocity v_0 at its center of mass at time $t = 0$. The inclined plane has a surface frictional coefficient μ and forms an angle θ relative to the ground, as shown below. At time $t = t_1$, the disk begins to roll down the plane without slipping. The local gravitational acceleration is g, pointing vertically down.

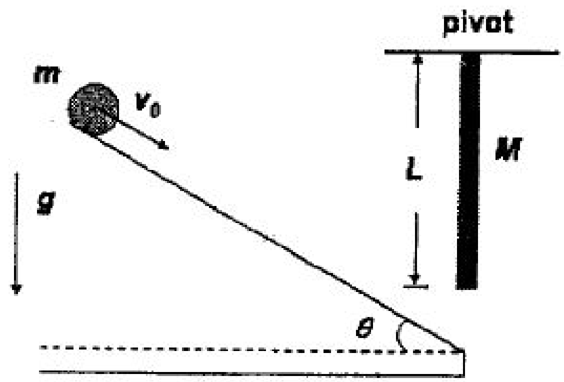


Figure 1A

a) (2 points) Express t_1 in terms of v_0, g, μ and θ.

b) (1 point) Find the minimal frictional coefficient μ (in terms of g and θ) required for the disk to achieve pure rolling motion?

At $t > t_1$ the disk reaches the end of the inclined plane with a final speed v_f at its center of mass, and it becomes stuck instataneously upon impact to the end of a uniform thin rod of length L and mass M hanging vertically from the ceiling. The rod-disk assembly swings to the right, as shown below.

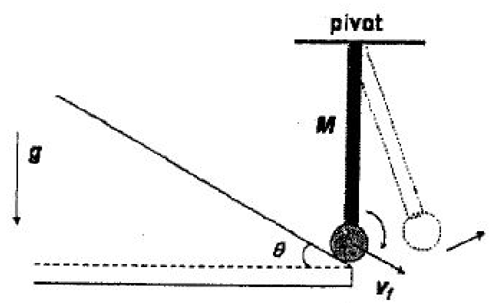


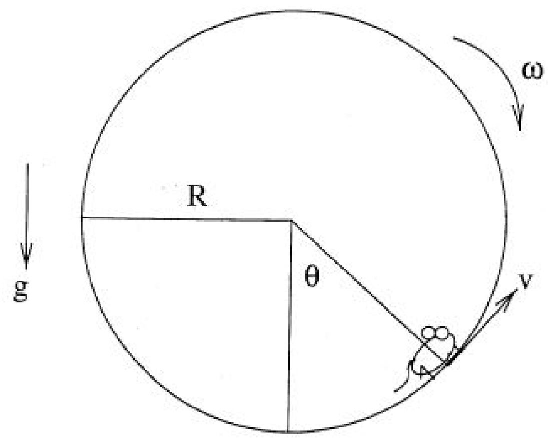
Figure 1B

c) (1 point) Find the moment of interia I of the rod-disk assembly about the axis through the pivot.

d) (2 points) Find the angular momentum (both the magnitude and direction) of the rod-disk assembly about the axis through the pivot after the impact. Express your answer in terms of v_f, m, M, r and L. Discuss the condition required for the rod-disk assembly to swing to the right.

QP33

A mouse of mass m runs in an exercise wheel of mass M and radius R. The mouse runs at constant speed v relative to the wheel. The wheel has a damping torque proportional to its angular velocity given by: $|\tau_{damp}| = k|\omega|$



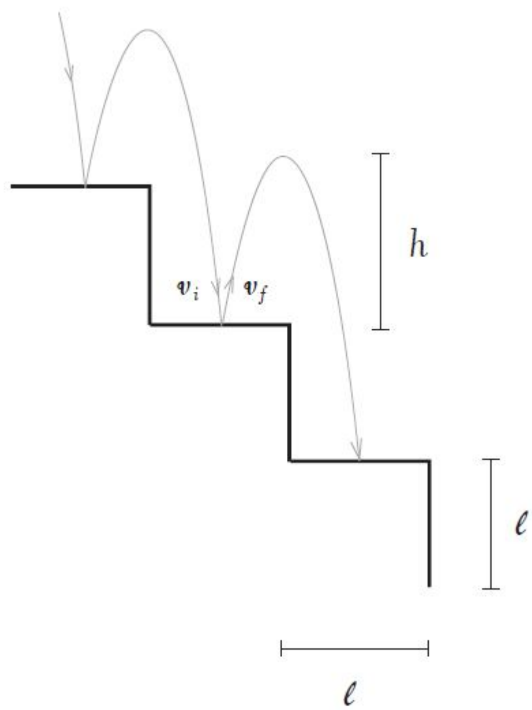
a) (1 point) If the mouse is not moving relative to the lab, what is the angular velocity ω of the wheel in terms of m, M, R, v and k?

b) (2 points) What is the angle of the mouse, θ?

c) (1 point) How much power is the mouse delivering?

QP34

A marble bounces regularly down a long flight of stairs, hitting each step with the same speed and at the same distance from the edge, and then bouncing up to the same height h above the step, as shown in the figure.



Each stair has the same height and depth ℓ. The horizontal compenent of the marble's velocity is constant throughout, but the stairs have the property that $-v_f/v_i = e$, where v_i and v_f are the vertical velocity components just before and just after the bounce respectively, and e is a constant $(0 < e < 1)$.

a) (2 points) Find an expression for v_i in terms of e, ℓ and the gravitational acceleration g.

b) (2 points) Find an expression for the time t between successive bounces, in term of e, ℓ, and g.

c) (2 points) Find an expression for the bounce height h in terms of e and ℓ.

QP35

The time derivative of the acceleration is called “jerk”, i.e. $j(t) = \frac{da(t)}{dt}$.

a) (1 point) For motion under constant jerk, j, derive equations for the acceleration $a(t)$, the velocity $v(t)$, and the position $x(t)$. Use x_0 for the initial position, v_0 for the initial velocity, and a_0 for the initial acceleration.

Two cars start a race at rest. Car A accelerates at constant rate a, while Car J moves with constant jerk j and zero initial acceleration. Part way through the race, at $t = 1\,s$, the cars are tied.

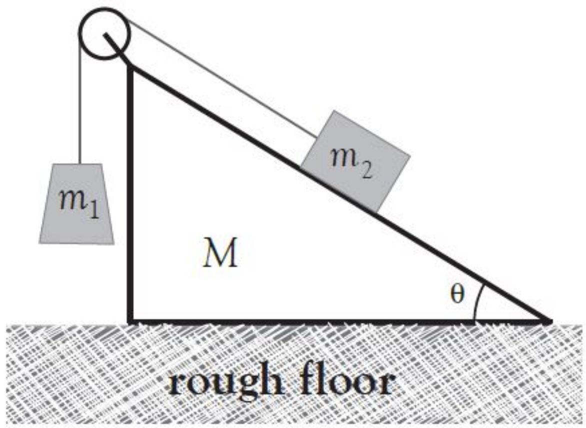
b) (1 point) In a single graph, sketch $x(t)$ for both Car A and Car J, and label the curves accordingly.

c) (1 point) Who was ahead at $t = 0.5\,s$?

d) (1 point) Which car will win the race?

QP36

An incline of mass M rests on a floor with a coefficient of static friction μ. A mass m_1 is suspended by a massless string that passes over a frictionless pulley of negligible mass, attached to the upper corner of the incline. The other end of the string is attached to a mass m_2 that slides without friction on the surface of the incline. The plane of the incline meets the horizontal at an angle θ, as shown in the figure.



Assume that the mass m_1 will move downward with respect to the pulley, and that the string can neither stretch nor break.

a) (2 points) For the case of very large μ use Newton's laws to determine the tension T of the string and the acceleration a of the masses. Express your answer in terms of M, m_1, m_2, θ and the gravitational acceleration g.

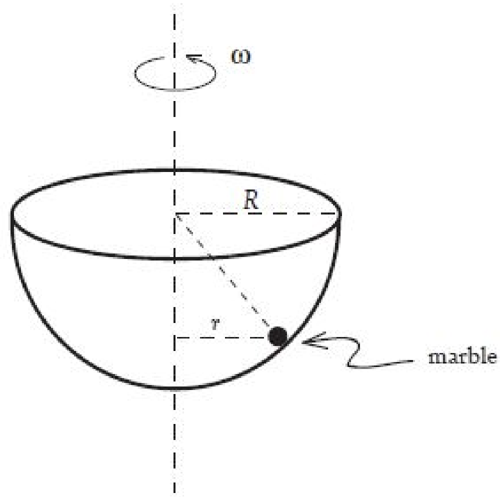
For the rest of this problem, you may express your answer in terms of the string tension T.

b) (1 point) Draw a free-body diagram for the incline. Hint: In order to determine the direction of the frictional force, think first of which way the incline woul move in the frictionless case.

c) (1 point) Find the smallest μ for which the incline will remain at rest. You don't need to simplify your answer.

QP37

A marble of mass m is deposited inside a hemispherical bowl of radius R, as shown in the figure. The bowl is then spun around its vertical axis with a constant angular velocity ω. The marble eventually settles at a distance r from the bowl's vertical axis while rotating around that axis with the same angular velocity ω as the bowl itself.



a) (3 points) Find the force that the bowl exerts on the marble. Give the total magnitude of the force as well as its angle with respect to the vertical axis of the bowl.

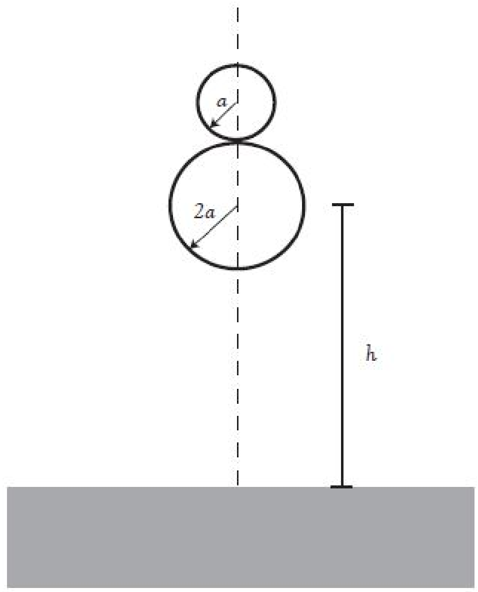
b) (2 points) Derive an expression for r in terms of R, ω and the gravitational acceleration g.

Note that, for small enough ω, the answer to part (b) does not make sense.

c) (1 point) Explain what happens physically when the angular velocity ω is too small.

QP38

Two balls, the lower one of radius $2a$ and the upper one of radius a, are dropped from a height h (measured from the center of the lower ball to the floor), as shown in the figure. The mass of the upper ball is m and the mass of the lower ball is $M = 3m$. Assume that the centers of the spheres always lie along the vertical line and that all collisions are perfectly elastic. You may neglect air resistance.



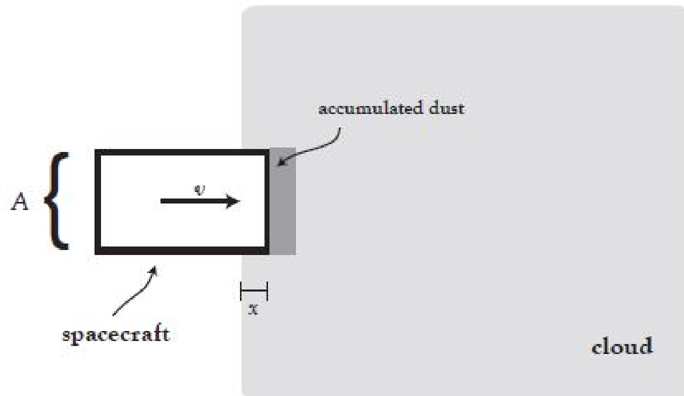
a) (1 point) Calculate the velocity v_0 of the balls immediately before they hit the floor. Assume there is a short interval between the lower ball bouncing on the floor and it hitting the upper ball. What is the velocity of the lower ball immediately after hitting the floor but before hitting the upper ball?

b) (3 points) Immediately after the lower ball hits the upper ball, what will the velocity v_1 be for the upper ball? Hint: It might be less cumbersome to compute this in terms of v_0, substituting the answer to part (a) only at the very end.

c) (2 points) How high will be upper ball bounce? Express the answer H in terms of h and a. (Measure H from the floor level to the upper ball's center at its highest position.)

QP39

A spacecraft of mass m_0 and cross sectional area A is moving at a constant velocity v_0 when it encounters a stationary cloud of dust of density ρ. Assume the dust sticks to the surface of the spacecraft and that A is constant over time.



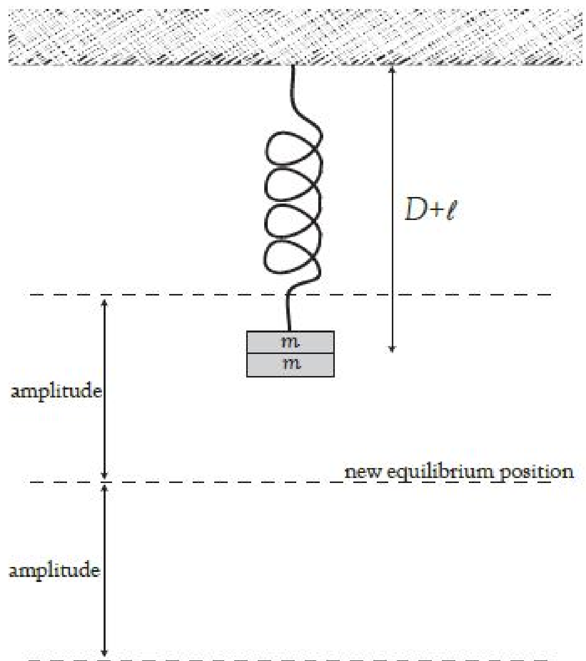
a) (1 point) Will the total mechanical energy be conserved as the spacecraft moves through the cloud? Will the momentum? Explain why.

b) (1 point) Find an expression for the time rate $\frac{dm}{dt}$ at which the duct-covered spacecraft gains mass. Express your answer in terms of A, ρ and the velocity v of the spacecraft.

c) (2 point) Write down a first-order differential equation for $v(x)$. Solve it by integration.

QP40

A linear spring has a free length D. When a mass m is hung on one end, the spring has an equilibrium length $D + \ell$. While it is hanging motionless with an attached mass m, a second mass m is dropped from a height ℓ onto the first one. The masses collide inelastically and stick together. The figure below shows the system at the time of the collision.



a) (1 point) What is the new equilibrium length of the spring?

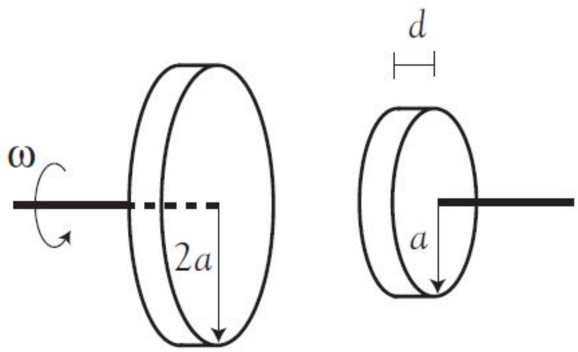
b) (1 point) What is the period of the resulting motion?

c) (2 points) Find the amplitude of the motion. Express your answer in terms of ℓ.

d) (2 points) How long after the collision do the joined masses reach the lowest point of their oscillation? Express your answer in terms of ℓ and g.

QP41

Two wheels are mounted on collinear frictionless shafts, initially without touching. The first wheel turns with angular velocity ω while the second wheel is stationary. Both wheels are uniform disks of thickness d and density ρ. The radii of the wheels are $2a$ and a respectively.



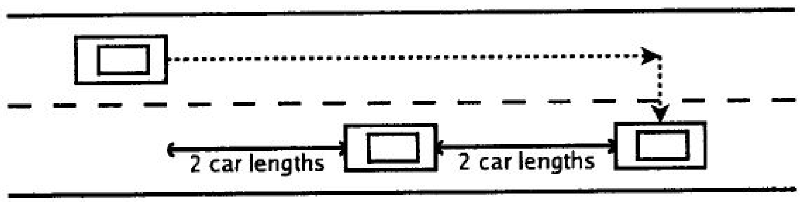
a) (1 point) Express the moment of interia of each wheel in terms of a, ρ and d. What is the ratio of the two moments of intertia?

Now imagine that the shafts are slowly moved until the two wheels come into contact. The axes of rotation remain collinear throughout. After a while, an equilibrium is achieved and the wheels turn without their surfaces slipping.

b) (2 points) Compute the final angular velocity of the second wheel in terms of ω.

c) (1 point) Is the kinetic energy of rotation conserved? Explain.

QP42



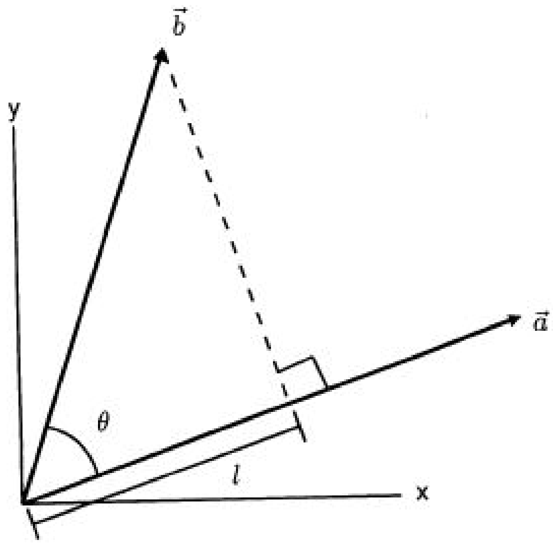
Traveling on a two-lane country road, you are behind a slowpoke going 40 miles per hour in a 45 mile-per-hour zone. The road straightens out and you decide to pass the car. You pull into the other lane exactly 2 car lengths behind the other car, traveling at the same speed. Pressing the gas pedal to the floor, you accelerate at 2 miles per hour per second. You ignore the speed limit and merge into your original lane exactly 2 car lengths ahead of the slowpoke without slowing down. Both of your cars are 15 feet long.

a) (2 points) How much distance have you traveled by the time you have passed the other car and returned to the original lane?

b) (2 points) How fast are you going at this point?

c) (2 points) What is the minimum distance a car travelling at the speed limit in the other direction should be from you when you begin passing in order to pass the slowpoke as described above and avoid a collision? Assume you can change lanes instantaneously.

QP43



Two vectors $\vec{a} = 3\hat{i} + \hat{j}$ and $\vec{b} = \hat{i} + 3\hat{j}$ lie in the $x - y$ plane as shown.

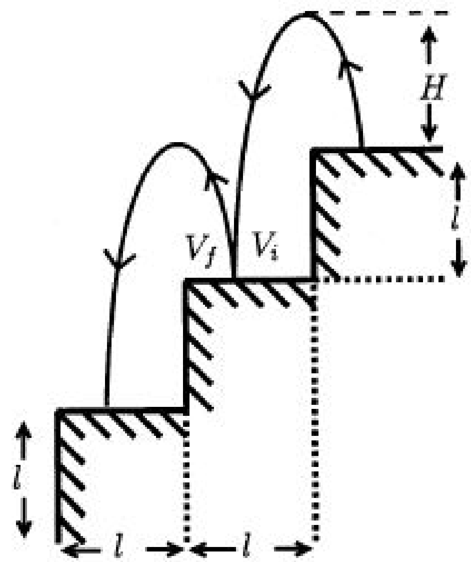
a) (1 point) What is the magnitude of the projection of $\vec{b}$ upon $\vec{a}$ indicated as l in the figure?

b) (1 point) What is the angle between $\vec{a}$ and $\vec{b}$ as indicated as θ in the figure?

c) (1 point) What is the magnitude and direction of the cross product $\vec{a} \times \vec{b}$ Hint: you might want to calculate the magnitude without calculating a determinant by using your answer to (b) and the fact that $sin^2\theta + cos^2\theta = 1$ for any angle θ.

d) (1 point) Draw a vector $\vec{c}$ connecting the endpoints of $\vec{a}$ and $\vec{b}$. What is the area of the triangle enclosed by the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$?

QP44



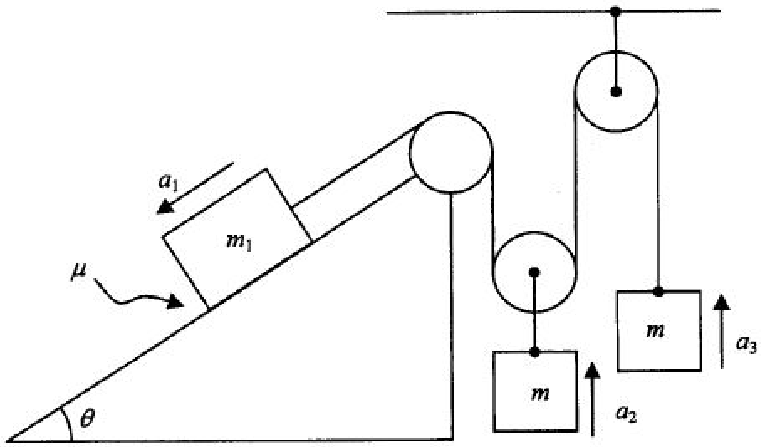
A marble bounces down a long flight of stairs in a regular manner, hitting each step vertically at the same speed and distance from the edge and bouncing up to the same height above each step, as shown in the figure below. Each stair has the same height and depth l, as shown. The hortizontal component of velocity V_h is unaffected, but the stairs have the property that $-V_f/V_i = e$, where V_i and V_f are the vertical velocity components just before and after the bounce respectively, and e is a constant ($0 < e < 1$).
Ignore the size of the marble and air resistance is answering the following questions. Assume the trajectory of the marble lies in the plane of the paper.

a) (2 points) Find an expression for V_i in terms of e, l and the acceleration of gravity g.

b) (2 points) Find the time between bounces in terms of e, l and g.

c) (1 point) Find an expression for the bouncing height, H, in terms of l and e.

QP45



The following questions refer to the diagram above. All pulleys are assumed to be massless and frictionless and the inclined plane is fixed in place. The static and kinetic coefficients of friction between m_1 and the plane are both μ.

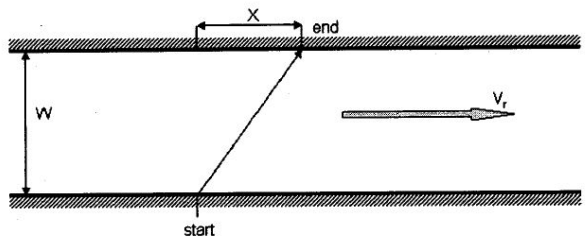
a) (1 point) Write the relationship between the accelerations a_1, a_2 and a_3 with the arrows above denoting the direction of positive acceleration.

b) (1 point) Sketch a free-body diagram for each of the three masses and write Newton's second law for each.

c) (2 points) Derive an expression for a_1 in terms of m_1, m, μ, θ, and g.

d) (1 point) In the case of $\mu < \tan\theta$, find the value of m_1 such that the block is just about to slide down the plane as pictured.

QP46



You decide to swim across a river. The current in the river has speed V_r left to right as seen from your point of view on the riverbank. The river has a constant width W. You want to arrive on the opposite bank a distance X from the point directly across from your starting point. Note that X may be positive or negative; call X positive if you end up downstream (i.e. to the right) of your starting point, negative if you manage to end up upstream (i.e. to the left).
Let V_s be your swimming speed, measured relative to the water and assume you swim in a straight line. Note: you may use vectors to solve this problem, but it is not necessary.

a) (2 points) The trip takes a time T. How far did you swim, in the frame of reference of the water? Express your answer two ways, once in terms of T and V_s and again in terms of W, X, V_r and T.

b) (1 point) At what angle are you moving, in the frame of reference of the water? A formula for the tangent of this angle will suffice. Define the angle so that zero mean perpendicular to the river banks and pointed to the opposite shore. Let positive angles denote downstream swimming, negative angles upstream swimming.

c) (1 point) What is the minimum swimming speed V_s you must attain to reach the opposite bank at $X = 0$? (i.e. directly opposite your starting point).

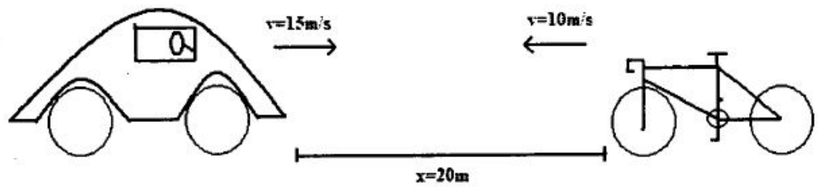
d) (2 points) Using the results of part (a), find an equation, quadratic in T, which relates T, X, W, V_s and V_r. Solve for T.

e) (1 point) Assuming you want to arrive at $X = 0$ and have sufficient speed, how long will the trip take?

f) (3 points) Your aim is to arrive on the opposite bank in the least amount of time. What is this minimum time and what value of X corresponds to it? In the reference frame of the water, what is the angle at which you are swimming?

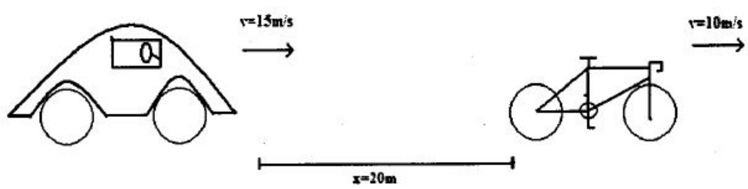
QP47

This problem is intended to help you answer the question of whether it is safer to ride your bike against or with traffic.
Suppose you are riding your bike against the flow of traffic (see below) at a speed of $v_b = 10\,m/s$ and a car comes around a blind curve directly towards you at a speed of $v_c = 15\,m/s$ only $x_0 = 20\,m$ ahead of your current position. Assume both you and the car begin braking instantaneously at this point, and both your bike and the car can each decelerate at a maximum rate of $5\,m/s^2$.



a) (4 points) Will you hit the car and if so what are the speeds of your bike and the car at impact? If you find you will not hit the car, how close did you come to colliding with the car (in meters)?

Now suppose you are riding your bike with the flow of traffic (see below) still at a speed of $v_b = 10\,m/s$. A car comes from behind you around a blind curve at a speed of $v_c = 15\,m/s$ only $x_0 = 20\,m$ from your current position. Since the car is behind you, and you do not see it, you continue riding at $v_b = 10\,m/s$. The car brakes immediately upon seeing you and it can again decelerate at a rate of $5\,m/s^2$.

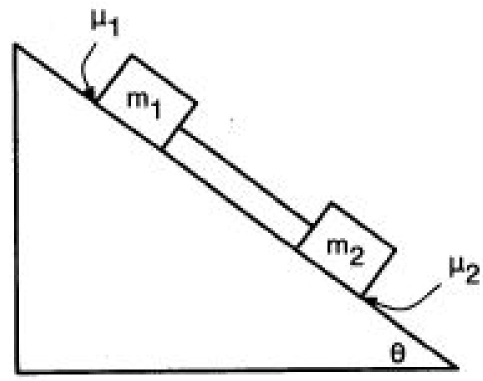


b) (4 points) Will the car hit you in this situation?

c) (2 points) What is the minimum deceleration the car needs to avoid hitting your bike?

QP48

Two masses connected by a string slide down a ramp making an angle θ with the horizontal, as shown in the figure below. The mass m_1 has a coefficient of kinetic friction μ_1 and the mass m_2 has a coefficient of kinetic friction μ_2. Assume the string is massless and remains taut as the masses slide down the incline.

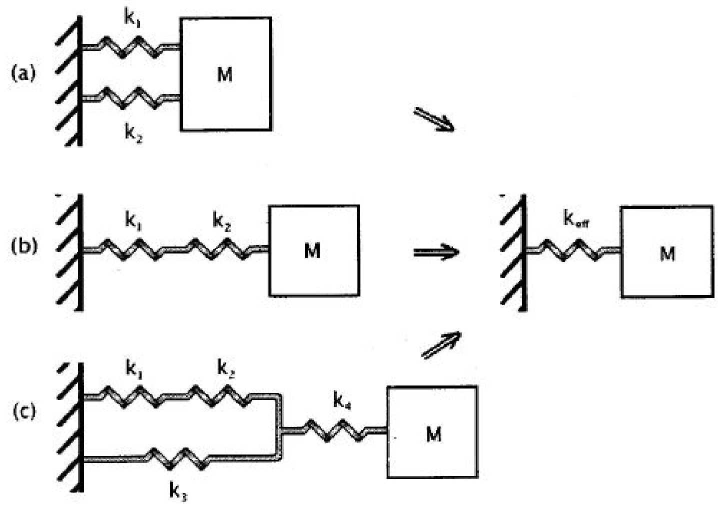


a) (4 points) Draw the free body diagrams for both masses, showing the forces acting on each as they slide down the ramp. Write down the equations of Newton's Second Law for both m_1 and m_2.

b) (4 points) Find the acceleration a of the masses and the tension T of the string. Give your answer in terms of m_1, m_2, μ_1, μ_2, g and θ.

c) (2 points) Find the condition on μ_1 and μ_2 such that the string indeed remains taut as the masses slide down the incline.

QP48



The resting force on a spring is governed by Hooke's Law, $F = -kx$. Springs are linear devices, so any combination of springs can be modelled as a single spring with an effective spring constant $F = -k_{\text{eff}}x$. Refer to the diagram for parts (a), (b) and (c). Your final answer will not depend on M.

a) (2 points) Derive the relation between k_1, k_2 and k_{eff} for springs attached in parallel. You should find $k_{\text{eff}} = k_1 + k_2$

b) (3 points) Derive the relation between k_1, k_2 and k_{eff} for springs attached in series. You should find:

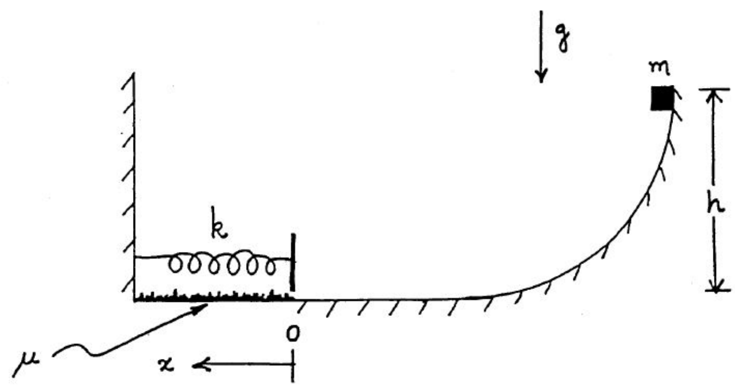
$$\frac{1}{k_{\text{eff}}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Hint: consider the balance of forces at the junction of the two springs.

c) (2 points) Find k_{eff} for the illustrated system. The springs are attached by rigid rods and do not bend.

d) Suppose you use your Caltech degree to get a job at a spring factory. The factory can produce springs of any length; springs of length l have spring constant k. A client orders a spring with constant $k/3$. What must be the length of this spring? If you cut a spring of length l in half, what will the spring constant be?

QP48



A block of mass m starts at rest and slides down a frictionless circular ramp from a height h. At the bottom, it hits a massless spring with spring constant k and in addition begins to experience a frictional force. The coefficient of kinetic friction is given by μ.

a) (1 point) What is the speed of the block at the bottom of the ramp just before it hits the spring?

b) (2 points) Find the total horizontal force on the mass after it hits the spring as a function of the coordinate x given in the diagram.

The mass first comes to rest instantaneously at $x = x_s$. It then rebounds back up the ramp, reaching a maximum height $h' < h$. In the following, express your answer in terms of x_s.

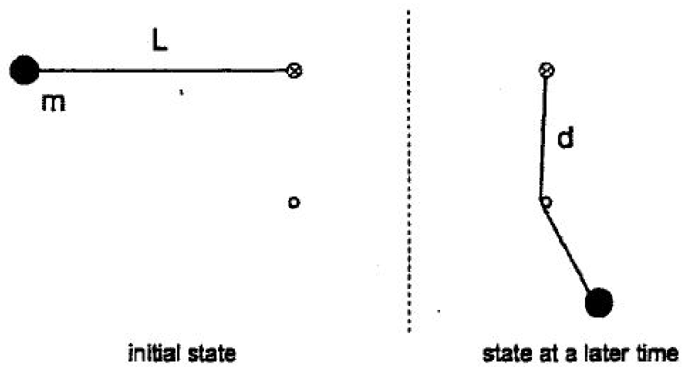
c) (2 points) What is the total work done on the mass by the spring and friction between $x = 0$ and $x = x_s$?

d) (2 points) What is the total energy W_f that has been dissipated by friction when the spring first returns to $x = 0$?

e) (2 points) Find the vertical height h' up the ramp to which the block rebounds. You may express your answer in terms of W_f.

f) (1 point) Find x_s in terms of the quadratic shown above.

QP51



A ball of mass m is attached to a massless string of length L. The ball is released from rest as shown at left, with the acceleration of gravity g pointing down, and travels along a circular arc. As the ball reaches the bottom of the arc, the string starts wrapping around a nail (having a negligible diameter) located a distance d below the center of the arc.

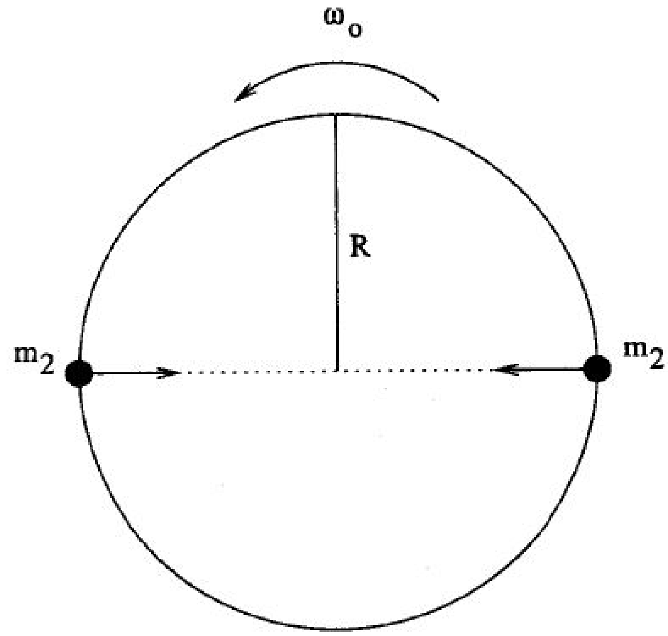
a) (3 points) What is the tension in the string just before it makes contact with the nail?

b) (3 points) What is the tension in the string just after it makes contact with the nail?

c) (4 points) What is the minimum value of d (expressed as a function of L, g and m) for which the ball executes a complete circle around the nail, with the string remains taut?

QP52

Upon a frictionless suspension sits a turntable with radius R and mass m_1 uniformly distributed over its area. The turntable is spinning at an angular speed ω_0 about its center. A man and a woman are standing on the spinning turntable. They are initially located on opposite sides of the turntable at the outer edge, but start walking toward each other at the same rate until the meet at the center. Assume that they are able to walk along radial trajectories with constant speed despite any coriolis and centrifugal effects they might feel. You may approximate the people as objects with the mass m_2 and negligible size as compared to the turntable radius R.



a) (4 points) Calculate the angular speed $\omega(r)$ of the turntable when the man and woman are at a radial distance r from the center. In the case that $m_1 = 2m_2$, what is the final angular speed when the people meet at the center?

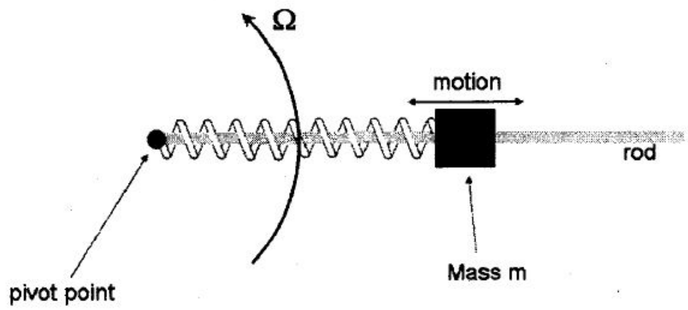
b) (4 points) In the case that $m_1 = 2m_2$, what is the initial rotational kinetic energy? What is the final rotational kinetic energy, when the people meet at the center? Express your answers as a numerical factor multiplying $m_1\omega_0^2R^2$. Is rotational kinetic energy conserved? Explain why or why not in a sentence or two.

c) (2 points) At what distance r from the center is the centrifugal force experienced by the man and woman the largest? What is this distance $r_{\max}$ in the case $m_1 = 2m_2$?

QP53

A long massless rod is attached at one end to a motor which rotates the rod at constant angular speed Ω. A massless spring of force constant k and equilibrium length r_0 is wound around the rod. One end of the spring is attached to the pivot point. At the other end of the spring a mass m is attached. This mass slides along the rod as the spring expands and contracts. There is no friction and no gravity and the motor is powerful enough to keep the angular speed Ω constant. In case you are worrying about the Coriolois force: don't. It is not relevant here.

The natural oscillation frequency of the spring and mass combination would be $\omega_0 = (k/m)^{1/2}$ were it not for the rotation of the rod. Let the radial position of the mass be r, measured from the pivot point.



a) (2 points) Write down Newton's second law for the mass m in the reference frame rotating with the rod. Show that simple harmonic oscillations are still expected, at least for sufficiently small Ω.

b) (1 point) Assuming no oscillations, what is the equilibrium radial position of the mass?

c) (2 points) Find a formula for the natural oscillation frequency ω of the mass/spring system. Show that your formula reduces to the right value when the rod is not rotating (i.e. when $\Omega = 0$).

d) (2 points) You'll see that your formula does something strange when Ω is too large. Describe what happens when this is the case. Are there still nice harmonic oscillations? What is the value of Ω when this strange behavior takes over? If you could not work part (c), try to answer this question based on your physical intuition.

e) (2 points) Sketch the potential energy $U(r)$ of the mass/spring system for Ω less than the critical value and for Ω greater than the critical value.

f) (1 point) Is angular momentum conserved in this problem? Explain your answer.