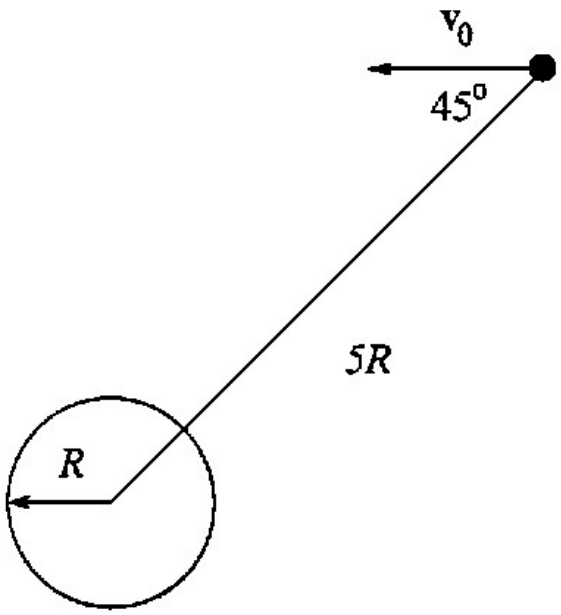
FP4

From a distance of $5R$ from the center of a planet of mass M and radius R, a satellite of mass m is launched with a speed $v_0 = \sqrt{\frac{GM}{5R}}$ at time $t = 0$ in the direction shown in the figure.



a) (2 points) Calculate the energy E of the satellite in terms of G, m, M and R. What type of orbit (i.e. what type of curve) does the satellite follow?

b) (2 points) Calculate the magnitude of the angular momentum in terms of G, m, M, and R.

c) (4 points) Using conservation of energy and angular momentum, calculate the speed of the satellite at its perigee (the point of closest approach) in terms of G, m, M and R. *Hint:* At perigee, the velocity v is perpendicular to the radius vector $\mathbf{r}$.

Suppose that a disturbance at the core of the planet suddenly reduces the planet's mass to $M/2$. The remainder of the planet's mass is vaporized and is quickly ejected to distances well outside the orbit of the satellite. The center of mass of the planet is not accelerated by the disturbance (the vaporized matter is ejected symmetrically) and the satellite is not struck by the flying debris.

d) (3 points) If the disturbance occurs at time t = 0 (the time depicted in the figure), what type of orbit does the satellite follow?