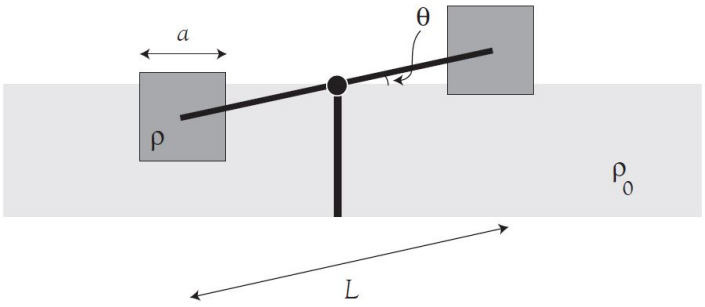
FP17

Two solid cubed of edge length a and uniform density ρ are partly immersed in an ideal fluid of density ρ_0. The cubes are connected at their centers by a rigid, massless rod of length L, with a fixed pivot point that lies on the surface of the liquid, as shown in the figure. The system is built so that the cubes remain upright even as the rod turns on its pivot.



Suppose that the system is tilted until the rod makes an angle θ with the surface of the fluid. Assume that for this value of θ both cubes remain in contact with the fluid.

a) (2 points) Calculate the net force on the rod's pivot. For what value of ρ/ρ_0 is this force zero? Hint: The value is independent of θ.

b) (2 points) Calculate the torque τ about the pivot as a function of θ and the other parameters specified in the problem.

c) (3 points) If the system is tilted by an angle θ_0 and then released, it will oscillate. Assume that $a << L$, so that the cubes may be treated as point masses for the purposes of computing the moment of interia. Find an expression for the angular acceleration $\ddot{\theta}$ of the rod.

d) (3 points) Now assume that θ_0 is very small. Express the period T of the oscillation in terms of the parameters in the problem.