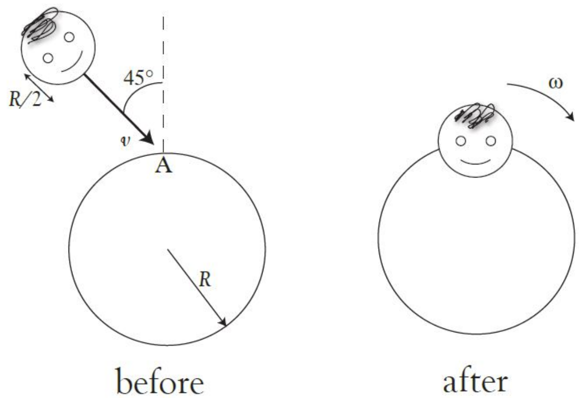
FP16

A playground merry-go-round of uniformly distributed mass M and radius R is initially at rest. A kid with uniformly distributed mass $2M$ (whose shape is remarkably close to that of a disk of radius $R/2$) runs at a constant velocity $\vec{v}$ towards point A on the edge of the merry-go-round. The angle between $\vec{v}$ and the line that runs from A to the center of the merry-go-round is $45°$, as shown in the figure.



At the instant when she jumps aboard, the child grabs hold of the edge so that she is fixed with respect to the merry-go-round, with her center of mass at point A. The merry-go-round then begins turning with angular velocity ω.

a) (2 points) Find the total moment of interia I_{tot} of the system about the axis of the merry-go-round, after the child has jumped on.

b) (3 points) Find the angular velocity ω of the system after the jump. Express your answer in terms of v, M, and R.

c) (2 points) Find the fraction of the initial kinetic energy that is lost in the "collision" between the kid and the merry-go-round.

d) (3 points) After a few revolutions at a constant angular velocity ω, the kid decides to move to the center of the merry-go-round with a constant radial speed $v_r = -dr/dt$. Find the angular acceleration α of the merry-go-round as a function of r. Express your answer in terms of M, R, v_r and the initial angular velocity ω.
Hint: You might want to save yourself some computation by first finding an expression for ω as a function of r, and then using the chain rule $\alpha = \frac{d\omega}{dt} = \frac{d\omega}{dr}\frac{dr}{dt}$.