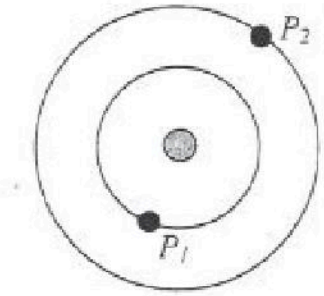
FP12

Two planets each of mass m are in counter-clockwise circular orbits around a star of mass M (see the figure below). The inner planet P_1 is in an orbit of radius R, while the outer planet P_2 is in an orbit of radius $2R$. Express your answer to all parts of this problem in terms of R, M, m and G, Newton's gravitational constant. Assume $m << M$.

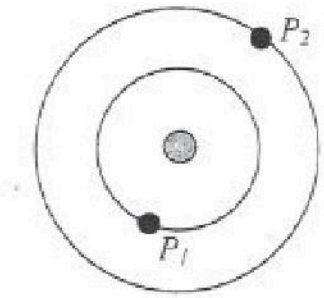


a) (2 points) What is the ratio T_2/T_1 of the orbital period of planet P_2 to that of P_1? What is the value of T_1?

b) (2 points) What is the ratio of E_2/E_1 of the total energies of the orbits? What is the value of E_1?

c) (2 points) What is the ratio L_2/L_1 of the orbital angular momenta of the orbits? What is the value of L_1?

Now suppose that we send a spacecraft of mass m_s from P_1 to P_2 along the elliptical transfer orbit shown below. The orbit has a "periastron" (the distance of closest approach to the star about which the planets are orbiting) of R and an "apastron" (the greatest distance to the star) of $2R$.



d) (4 points) What are the semi-major axis a, the energy E, the angular momentum L and the eccentricity e of the transfer orbit?