DUET FOR THE END OF MATH

MATILDE MARCOLLI

Is the rise of mathematical proficiency in the latest corporate models of AI a harbinger of the end of mathematical research as a profession, as some prominent members of our community have recently argued? In the middle of a deluge of claims of vanquished Famous Conjectures™ and solved Big Problems™, as mathematicians we are increasingly and loudly asking (and being asked) how to justify our existence. Does the rise of AI-generated mathematics imply that humans and machines are engaged in a dramatic duet for the end of time? (Mathematical time at the least, if not the civilizational collapse that some AI developers have repeatedly been warning us about.)

There seem to be several strange assumptions behind this reasoning and these conclusions, which reveal that something must have gone badly wrong, since a very long time, in how our mathematical community perceives the goals and the values in our profession.

Why is the automated solving of some well known open mathematical conjectures sending so many shock waves of panic in our professional community? This points, first and foremost, to the disproportionate role that the idea of Famous Conjectures™ and Big Problems™ has come to play in the field. When and how did we start viewing mathematics as a zero-sum game, with some predetermined scarce trophies out there that one alone will conquer while all the others will fail? Who convinced *you* of that? When? And why did you let it happen?

The process of mathematical creativity is incredibly varied, rich, and complex, and the solving of certain specific narrowly defined outstanding problems is but one particular aspect of it. It is, however, an aspect that has been endowed with a disproportionately large attribution of value. Career advancement, prizes, professional status, and ultimately power are tied up primarily to conquering one of a narrow set of goals that someone else (those who have priorly acquired that kind of power status in the field) have singled out as valuable. These goals are achievable, generally, through a technical hyperspecialization within a very clearly defined subdiscipline (and therefore a very narrow subcommunity) of mathematics. Since these became the primary and unquestionable goals, we reoriented the whole functioning of our community, training of students and postdocs, hierarchies of journal publishing, control of reward attribution, around this particular system of values. We are getting to the point, to paraphrase a famous aphorism, where it is becoming easier to envision the end of mathematics than the end mathematicians' obsession with Famous Conjectures™.

And yet, if we take a look at another field of science that is very close in methods and in spirit to mathematics, namely theoretical physics, we begin to see how it is actually possible for things to look very different. In theoretical physics, for obvious reasons, what someone else was not able to do one or two centuries ago has absolutely no relevance on what drives the development of the field today. This is of course because our understanding of the physical universe has so dramatically changed with respect to a century ago (in 1926 it was not even known yet that neutrons existed) to make such comparisons irrelevant. Mathematics by contrast has a long persistence, so that it is indeed possible to have open problems that

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trail along with the community for centuries without losing relevance. The question remains though, on why they should catalyze so much value attribution. So let's look a little more at how our next of kin in theoretical physics view the question of value attribution. Since I have been most of my life in between these two communities, I have been observing since long time these profound sociological differences. In theoretical physics I see a system of value attribution that focuses very strongly on a balance between the originality of ideas and their degree of connectedness, meaning that an idea that is highly novel and that at the same time connects to many genuinely different questions and problems that many other people are thinking about typically receives very high value attribution. This points in a completely different direction from the value attribution that we have been forcing upon the mathematical community, because it privileges broadness over hyperspecialization, it privileges abundance-driven collaboration over scarcity-driven competitiveness, and it regards technical muscle-flexing as secondary to ideation, so that the perspective of automating the first is not perceived as threatening the latter.

Most importantly, these different systems of value are not an intrinsic reality, ordained by the gods or by the natural order of the universe, but are fairly arbitrary choices that are made by our human communities, and that can and should be questioned and not given for granted. It is an act of propaganda, fueled like all such acts by an entrenched system of power, that maintains the view of goals and values as immutable and unquestionable. We have often heard mathematicians speak of some kind of "mathematical Platonism", where mathematical concepts and their properties have ontological existence in some unspecified "world of ideas" and are simply being "discovered", not invented, by the human mind. Depending on our personal background, we may have different reactions to such claims, from just viewing them as an exceedingly metaphysical rephrasing of the undeniable "unreasonable effectiveness" of mathematics in the sciences, to regarding them as a benign delusion, to be viewed with a sympathetic eye if not enthusiastically embraced. This is not so harmless and inconsequential, though, and rather than a mere delusion, it is akin to a subtle propaganda instrument. Because if mathematics is indeed predetermined and enshrined in stone in some olympian other-world, determined by more-than-human forces, and all we can do about it is uncover it, then it really is a zero-sum game of competing armed conquerors over a virgin but immutable territory. Ironically, the ultimate conqueror of this other-than-human landscape may turn out to be itself other-than-human, hence the widespread panic about artificial minds out-counter-exampling human mathematicians in a conquering spree worthy of Napoleon, Alexander the Great, or Genghis Khan, laying our precious beloved Famous Conjectures™ in ruins.

Overtime a small set of problems with a varyingly long history came to dominate the mathematical imagination. Some of them, like Fermat's last theorem, spanned the course of centuries until a final bout of virtuoso arithmetic geometry brought about its triumphant resolution. Others, like the Riemann hypothesis, still haunt the mathematical night, amidst hopeful claims and long programs. In addition to these widely known cases there is a fair number of other conjectures which you are likely to consider crucially important problems if you happen to belong to one specific narrow subfield of math, and you have likely never heard of if you don't. What is it about all of these famous and less famous, invariably hard, Big Problems™? There is an undeniable sense in which these questions are interesting. Typically they are because, in the process of thinking about a particularly hard question, a lot of new mathematics gets developed that advances the field as a whole. The Riemann hypothesis is

currently unsolved (at the time of this writing), but many people, in the process of thinking about the problem, have developed a very significant amount of very interesting mathematics. That is certainly a valuable goal. But is the Famous Conjecture™, invested with its aura of sacred object, actually needed for that goal? The idea of kettling mathematical research into narrow streets overseen by the dominant presence of Big Problems™ gained prominence with programmatic efforts like the Hilbert problem-list and its more recent millennial revival. While some of the problems in Hilbert's list were broad in scope, the understanding of what constitute a Big Problem™ is increasingly reflecting a competition system of pain and rewards that is simply a system of power, and that can easily become detrimental to our creativity and to the inner life of the mind.

Now we are all faced with a new reality in which sudden unpredictable burps of artificial intelligence, often with unauthorized access to ongoing unpublished work of human mathematicians gobbled into their garagantuan training sets, can liquidate our centenarian Big Problems™ in a moment's time. One may be tempted to say, to quote another famous aphorism, that these AI-solutions "fill a much needed gap" since probably quite a bit of very interesting mathematics that would have been developed while thinking about these questions will now no longer be pursued once it becomes detached from its value attributor. Is having sizable parts of the development of mathematics entangled with a small number of very specific questions really helpful to the field? Doing mathematics consists of many different activities and the creativity is everywhere along that process, but we choose to reward it in a very uneven way. It is our right as a community to decide what we want to value, why do we exercise this right so narrowly? What if, instead of attributing importance to a mathematical result solely on the basis of whether it cracks a question on which N people have previously failed for M years, with N and M sufficiently large, we would attribute value more like theoretical physics does, on the basis of originality of ideas, collaboration, and broad connectedness between different areas of mathematics. It is possible to envision a genuine plurality of values that much more closely represents the richness of mathematical thinking. Why do you care about what someone did *not* prove a century ago? Why is that the only source of value? Why do we grow into the world of professional mathematicians, and in turn educate the next generation to prioritize working on what *others* may attribute value to, rather than listening to our own inner voice. How do we become enablers of a vision of mathematics as a conquering battle field rather than as a shared voyage of self discovery?

What is for us humans the act of doing mathematics? I'd say mathematics is two very important things at once: a language to make sense of reality and a form of self-expression, much like the arts. When seen from this perspective, the existence of artificial intelligences that can generate solutions to mathematical problems becomes essentially irrelevant, and is just left to individuals and their ethical considerations whether to regard AI as a useful scientific tool, an opaque proprietary instrument of corporate greed and oppression, a new possibility for increased accessibility, an uncontrollable danger, or all of the above at once. If you are a poet, the fact that a large language model can compose a dochmiac metered poem in Ancient Greek upon demand (I checked) is entirely irrelevant, because that would simply not be *your* poem. So don't ask yourself which are the Big Problems™ and Famous Conjectures™ you should be working on and what is your reason of existence now that an artificial intelligence can get them before you've even finished your morning coffee. Ask yourself rather what is the mathematics that lives only inside you and nowhere else. The one that makes you dream, like the music that no-one has ever heard, that wakes you up

in the middle of the night, playing itself inside your head, and demanding to be written. Your mathematics, that is like the poem that composes itself in the mind and explodes onto the page because your life demands it, the gesture on canvas that crosses the boundaries of thought and action. Mathematics, like all of these other activities, the music, the poetry, the painting, is a form of self-expression, that we all collectively share because we are human, like we share our appreciation and performance of the arts and our profound need for them.

Current forms of artificial intelligence have become very proficient at simulating human products, including the arts and including mathematics, because they have been fed every single product of human civilization that could be shoveled into their training data. What comes out of it may be good or evil, useful or not, dangerous or tame, but there is one thing it will not be: the thing that lives right now inside your mind and that is asking to be expressed, the thing that only you can give a voice to.

You don't normally worry that the existence of an AI capable of writing acceptable texts in human languages will imply that every single novel in the world will end up having already been written. You do (I hope) recognize the absurdity of such a thought. And yet large numbers of mathematicians now worry that every theorem will have been proven by AI, leaving the mathematical profession in tatters. This narrowing of the horizon that we have all implicitly accepted is astonishing! Yes, if all we see in mathematics is a few shiny trophies consisting of Famous Conjectures™ (old and well seasoned) sanctioned by a small number of established individuals (old and well seasoned too, if possible) then we may surely risk running out of all such rewards, through the intervention of new technologies. Well, the sooner the better! Good riddance! Because with that way of envisioning mathematics being shattered before our eyes, maybe we can finally start to have a conversation that the mathematical community should have had a long time ago. We can and should come to accept the idea that the work of a mathematician can be more like that of an orchestra conductor or of a music composer and less like the virtuoso violinist, single instrument performer, whose enormous skills and creativity spin to interpretive perfection music composed by someone else. The skills of the orchestra conductor are different from those of the solo instrumentalist, with a less hyperspecialized technique and a broader sense of the orchestra as a whole, of how sound circulates and different instruments complement each other. The world of music knows how to distribute value more evenly to such different skills. It is not difficult to imagine a path of mathematical education and professional performance that is more like the conductor and the composer, and less like the virtuoso single instrument performer, or at least that can acknowledge that such different roles exist. If we insist on focusing our entire profession solely on the tackling of conjectures as the unique goal and reward, then we may be out of business quite fast, not because of the end of mathematics but because of the end of our imagination.

Mathematics Department, Caltech, Pasadena, CA 91125, USA
Email address: matilde@caltech.edu