You Spin Me Round (Like a Record)

Upon a frictionless suspension sits a turntable with radius $R$ and mass $m_1$ uniformly distributed over its area. The turntable is spinning at an angular speed $\omega_0$ about its center. A man and a woman are standing on the spinning turntable. They are initially located on opposite sides of the turntable at the outer edge, but start walking toward each other at the same rate until they meet at the center. Assume that they are able to walk along radial trajectories with constant a speed despite any coriolis and centrifugal effects they might feel. You may approximate the people as objects with the mass $m_2$ and negligible size as compared to the turntable radius $R$.

(4 points) (a) Calculate the angular speed $\omega(r)$ of the turntable when the man and woman are at a radial distance $r$ from the center. In the case that $m_1 = 2m_2$, what is the final angular speed when the people meet at the center?

(4 point) (b) In the case that $m_1 = 2m_2$, what is the initial rotational kinetic energy? What is the final rotational kinetic energy, when the people meet at the center? Express your answers as a numerical factor multiplying $m_1 \omega_0^2 R^2$. Is rotational kinetic energy conserved? Explain why or why not in a sentence or two.

(2 point) (c) At what distance $r$ from the center is the centrifugal force experienced by the man and woman the largest? What is this distance $r_{\text{max}}$ in the case $m_1 = 2m_2$?