System of Springs

The restoring force on a spring is governed by Hooke's Law, $F = -kx$. Springs are linear devices, so any combination of springs can be modelled as a single spring with an effective spring constant $F = -k_{\text{eff}}x$. Refer to the diagram for parts (a), (b), and (c). Your final answers will not depend on $M$.

(a) (2 points) Derive the relation between $k_1$, $k_2$, and $k_{\text{eff}}$ for springs attached in parallel. You should find:

$$k_{\text{eff}} = k_1 + k_2$$  \hfill (1)
(b) (3 points) Derive the relation between $k_1$, $k_2$, and $k_{\text{eff}}$ for springs attached in series. You should find:

$$\frac{1}{k_{\text{eff}}} = \frac{1}{k_1} + \frac{1}{k_2} \tag{2}$$

Hint: Consider the balance of forces at the junction of the two springs.

(c) (2 points) Find $k_{\text{eff}}$ for the illustrated system. The springs are attached by rigid rods and do not bend.

(d) (3 points) Suppose you use your Caltech degree to get a job at a spring factory. The factory can produce springs of any length; springs of length $l$ have spring constant $k$. A client orders a spring with constant $k/3$. What must be the length of this spring? If you cut a spring of length $l$ in half, what will the spring constant be?